Some large time behaviors of surface water waves

Sijue Wu
University of Michigan, Ann Arbor
sijue@umich.edu

The mathematical problem of n-dimensional water wave concerns the motion of the interface separating an inviscid, incompressible, irrotational fluid, under the influence of gravity, from a region of zero density (i.e. air) in n-dimensional space. It is assumed that the fluid region is below the air region. Assume that the density of the fluid is 1, the gravitational field is $-\mathbf{k}$, where $\mathbf{k}$ is the unit vector pointing in the upward vertical direction, and at time $t \geq 0$, the free interface is $\Sigma(t)$, and the fluid occupies region $\Omega(t)$. When surface tension is zero, the motion of the fluid is described by

$$\begin{cases} \mathbf{v}_t + \mathbf{v} \cdot \nabla \mathbf{v} = -\mathbf{k} - \nabla P & \text{on } \Omega(t),\ t \geq 0, \\ \operatorname{div} \mathbf{v} = 0, \qquad \operatorname{curl} \mathbf{v} = 0, & \text{on } \Omega(t),\ t \geq 0, \\ P = 0, & \text{on } \Sigma(t) \\ (1, \mathbf{v}) \text{ is tangent to the free surface } (t, \Sigma(t)), \end{cases} \tag{1}$$

where $\mathbf{v}$ is the fluid velocity, P is the fluid pressure.

In this talk, we will survey results and ideas concerning the local and global wellposedness of the Cauchy problem of equation (1), and present some recent work concerning singularities of the solutions.