Efficient numerical methods for quantifying uncertainty in solutions of systems of conservation laws

Siddhartha Mishra
Center of Mathematics for Applications, University of Oslo, Norway and Seminar for Applied Mathematics, ETH Zürich, Switzerland.
smishra@sam.math.ethz.ch

Inputs to systems of conservation laws such as initial data, boundary conditions, source terms, flux and diffusion coefficients are characterized by uncertainty, due to measurement errors. This input uncertainty results in uncertainty in the solutions of the underlying systems. We model input uncertainty as well as the resulting solutions by random fields. The well-posedness theory of random entropy solutions for scalar conservation laws with random initial data, sources and fluxes is presented and possible extensions to systems indicated. The focus of the lecture will be on reviewing state of the art numerical methods for quantifying uncertainty in random conservation laws. We will consider statistical sampling methods such as the Monte Carlo methods and the recently developed Multi-level Monte Carlo (MLMC) methods, present the underlying convergence and computational complexity theories and describe various numerical experiments that show the robustness and efficiency of these methods. In particular, Euler and MHD equations with random initial data, shallow water equations with random bottom topography, Euler equations with uncertain equations of state and two-phase flow equations with random relative permeabilities will be presented. The experiments will demonstrate that MLMC methods are totally non-intrusive, can handle large number of sources of uncertainty and scale to a very large number of processors in a parallel computing architecture. Some open issues regarding statistical sampling methods will be discussed and these methods will be compared with a novel deterministic class of numerical methods, the so called stochastic finite volume (SFV) methods. The convergence theory of SFV methods will be indicated and possible advantages of these methods, on problems with small number of sources of uncertainty, will be highlighted.

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