Optimal placement of sensors, actuators and dampers for waves

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In this lecture we address the problem of the optimal placement of sensors, actuators and dampers for wave equations. We first discuss the dissipative wave equation where, due to the non-selfadjoint nature of the generator of the dynamics, characterizing the decay rate of solutions as time tends to infinity needs to take into account both spectral properties and the propagation of bicharacteristic rays. We present the state of the art in what concerns the optimal placement of dampers. We then turn our attention to the conservative wave equation and the optimal placement of sensors and actuators, both fundamental problems from a control theoretical point of view, with many potential applications. Using Fourier series representations the problem can be recast as an optimal design one involving all the spectrum of the laplacian. We develop a complete theory allowing to distinguish, depending on the complexity of the data to be observed/controlled, cases in which the solution is a classical set constituted by a finite number of subdomains, from others in which the optimal set is of Cantor type or those when relaxation occurs. These results will be illustrated by numerical simulations. Most of the work presented in this lecture is part of ongoing research in collaboration with Y. Privat (ENS Cachan, Antenne de Bretagne, France)and E. Trélat (Université Pierre et Marie Curie (Paris 6), Laboratoire Jacques-Louis Lions, Paris, France).

References

[1] Y. Privat, E. Trélat and E. Zuazua, Optimal observation of the one-dimensional wave equation, preprint (2012).

[2] Y. Privat, E. Trélat and E. Zuazua, Optimal observation of the one-dimensional wave equation, preprint (2012).