Lipschitz stability for the Hunter-Saxton and Camassa-Holm equation

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In this talk, we will present the construction of a metric for two related nonlinear partial differential equations, the Hunter-Saxton equation and the Camassa-Holm equation. This metric makes the semigroup of solutions Lipschitz continuous. The solutions typically break down in finite time. After breakdown, they are no longer unique and can be prolongated in several consistent ways. By using a change of variable - from Eulerian to Lagrangian coordinates - and introducing an extra energy density variable, we obtain an equivalent system which is well-posed as a system of ordinary differential equations in a Banach space. Going back to the original Eulerian variable, we can construct a semigroup of conservative solutions (solutions which for almost every time conserve the total energy). However, we observe that this semigroup of solutions is not stable with respect to any standard norm. To construct a metric which yields stability, we transport the topology of the equivalent system in Lagrangian variables to the Eulerian setting. The difficulty is that the mapping between Lagrangian and Eulerian variables is not a bijection. However, by precisely identifying the discripency between the two sets as the action of a group (group of diffeomorphism or relabelling group), we can eliminate the redundancy which is introduced by the Lagrangian variables.

References

[1] Bressan, Alberto and Holden, Helge and Raynaud, Xavier, Lipschitz metric for the Hunter-Saxton equation *J. Math. Pures Appl.*, **1** (2010).

[2] Grunert Katrin and Holden, Helge and Raynaud, Xavier, Lipschitz metric for the periodic Camassa-Holm equation, *J. Differential Equations*, **250** (2011).

[3] Grunert, Katrin and Holden, Helge and Raynaud, Xavier , Lipschitz metric for the Camassa–Holm equation on the line *arXiv-1010.0561v1* (2010).

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