On the finite-time splash and splat singularities for the 3-D free-surface Euler equations

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We prove that the 3-D free-surface incompressible Euler equations with regular initial geometries and velocity fields have solutions which can form a finite-time "splash" (or "splat") singularity first introduced in [1], wherein the evolving 2-D hypersurface, the moving boundary of the fluid domain, self-intersects at a point (or on surface). Such singularities can occur when the crest of a breaking wave falls unto its trough, or in the study of drop impact upon liquid surfaces. Our approach is founded upon the Lagrangian description of the free-boundary problem that we used in [2], combined with a novel approximation scheme of a finite collection of local coordinate charts; as such we are able to analyze a rather general set of geometries for the evolving 2-D free-surface of the fluid. We do not assume the fluid is irrotational, and as such, our method can be used for a number of other fluid interface problems, including compressible flows, plasmas, as well as the inclusion of surface tension effects.

References

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