Singular behavior of a rarefied gas on a planar boundary

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We will discuss some singularities in a rarefied gas that should be observed on a planar boundary.

It has already been shown in 1960s and 1970s by Sone [1] and Sone & Onishi [2] that the slope of a macroscopic quantity diverges logarithmically in a rarefied gas on a planar boundary, by using the Bhatnagar-Gross-Krook-Welander (BGK or BKW) model of the Boltzmann equation. Recently, Lilley & Sader [3] have also numerically pointed out the slope divergence of macroscopic quantities on the boundary for the Boltzmann equation. However, the divergence rate is not clear in their discussion. With I-Kun Chen and Tai-Ping Liu [4], we have recently proved for a highly rarefied gas on the basis of the Boltzmann equation that the slope of flow velocity diverges logarithmically in the thermal transpiration between two parallel plates. In this talk, we will show on the basis of the Boltzmann equation that, irrespective of the Knudsen number,

(i) the slope of a macroscopic quantity diverges logarithmically on a planar boundary;

(ii) the logarithmic behavior of (i) induces a microscopic divergence on the boundary, namely the derivative of velocity distribution function with respect to the normal component of the molecular velocity diverges logarithmically for the molecular velocities parallel to the boundary;

(iii) the singularity (i) is related quantitatively to the discontinuity of the velocity distribution function on the boundary.

References

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