Two-Phase Flow in Porous Media: Shock Waves and Stability

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In this talk, I discuss a variety of contexts in which undercompressive shock waves have been discovered recently. The main focus will be on models of two-phase flow in porous media. Plane waves are modeled by the one-dimensional Buckley-Leverett equation, a *scalar* conservation law. The Gray-Hassanizadeh model for rate-dependent capillary pressure adds dissipation and a BBM-type dispersion, giving rise to undercompressive waves. Two-phase flow in porous media is notoriously subject to fingering instabilities, related to the classic Saffman-Taylor instability. However, a two dimensional linear stability analysis of sharp planar interfaces reveals a criterion predicting that weak Lax shocks may be stable or unstable to long-wave two-dimensional perturbations. This surprising result depends on the hyperbolic-elliptic nature of the *system* of linearized equations. Numerical simulations of the full nonlinear system of equations, including dissipation and dispersion, verify the stability predictions at the hyperbolic level.

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