Viscek flocking dynamics and phase transition

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Consider the following nonlinear Fokker-Planck equation describing self-propelled dynamics for some large biological system such as flocking of birds and schooling of fishes,

$$f_t + \omega \cdot \nabla_x f = d\Delta_\omega f + \nabla_\omega \cdot (f \nabla_\omega \phi)$$

where $f(x, \omega, t)$ is a distribution function on $\mathbb{R}^n \times \mathbb{S}_{n-1}$. The self-propelled speeds for all biological agents are assumed to be uniform and take value one for simplest. ϕ is the interaction potential describing Viscek self-alignment of the orientation towards its local averaged orientation [2, 5]

$$\phi(x, \omega) = \nu(\rho) \int_{\mathbb{R}^n \times \mathbb{S}_{n-1}} \psi(|x - x'|)\omega \cdot \omega' f(x', \omega', t) dx' d\omega'$$

Δ_ω, $\nabla_\omega \cdot$, ∇_ω are the laplican, divergence and gradient operator on $\mathbb{S}_{n-1}$. $\rho = \int f d\omega$ is density.

The flocking behavior in the above Fokker-Planck equation appears in a similar form as orientational phase transitions as in liquid crystal and ferromagnetism at a critical norse level d_c or equivalently at a critical mass density ρ_c. Below this value, the only equilibrium distribution is isotropic for orientations and is stable. Any initial distribution relaxes exponentially fast to this isotropic equilibrium state. By contrast, when the density is above the threshold, a second class of anisotropic equilibria formed by Von-Mises-Fischer distributions of arbitrary orientation appears. The isotropic equilibria become unstable and any initial distribution relaxes towards one of these anisotropic states with exponential speed of convergence.

In this talk, I will present a joint work with Amic Frouvelle [4] on rigorous analysis of the phase transition for the the spatial homogeneous dynamics and a joint work with Pierre Degond and Amic Frouvelle [1] on asymptotic analysis for the spatial inhomogeneous case on the hydrodynamics limit for the anisotropic region and nonlinear diffusion approximation in the isotropic region.

References

[1] P. Degond, A. Frouvelle and J.-G. Liu, Macroscopic limits and phase transition in a system of self-propelled particles, submitted.

[2] P. Degond and S. Motsch, Continuum limit of self-driven particles with orientation interaction, *Mathematical Models and Methods in Applied Sciences*, **18** (2008) pp. 1193–1215.

[3] P. Degond and J.-G. Liu, Hydrodynamics of self-alignment interactions with precession and derivation of the Landau-Lifschitz-Gilbert equation *Math. Models Methods Appl. Sci.*, **22** (2012), pp. 1114001-18

[4] A. Frouvelle and J.-G. Liu, Dynamics in a kinetic model of oriented particles with phase transition *SIAM J. Math Anal*, **44** (2012), pp. 791–826.

[5] T. Vicsek, A. Czirk, E. Ben-Jacob, I. Cohen, and O. Shochet, Novel type of phase transition in a system of self-driven particles, *Phys. Rev. Lett.* **75**, 1226–1229 (1995).

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