Entropy viscosity for hyperbolic systems and questions regarding parabolic regularization

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A numerical method for approximating nonlinear conservation laws is described [1]. The technique consists of augmenting the numerical discretization at hand with a viscous regularization where the nonlinear viscosity is based on the local size of a discrete entropy production. This method is simple to program and does not use any flux or slope limiters. The method can reasonably be justified for scalar conservation equations. Stability results are established for scalar conservation equations using some explicit Runge Kutta techniques.

The implementation is not so clear when dealing with systems, since the question of parabolic regularization is mainly open in this case. The particular question of the Euler system and the Navier-Stokes regularization is addressed. A nonstandard (non-diagonal) regularization is proposed that, in addition to stabilizing the velocity, acts on the density and the internal energy and is such that the entropy sets $\{s(e, \rho) \geq r\}$ are positively invariant.

The technique is illustrated on various benchmark problems using continuous finite elements, discontinuous finite elements, spectral elements, and Fourier series.

References

[1] Jean-Luc Guermond and Richard Pasquetti and Bojan Popov, Entropy viscosity method for nonlinear conservation laws, *Journal of Computational Physics*, **230** (2011), pp. 4248–4267

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