Large time step and asymptotic preserving numerical schemes for the gas dynamics equations with source terms

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We are interested in the simulation of subsonic compressible flows in a specific regime where the (main) driving phenomena are stiff source terms and material transport. More precisely, we consider the system of gas dynamics with external body forces and friction

\[
\begin{align*}
\partial_t \rho + \partial_x (\rho u) &= 0, \\
\partial_t (\rho u) + \partial_x (\rho u^2 + p) &= \rho (g - \alpha u), \\
\partial_t (\rho E) + \partial_x ((\rho E + p)u) &= \rho u (g - \alpha u),
\end{align*}
\]

where \(\rho\), \(u\) and \(E\) denote the density, the velocity and the total energy of the fluid, \(g\) the gravitational acceleration and \(\alpha\) the friction parameter. The pressure law \(p = p(\rho, e)\) is assumed to be a given function of the density \(\rho\) and the internal energy \(e\) defined by \(e = E - \frac{u^2}{2}\). Such flow configuration may be encountered in several industrial processes like the flows involved within the core of a nuclear power plant. We propose a large time step and asymptotic preserving scheme for the gas dynamics equations with external forces and friction terms.

By asymptotic preserving, we mean that the numerical scheme is able to reproduce at the discrete level the parabolic-type asymptotic behaviour satisfied by the continuous equations [2]. Indeed, when one considers the asymptotic regime obtained for both long time and large friction coefficients, the solution of the system is formally expected to behave like the solution of a typical parabolic system

\[
\begin{align*}
\partial_t \rho + \partial_x (\rho u^1) &= 0, \\
\partial_x p &= \rho (g - \alpha u^1), \\
\partial_t (\rho e) + \partial_x ((\rho e + p)u^1) &= \rho u^1 (g - \alpha u^1),
\end{align*}
\]

where \(u = \epsilon u^1 + O(\epsilon^2)\). We aim at deriving a scheme that preserves this property for the discrete approximate of the solution.

By large time-step, we mean that the scheme is stable under a CFL stability condition driven by the (slow) material waves, and not by the (fast) acoustic waves as it is customary in Godunov-type schemes. We propose a mixed implicit-explicit strategy: the terms responsible for the acoustic waves receive a time implicit treatment while the ones responsible for the transport waves are treated by an explicit update. This task is achieved by means of a Lagrange-Projection algorithm as in [3]. An approximation based on a relaxation strategy [4,1,5] provides a simple mean to circumvent the nonlinearities involved with the equation of state of the fluid.

Numerical evidences are proposed and show a gain of several orders of magnitude in both accuracy and efficiency.
References


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