We present two results on the regularity of viscosity solutions of Hamilton-Jacobi equations described in [1], [2].

When the Hamiltonian is strictly convex, viscosity solutions of Hamilton-Jacobi equations are semiconcave, hence their gradient is BV. It is therefore of interest to see when it is SBV. The first result in this direction was obtained by Cannarsa, Mennucci and Sinestrari in [3] but requires a very regular initial datum for the Hamilton-Jacobi equation. First we prove the SBV regularity of the gradient of a viscosity solution of the Hamilton-Jacobi equation
\[ \partial_t u + H(t, x, D_x u) = 0 \quad \text{in a open } \Omega \subset [0,T] \times \mathbb{R}^n, \]
under the hypothesis of uniform convexity of the Hamiltonian $H$ in the last variable. Thus the SBV regularity holds even in the case of a bounded Lipschitz initial datum. Secondly we remove the uniform convexity hypothesis on the Hamiltonian, considering a viscosity solution $u$ of the Hamilton-Jacobi equation
\[ \partial_t u + H(D_x u) = 0 \quad \text{in } \Omega \subset [0,T] \times \mathbb{R}^n, \]
where $\Omega$ is open and $H$ is smooth and convex. In this case the viscosity solution is only locally Lipschitz. However when the vector field $d(t, x) := H_p(D_x u(t, x))$, here $H_p$ is the gradient of $H$, is BV for all $t$ in $[0,T]$ and suitable hypotheses on the Lagrangian $L$ hold, the divergence of $d(t, \cdot)$ can have Cantor part only for a countable number of $t$'s in $[0,T]$.

These results extend a result of Bianchini, De Lellis and Robyr in [4] for a uniformly convex Hamiltonian which depends only on the spatial gradient of the solution.

References


Joint work with: Stefano Bianchini (SISSA, Trieste, Italy)