A dispersive property of the Euler-Korteweg model

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The Euler-Korteweg system consists in a quasi-linear dispersive perturbation of
the Euler equations by the so-called Korteweg tensor which is intended to take
into account capillary effects. The system reads

\[
\begin{aligned}
\partial_t \rho + \text{div}(\rho u) &= 0, \\
\partial_t u + (u \cdot \nabla)u + \nabla g_0(\rho) &= \nabla \left( K(\rho) \Delta \rho + \frac{1}{2} K'(\rho)|\nabla \rho|^2 \right),
\end{aligned}
\quad (EK)
\]

The Cauchy problem has been studied in any dimension \( d \geq 1 \) by Benzoni-
Danchin-Descombes [1], who obtained local well-posedness results when the
velocity is in \( H^s(\mathbb{R}^d) \) for \( s > d/2 + 1 \). They noticed that one may expect to find
some smoothing effect due to the dispersion (more precisely the local gain of 1/2
derivative). Our aim here is to give such results in any dimension under their
local existence assumptions. Though dispersive smoothing is well known for the
-possibly quasi-linear- Schrödinger [3] or Korteweg de Vries equation [4], \((EK)\)
exhibits several singular features. Besides technical difficulties arising from its
quasi-linear nature, special attention is devoted to two points

- Not all Cauchy data produce a solution satisfying dispersive estimates.
  Namely we will describe why the irrotionality of \( u(t=0) \) is essential,

- The system admits traveling waves solutions [2] whose profile do not satisfy
  the decay assumptions usually required in dispersive smoothing results.
  We give a sufficient condition ensuring that smoothing occurs for solutions
  of \((EK)\) linearized near a traveling profile.

The main technics involved are the construction of a symbol in Doi’s spirit
[5] that leads formally to dispersive estimates, and the use of para-differential
calculus to tackle the non-linearities.

References

for the Euler-Korteweg model in several space dimensions, Indiana Univ.

[2] Benzoni-Gavage, Sylvie and Danchin, Raphaël and Descombes, Stéphane
pp. 103-127
