A model for shock wave chaos

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We propose the following simple model equation that describes chaotic shock waves:

\[ u_t + \frac{1}{2} \left( u^2 - uu_s \right)_x = f(x, u_s). \]

It is given on the half-line \( x < 0 \) and the shock is located at \( x = 0 \) for any \( t \geq 0 \). Here \( u_s(t) \) is the shock state and \( f \) is a given source term [1]. The equation is a modification of the Burgers equation that includes non-locality via the presence of the shock-state value of the solution in the equation itself. The model predicts steady-state solutions, their instability through a Hopf bifurcation, and a sequence of period-doubling bifurcations leading to chaos. This dynamics is similar to that observed in the one-dimensional reactive Euler equations that describe detonations. We present nonlinear numerical simulations as well as a complete linear stability theory for the equation.

References


Joint work with: Luiz Faria (KAUST, Thuwal, Saudi Arabia), Rodolfo R. Rosales (MIT, Cambridge, MA)