Existence and stability of relativistic plasma-vacuum interfaces

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Relativistic magnetohydrodynamics (RMHD) in the form of conservation laws (special relativity):

$$\partial_t(\rho\Gamma) + \mathrm{div}\,(\rho u) = 0,$$
$$\partial_t\left(\rho h\Gamma u + |H|^2 v - (v,H)H\right) + \mathrm{div}\left((\rho h + B^2)u\otimes u - b\otimes b\right) + \nabla q = 0,$$
$$\partial_t(\rho h\Gamma^2 + |H|^2 - q) + \mathrm{div}\left(\rho h\Gamma u + |H|^2 v - (v,H)H\right) = 0,$$
$$\partial_t H - \nabla\times(v{\times}H) = 0,$$

where v and H are the 3-vectors od velocity and magnetic field, $u = \Gamma v$, $\Gamma = (1-|v|^2)^{-1/2}$, ρ is the proper rest-mass density, $h = 1 + e + (p/\rho)$, p the pressure, $e = e(\rho, S)$ the specific internal energy, S the entropy, $q = p + \frac{1}{2}B^2$ the total pressure, $b = H/\Gamma + (u,H)v$, $B^2 = |H|^2/\Gamma^2 + (v,H)^2 = |H|^2 - |E|^2$, with $E = -v\times H$.

Divergence constraint

$$\mathrm{div}\,H = 0$$

on the initial data $U_0(x)$ for $U = (p,u,H,S)$.

Maxwell equations in vacuum:

$$\partial_t \mathcal{H} + \nabla \times E = 0, \qquad \partial_t E - \nabla \times \mathcal{H} = 0,$$

where

$\mathcal{H}$ and E are the vacuum magnetic and electric fields respectively, and the equations

$$\operatorname{div} \mathcal{H} = 0, \qquad \operatorname{div} E = 0$$

are the divergence constraints on the initial data $V_0(x)$ for $V = (\mathcal{H}, E)$.

Problem statement

The FBP on the motion of a relativistic plasma body in vacuum can be used for modeling the motion of a (neutron) star.

Locally the interface $\Sigma(t)$ has the form of a graph and for technical simplicity we consider the case of unbounded plasma and vacuum domains:

$$\Sigma(t) = \{x^1 = \varphi(t,x')\}, \quad x' = (x^2, x^3), \quad \Omega^{\pm}(t) = \left\{x^1 \gtrless \varphi(t,x')\right\}.$$

The free boundary $\Sigma(t)$ moves with the velocity of the plasma particles at the boundary:

$$\partial_t \varphi = v_N \qquad \text{on} \quad \Sigma(t)$$

(for all $t \in [0,T]$). The rest boundary conditions are

$$q = \frac{|\mathcal{H}|^2 - |E|^2}{2}, \quad v_N \mathcal{H} = N \times E, \quad H_N = 0, \quad \mathcal{H}_N = 0 \qquad \text{on} \quad \Sigma(t),$$

where $N = (1, -\partial_2\varphi, -\partial_3\varphi)$, $v_N = (v, N)$, etc.

Our main goal

Our goal is to prove the **local-in-time existence and uniqueness of a smooth solution** (U, V, φ) of the relativistic plasma-vacuum interface problem under certain restrictions on the initial data (U_0, V_0, φ_0).

Moreover, we should find these restrictions (at least, sufficient conditions). The main restrictions will be sufficient conditions for the **linear stability of a planar interface** (sufficient conditions for the fulfillment of the Kreiss-Lopatinski condition). For the nonlinear problem these conditions should be satisfied at each point of the initial interface.

Symmetrization of RMHD

Using the div. constraint and the **additional conservation law**

$$\partial_t(\rho\Gamma S) + \mathrm{div}\,(\rho S u) = 0$$

which holds on smooth solutions of the RMHD system, and following Godunov's symmetrization procedure, we can symmetrize the conservation laws in terms of a vector of canonical variables $Q = Q(U)$.

This was done by Ruggeri & Strumia (1981) and also by Anile & Pennisi (1987). However, they have not found a concrete form of symmetric matrices. Moreover, if we deal with an initial-boundary value problem it is very inconvenient to work in terms of the vector Q.

The return from Q to the vector of primitive (physical) variables U (with keeping the symmetry property) is connected with unimaginable (or even almost unrealizable in practice) calculations.

Symmetrization of RMHD [Freistühler & T., 2010]

For the fluid rest frame ($v = 0$) we rewrite RMHD in a nonconservative form which will be already a symmetric system. Applying properly the Lorentz transformation we get the symmetric system

$$A_0(U)\partial_t U + \sum_{j=1}^{3} A_j(U)\partial_j U = 0$$

in the LAB-frame, where

$$A_0 = \begin{pmatrix} \frac{\Gamma}{\rho a^2} & v^{\mathsf{T}} & 0 & 0 \\ v & \mathcal{A} & 0 & 0 \\ 0 & 0 & \mathcal{M} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad A_j = \begin{pmatrix} \frac{u_j}{\rho a^2} & e_j^{\mathsf{T}} & 0 & 0 \\ e_j & \mathcal{A}_j & \mathcal{N}_j^{\mathsf{T}} & 0 \\ 0 & \mathcal{N}_j & v_j\mathcal{M} & 0 \\ 0 & 0 & 0 & v_j \end{pmatrix},$$

$a^2 = p_\rho(\rho, S)$, the unit column vectors $e_j = (\delta_{1j}, \delta_{2j}, \delta_{3j})$,

Symmetrization of RMHD [Freistühler & T., 2010]

$$\mathcal{A} = \left(\rho h \Gamma + \frac{|H|^2}{\Gamma}\right) I - \left(\rho h \Gamma + \frac{|H|^2 + B^2}{\Gamma}\right) v \otimes v - \frac{1}{\Gamma} H \otimes H + \frac{(v, H)}{\Gamma} (v \otimes H + H \otimes v),$$

$$\mathcal{M} = \frac{1}{\Gamma} (I + u \otimes u), \qquad \mathcal{N}_j = \frac{1}{\Gamma} b \otimes e_j - \frac{v_j}{\Gamma} b \otimes v - \frac{H_j}{\Gamma^2} I,$$

$$\mathcal{A}_j = v_j \left\{ \left(\rho h \Gamma + \frac{|H|^2}{\Gamma}\right) I - \left(\rho h \Gamma + \frac{|H|^2 - B^2}{\Gamma}\right) v \otimes v - \frac{1}{\Gamma} H \otimes H \right\} + \frac{H_j}{\Gamma} \left\{ \frac{1}{\Gamma^2} (v \otimes H + H \otimes v) - 2(v, H)(I - v \otimes v) \right\} + \frac{(v, H)}{\Gamma} (H \otimes e_j + e_j \otimes H) - \frac{B^2}{\Gamma} (v \otimes e_j + e_j \otimes v) .$$

Hyperbolicity condition $(A_0 > 0)$:

$$\rho > 0, \quad p_\rho > 0, \quad 0 < c_s^2 < 1$$

(of course, $|v| < 1$), where

c_s is the relativistic speed of sound, $\quad c_s^2 = a^2/h = p_\rho/h.$

The last inequality above is the relativistic causality condition.

For the plasma-vacuum interface problem we will assume that the hyperbolicity condition holds up to the boundary, i.e., the density ρ does not go to zero continuously, but jumps: $0 < \varepsilon \leq \rho|_\Sigma \ll 1$.

Secondary symmetrization of the vacuum Maxwell equations

The vacuum Maxwell equations form the symmetric system

$$\partial_t V + \sum_{j=1}^{3} B_j \partial_j V = 0$$

(B_j can be easily written down). For this system we have not only the standard energy integral

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathbb{R}^3} |V|^2 \, \mathrm{d}x = 0$$

but also

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathbb{R}^3} (\mathcal{H}_2 E_3 - \mathcal{H}_3 E_2) \, \mathrm{d}x = 0,$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathbb{R}^3} (\mathcal{H}_3 E_1 - \mathcal{H}_1 E_3) \, \mathrm{d}x = 0, \quad \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathbb{R}^3} (\mathcal{H}_1 E_2 - \mathcal{H}_2 E_1) \, \mathrm{d}x = 0.$$

Secondary symmetrization of the vacuum Maxwell equations

Then, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}^3}\{|V|^2+\nu_1(\mathcal{H}_2E_3-\mathcal{H}_3E_2) \\ +\nu_2(\mathcal{H}_3E_1-\mathcal{H}_1E_3)+\nu_3(\mathcal{H}_1E_2-\mathcal{H}_2E_1)\}\,\mathrm{d}x=0$$

or

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathbb{R}^3}(\mathcal{B}_0V,V)\,\mathrm{d}x=0,$$

where

$$\mathcal{B}_0=\begin{pmatrix} 1 & 0 & 0 & 0 & \nu_3 & -\nu_2 \\ 0 & 1 & 0 & -\nu_3 & 0 & \nu_1 \\ 0 & 0 & 1 & \nu_2 & -\nu_1 & 0 \\ 0 & -\nu_3 & \nu_2 & 1 & 0 & 0 \\ \nu_3 & 0 & -\nu_1 & 0 & 1 & 0 \\ -\nu_2 & \nu_1 & 0 & 0 & 0 & 1 \end{pmatrix} > 0 \quad \text{if} \quad \nu_1^2+\nu_2^2+\nu_3^2<1.$$

Secondary symmetrization of the vacuum Maxwell equations

Let ν_i be arbitrary functions $\nu_i(t,x)$. Then

$$\mathcal{B}_0\partial_t V + \sum_{j=1}^{3}\mathcal{B}_0 B_j\partial_j V + R_1 \mathrm{div}\,\mathcal{H} + R_2 \mathrm{div}\,E = \mathcal{B}_0\partial_t V + \sum_{j=1}^{3}\mathcal{B}_j\partial_j V = 0,$$

where

$$\mathcal{B}_1 = \begin{pmatrix} \nu_1 & \nu_2 & \nu_3 & 0 & 0 & 0 \\ \nu_2 & -\nu_1 & 0 & 0 & 0 & -1 \\ \nu_3 & 0 & -\nu_1 & 0 & 1 & 0 \\ 0 & 0 & 0 & \nu_1 & \nu_2 & \nu_3 \\ 0 & 0 & 1 & \nu_2 & -\nu_1 & 0 \\ 0 & -1 & 0 & \nu_3 & 0 & -\nu_1 \end{pmatrix}, \quad \mathcal{B}_2 = \begin{pmatrix} -\nu_2 & \nu_1 & 0 & 0 & 0 & 1 \\ \nu_1 & \nu_2 & \nu_3 & 0 & 0 & 0 \\ 0 & \nu_3 & -\nu_2 & -1 & 0 & 0 \\ 0 & 0 & -1 & -\nu_2 & \nu_1 & 0 \\ 0 & 0 & 0 & \nu_1 & \nu_2 & \nu_3 \\ 1 & 0 & 0 & 0 & \nu_3 & -\nu_2 \end{pmatrix}$$

$$\mathcal{B}_3 = \begin{pmatrix} -\nu_3 & 0 & \nu_1 & 0 & -1 & 0 \\ 0 & -\nu_3 & \nu_2 & 1 & 0 & 0 \\ \nu_1 & \nu_2 & \nu_3 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\nu_3 & 0 & \nu_1 \\ -1 & 0 & 0 & 0 & -\nu_3 & \nu_2 \\ 0 & 0 & 0 & \nu_1 & \nu_2 & \nu_3 \end{pmatrix}, \quad R_1 = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad R_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$

Another use of the secondary symmetrization

The secondary symmetrization was also recently used in the joint work with Paolo Secchi to prove the existence of solutions for the nonrelativistic version of the linearized plasma-vacuum interface problem by a hyperbolic regularization of the elliptic system $\operatorname{div} \mathcal{H} = 0$, $\nabla \times \mathcal{H} = 0$.

Attend **the talk by P. Secchi on Thursday, June 28** for more details.

It was used in the joint work with Alessandro Morando and Paola Trebeschi for the plasma-vacuum interface problem in incompressible MHD (we additionally apply a "compressible" regularization).

Attend **the talk of P. Trebeschi on Tuesday, June 26** for more details.

Reduction to a fixed domain

By the simple change of variables

$$\tilde{x}^1 = x^1 - \varphi(t, x') \qquad \text{for the RMHD system}$$

and

$$\tilde{x}^1 = -x^1 + \varphi(t, x') \qquad \text{for the Maxwell equations,}$$

we reduce our FBP to that in the half-space $\mathbb{R}^3_+ = \{x^1 > 0,\ x' \in \mathbb{R}^2\}$.

In principle, to avoid assumptions about compact support of the initial data and work globally in the normal direction we should use a cut-off function $\chi(x^1) \in C_0^\infty(\mathbb{R})$:

$$\tilde{x}^1 = \pm x^1 + \chi(x_1)\varphi(t, x').$$

But, here for simplicity we don't use a cut-off.

Problem in a fixed domain

Main (nonlinear) problem:

$$L(U,\varphi)U=0,\quad M(\varphi)V=0\quad \text{in } [0,T]\times\mathbb{R}^3_+,$$

$$\mathbb{B}(U,V,\varphi)=0\quad \text{on } [0,T]\times\{x^1=0\}\times\mathbb{R}^2,$$

$$(U,V)|_{t=0}=(U_0,V_0)\quad \text{in } \mathbb{R}^3_+,\qquad \varphi|_{t=0}=\varphi_0\quad \text{in } \mathbb{R}^2,$$

where $L(U,\varphi)=A_0(U)\partial_t+\widetilde{A}_1(U,\varphi)\partial_1+A_2(U)\partial_2+A_3(U)\partial_3$,

$$\widetilde{A}_1(U,\varphi)=A_1(U)-A_0(U)\partial_t\varphi-A_2(U)\partial_2\varphi-A_3(U)\partial_3\varphi,$$

$$M(\varphi)=I\,\partial_t-\widetilde{B}_1(\varphi)\partial_1+\sum_{k=2}^{3}B_k\partial_k,\quad \widetilde{B}_1(\varphi)=B_1-I\,\partial_t\varphi-\sum_{k=2}^{3}B_k\partial_k\varphi,$$

$$\mathbb{B}(U,V,\varphi)=\begin{pmatrix} v_N-\partial_t\varphi \\ q-\frac{1}{2}(|\mathcal{H}|^2-|E|^2) \\ E_{\tau_2}-\mathcal{H}_3\partial_t\varphi \\ E_{\tau_3}+\mathcal{H}_2\partial_t\varphi \end{pmatrix},\qquad \begin{aligned} &E_{\tau_i}=E_1\partial_i\varphi+E_i,\quad i=2,3,\\ &v_N=v_1-v_2\partial_2\varphi-v_3\partial_3\varphi. \end{aligned}$$

Restrictions on the initial data for the "plasma" unknown

Proposition

Let the initial data satisfy

$$H_N|_{x^1=0} = 0 \qquad \textit{and} \qquad \operatorname{div} \mathsf{h} = 0, \qquad (\star)$$

with $\mathsf{h} = (H_N, H_2, H_3)$. *If the IBVP has a solution* (U, V, φ), *then this solution satisfies* $(\star)$ *for all* $t \in [0, T]$.

Case 1) Plasma expansion

Proposition

Let the initial data satisfy

$$\mathcal{H}_N|_{x^1=0} = 0, \qquad \mathrm{div}^- \mathfrak{h} = 0, \qquad \mathrm{div}^- \mathfrak{e} = 0, \qquad (\star\star)$$

where $\mathfrak{h} = (\mathcal{H}_N, \mathcal{H}_2, \mathcal{H}_3)$, $\mathfrak{e} = (E_N, E_2, E_3)$, $E_N = (E, N)$, *and* $\mathrm{div}^- a = -\partial_1 a_1 + \partial_2 a_2 + \partial_3 a_3$ *for any vector* $a = (a_1, a_2, a_3)$. *If the IBVP has a solution* (U, V, φ) *with the property*

$$\partial_t \varphi < 0 \qquad \textit{(plasma expansion)},$$

then this solution satisfies $(\star\star)$ *for all* $t \in [0, T]$.

If $\partial_t \varphi \leq 0$, then the whole system for U and V has three incoming characteristics and, therefore, the number of boundary conditions is correct.

Case 2) Vacuum expansion

If

$$\partial_t \varphi > 0 \qquad \text{(vacuum expansion)},$$

the problem is formally underdetermined. We miss 2 boundary conditions. However, if as these 2 boundary missing conditions we take

$$\operatorname{div}^- \mathfrak{e}|_{x^1=0} = 0 \qquad \text{and} \qquad \operatorname{div}^- \mathfrak{h}|_{x^1=0} = 0,$$

then we can show that $\operatorname{div}^- \mathfrak{e} = 0$ and $\operatorname{div}^- \mathfrak{h} = 0$ hold in the whole domain $\mathbb{R}^3_+$ and $\mathcal{H}_N|_{x^1=0} = 0$ if these conditions were satisfied for $t = 0$.

Instead of the condition $\operatorname{div}^- \mathfrak{h}|_{x^1=0} = 0$ we can alternatively take

$$\mathcal{H}_N|_{x^1=0} = 0.$$

Mixed case

Locally, for each portion of the initial interface we may assume that either the plasma expands into the vacuum or the vacuum expands into the plasma.

If we consider the interface globally in space, then a mixed case is possible. However, mathematically it corresponds to a hyperbolic problem with characteristic boundary of variable multiplicity. We don't study yet this difficult problem.

Restrictions on the basic state in the linearized problem

The proof of the existence theorem is based on Nash-Moser iterations.

The first step is careful study of the linearized problem. We linearize the problem with respect to a sufficiently smooth **basic state** $(\widehat{U}, \widehat{V}, \hat{\varphi})$.

The main assumptions on the basic state:

1) The hyperbolicity conditions hold,

2) Plasma expansion (or alternatively, vacuum expansion),

3) The boundary conditions hold.

Linear problem in terms of the "good unknowns" $\dot{U} = U - \varphi\,\partial_1 \widehat{U}$ and $\dot{V} = V + \varphi\,\partial_1 \widehat{V}$

$$L(\widehat{U},\hat{\varphi})\dot{U} + \mathcal{C}(\widehat{U},\hat{\varphi})\dot{U} = f_{\rm I} \qquad \text{in } \Omega_T,$$

$$M(\hat{\varphi})\dot{V} = f_{\rm II} \qquad \text{in } \Omega_T,$$

$$\begin{pmatrix} \dot{v}_N - \partial_t\varphi - \hat{v}_2\partial_2\varphi - \hat{v}_3\partial_3\varphi + \varphi\,\partial_1\hat{v}_N \\ \dot{q} - (\widehat{\mathcal{H}},\dot{\mathcal{H}}) + (\widehat{E},\dot{E}) + [\partial_1\hat{q}]\varphi \\ \dot{E}_{\tau_2} - \partial_t(\widehat{\mathcal{H}}_3\varphi) - \varkappa\dot{\mathcal{H}}_3 + \partial_2(\widehat{E}_1\varphi) \\ \dot{E}_{\tau_3} + \partial_t(\widehat{\mathcal{H}}_2\varphi) + \varkappa\dot{\mathcal{H}}_2 + \partial_3(\widehat{E}_1\varphi) \end{pmatrix} = g \qquad \text{on } \partial\Omega_T,$$

$$(\dot{U},\dot{V},\varphi) = 0 \qquad \text{for } t < 0,$$

where $\Omega_T = (-\infty, T] \times \mathbb{R}^3_+$ and

$$\varkappa = \partial_t\hat{\varphi}, \quad \dot{v}_{\rm N} = \dot{v}_1 - \dot{v}_2\partial_2\hat{\varphi} - \dot{v}_3\partial_3\hat{\varphi}, \quad \dot{E}_{\tau_j} = \dot{E}_1\partial_j\hat{\varphi} + \dot{E}_j,$$

$$[\partial_1\hat{q}] = (\partial_1\hat{q})|_{x^1=0} - (\widehat{\mathcal{H}},\partial_1\widehat{\mathcal{H}})|_{x^1=0} + (\widehat{E},\partial_1\widehat{E})|_{x^1=0},$$

and $\dot{v}$ and $\dot{q}$ are determined similarly to v and q.

Functional setting

Anisotropic weighted Sobolev spaces H^m_*, studied by Ohno, Shizuta,..., and Secchi:

$$H^m_*(\mathbb{R}^3_+) = \left\{ u \in L_2(\mathbb{R}^3_+) \mid \partial^\alpha_* \partial_1^k u \in L_2(\mathbb{R}^3_+), \quad \text{and} \quad |\alpha| + 2k \leq m \right\},$$

$$\|u\|^2_{m,*} = \sum_{|\alpha|+2k\leq m} \|\partial^\alpha_* \partial_1^k u\|^2_{L_2(\mathbb{R}^3_+)}.$$

where $m \in \mathbb{N}$, $\partial^\alpha_* = (\sigma\partial_1)^{\alpha_1}\partial_2^{\alpha_2}\partial_3^{\alpha_3}$ and

$$\sigma = \begin{cases} x_1 & \text{in a neighborhood of } x_1 = 0 \\ 1 & \text{for } x_1 \text{ large enough} \end{cases} \in C^\infty(\mathbb{R}_+).$$

We also define: $H^m_*(\Omega_T) = \bigcap_{k=0}^m H^k((-\infty, T], H^{m-k}_*(\mathbb{R}^3_+))$,

$$[u]^2_{m,*,T} = \int_0^T |||u(t)|||^2_{m,*} dt, \quad |||u(t)|||^2_{m,*} = \sum_{j=0}^m \|\partial_t^j u(t)\|^2_{m-j,*}.$$

Theorem (Basic a priori estimate for the linearized problem)

Let the basic state $(\widehat{U}, \widehat{V}, \hat{\varphi})$ *satisfies all the assumptions above. Let also*

$$|\widehat{H}_2\widehat{\mathcal{H}}_3 - \widehat{H}_3\widehat{\mathcal{H}}_2|_{|x^1=0} \geq \epsilon > 0, \qquad \left((\widehat{H} \nparallel \widehat{\mathcal{H}})|_{x^1=0}\right)$$

where ϵ *is a fixed constant. Then there exists a positive constant* μ^* *such that for all* $(f,g) \in H^3_*(\Omega_T) \times H^3(\partial\Omega_T)$ *which vanish in the past the linear problem has a unique solution* $(\dot{U}, \dot{V}, \varphi) \in H^1_*(\Omega_T) \times H^1(\Omega_T) \times H^1(\partial\Omega_T)$ *for all* $\mu < \mu^*$, *where*

$$\mu = |\widehat{E}_1 + \hat{v}_2\widehat{\mathcal{H}}_3 - \hat{v}_3\widehat{\mathcal{H}}_2|_{|x^1=0}\,.$$

Moreover, this solution obeys the a priori estimate

$$[\dot{U}]_{1,*,T} + \|\dot{V}\|_{H^1(\Omega_T)} + \|\varphi\|_{H^1(\partial\Omega_T)} \leq C\left\{[f_{\mathrm{I}}]_{3,*,T} + \|f_{\mathrm{II}}\|_{H^3(\Omega_T)} + \|g\|_{H^3(\partial\Omega_T)}\right\},$$

where $C = C(K,T) > 0$ *is a constant independent of the data* (f,g).

Main idea of the proof

Using the secondary symmetrization of the vacuum Maxwell equation for the choice $\vec{\nu} = (\hat{v}_1, \hat{v}_2, \hat{v}_3)$, we get a dissipative energy integral for the prolonged system for tangential derivatives provided that

$$\mu = |\widehat{E}_1 + \hat{v}_2 \widehat{\mathcal{H}}_3 - \hat{v}_3 \widehat{\mathcal{H}}_2|_{|_{x^1=0}} \ll 1.$$

If we want the last condition can be refined by analyzing (e.g., numerically) the conditions for positive definiteness of some matrix of order 42 depending on the basic state (unperturbed flow).

Planar interface can be violently unstable!

The condition

$$|\widehat{E}_1 + \hat{v}_2\widehat{\mathcal{H}}_3 - \hat{v}_3\widehat{\mathcal{H}}_2| \ll 1 \quad \text{and} \quad \widehat{H} \nparallel \widehat{\mathcal{H}}$$

is a sufficient stability condition for a planar relativistic plasma-vacuum interface.

Can a planar interface be violently unstable???

Yes, it can!!!
We show this by the spectral method (by constructing Hadamard-type ill-posedness examples) for very particular cases (for which we can treat **terribly** complicated RMHD analytically).

At least for the considered particular cases, we can say that, roughly speaking, the electric field plays a destabilizing role whereas the magnetic field stabilizes the interface.

Local-in-time existence and uniqueness of a smooth stable (nonplanar) interface

Theorem

Let $m \geq 12$ *and the initial data*

$$((U_0 - \check{U}, V_0 - \check{V}), \varphi_0) \in H_*^{2m+19}(\mathbb{R}^3_+) \times H^{2m+19}(\mathbb{R}^2),$$

where $\check{U} = (\check{p}, 0, \check{H}, 0)$ *and* $\check{V} = (\check{\mathcal{H}}, \check{E})$ *are constant vectors, satisfy compatibility conditions of order* $m + 9$ *(in a suitable sense) and the same requirements as the basic state in the linearized problem (in particular, the sufficient stability condition for all* $x' \in \mathbb{R}^2$*). Then, there exists a sufficiently short time* $T > 0$ *such that the relativistic plasma-vacuum interface problem (reduced to the fixed domain* $\mathbb{R}^3_+$*) has a unique solution*

$$((U - \check{U}, V - \check{V}), \varphi) \in H_*^m([0, T] \times \mathbb{R}^3_+) \times H^m([0, T] \times \mathbb{R}^2).$$

Scheme of the proof

1. Derivation of a tame estimate for the linearized problem (an estimate "linear in high norms").

2. We solve the nonlinear problem by Nash-Moser iterations whose convergence is proved by using the tame estimate.

3. Uniqueness follows from the basic a priori estimate for the linearized problem .