Riemann solutions without intermediate constant states for a system in thermal multiphase flow in porous media

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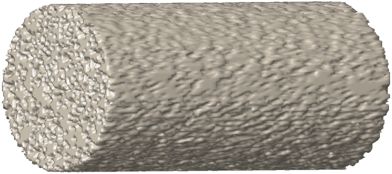
Physical model

Injection of gaseous volatile oil into a cylindrical horizontal core, following Bruining and Marchesin (2007).



The rock is filled with a mixture of oil s_o and gas s_g, i.e.:

$$s_g + s_o = 1.$$

Qualitative behavior

Simplifications:

- Physical quantities evaluated at a representative pressure;
- No thermal expansion for liquids;
- Darcy law for two-phase flow;
- No gravitational segregation.

The oil consists of a mixture of dead and volatile oil.

Mass balance equations

Balance of volatile alkane in oil:

$$\varphi \frac{\partial}{\partial t}(\rho_{ov} s_o) + \frac{\partial}{\partial x}(\rho_{ov} u f_o) = +q_{g\to o,v}.$$

Balance of volatile alkane in gas:

$$\varphi \frac{\partial}{\partial t}(\rho_{gV} s_g) + \frac{\partial}{\partial x}(\rho_{gV} u f_g) = -q_{g\to o,v}.$$

Balance of dead oil:

$$\varphi \frac{\partial}{\partial t}(\rho_{od} s_o) + \frac{\partial}{\partial x}(\rho_{od} u f_o) = 0.$$

Volatile vapor condensation rate $q_{g\to o,v}$, denotes mass transfer from the gaseous to the liquid phase.

Fundamentals of T. Equilibrium in the two-phase region

Gibb's phase rule gives two degrees of freedom:

temperature T, and pressure P.

Recall that P is fixed!

Clausius-Clapeyron and Raoult's laws:

Volatile oil concentration is a function of temperature, $\rho_{ov}(T)$.

Ideal mixing:

We disregard any volume contraction due to mixing.

$$\frac{\rho_{ov}}{\rho_V} + \frac{\rho_{od}}{\rho_D} = 1.$$

Conservation laws in $\mathbf{TP}$

Mass and energy conservation.

$$\begin{cases} \varphi \dfrac{\partial}{\partial t}\left(\rho_{gV} s_g + \rho_{ov} s_o\right) + \dfrac{\partial}{\partial x} u\left(\rho_{gV} f_g + \rho_{ov} f_o\right) &= 0, \\ \varphi \dfrac{\partial}{\partial t}(\rho_{od} s_o) + \dfrac{\partial}{\partial x} u(\rho_{od} f_o) &= 0, \\ \varphi \dfrac{\partial}{\partial t}\left(\widehat{H}_r + s_o H_o + s_g H_g\right) + \dfrac{\partial}{\partial x} u\left(f_o H_o + f_g H_g\right) &= 0, \end{cases}$$

$$(s_o, T, \rho_{od}(T), u) \in \Omega_{\mathbf{TP}} \times \mathbb{R}^+,$$

where:

$$\Omega_{\mathbf{TP}} = \{\ (s_o, T, \rho_{od}(T)) \mid 0 \le s_o \le 1,\ T > T_{bV}\ \}.$$

The Riemann problem

Self-similar weak solutions of

$$\frac{\partial}{\partial t}G(\mathbf{w}) + \frac{\partial}{\partial x}uF(\mathbf{w}) = 0,$$

for

$$\mathbf{w}(x,0) = \begin{cases} \mathbf{w}_L, & \text{if} \quad x < 0, \\ \mathbf{w}_R, & \text{if} \quad x > 0, \end{cases}$$

and

$$u(x,0) = u_L, \quad \text{if} \quad x < 0,$$

where $\mathbf{w}_R, \mathbf{w}_L \in \Omega$ and $u_L \in \mathbb{R}^+$ are constants.

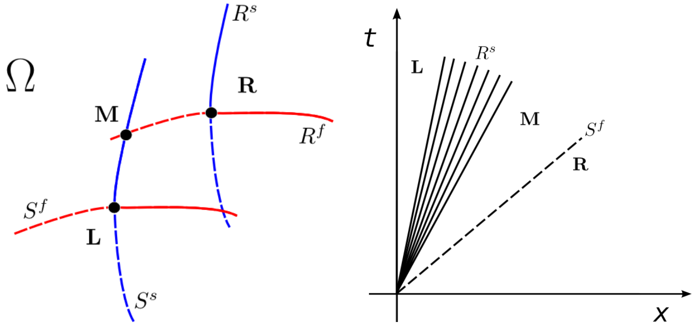
Works of Lax (1957) and Glimm (1965) main hypotheses: strict hyperbolicity and genuine nonlinearity.

Riemann solutions

Built by concatenations of fundamental waves (and constant states):

- Smooth rarefaction waves;
- Discontinuous shock solutions.

Selection of admissible shocks is a delicate issue.



Wave curve method

Strict hyperbolicity and genuine nonlinearity typically are violated in multiphase flow systems of conservation laws.

Structures that introduce bifurcations in Riemann solutions:

- Coincidences;
- Inflections;
- Self-intersections;
- Double contacts;
- etc.

The wave curve method was developed as a systematic way to solve Riemann problems.

Remarkable work of Liu (1974) and many others...

Singular points

In our case the coincidence locus of characteristic speeds is a pair of curves.

Generically, the eigenspace associated to points inside the coincidence locus is one dimensional. Except:

Definition

The singular points inside the coincidence curves are the points where the eigenspace of the characteristic equation:

$$\big(u\mathrm{D}F(\mathbf{w}) - \lambda \mathrm{D}G(\mathbf{w}),\ F(\mathbf{w})\big) r(\mathbf{w}, u) = 0.$$

is two dimensional.

Similar to Keyfitz, Kranzer, Isaacson, Temple; de Souza and Marchesin (1998).

A Riemann solution near the singular point

"Generically" systems of conservation laws with two distinct families possess solutions with two distinct wave groups.

But this class of models allows the generic existence of Riemann solutions with a single wave group.

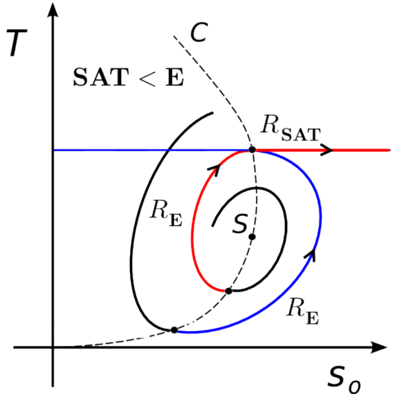
Example (In a neighborhood of $\mathcal{S}$)

The $\mathbf{w}^R$ state is above the singular point, on the right side of the coincidence locus. The $\mathbf{w}^L$ state is below the singular point in a "suitable" chosen open set.

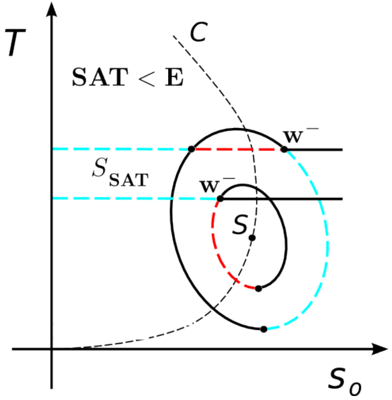
Elementary waves near the singular point

Two families:

Pure saturation transport **SAT** and thermal transport **E**.



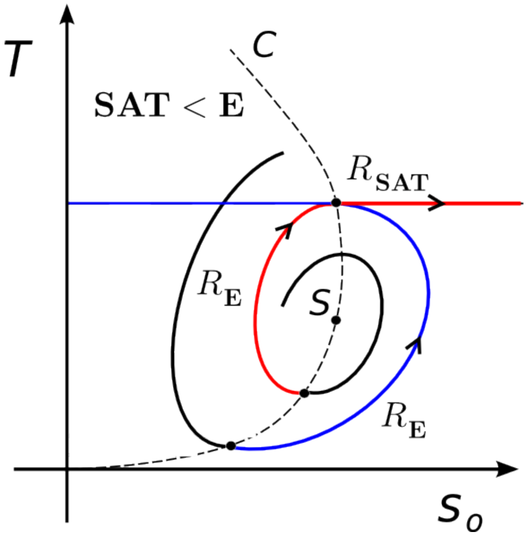
Rarefaction curves



Shock branches

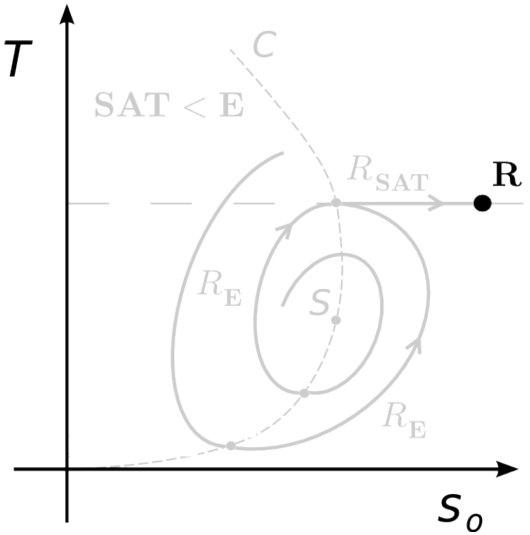
A Riemann solution without intermediate states

Rarefaction curves near the singular point:



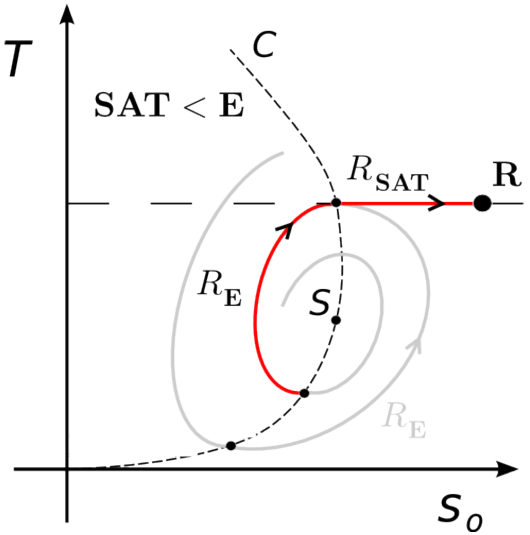
A Riemann solution without intermediate states

State **R**:



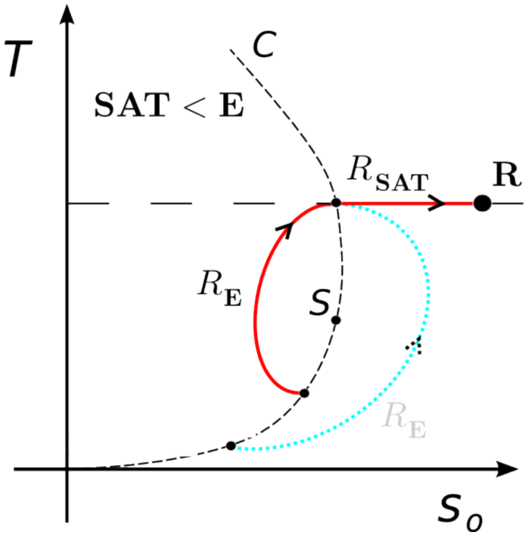
A Riemann solution without intermediate states

Fast wave curve reaching state **R**:



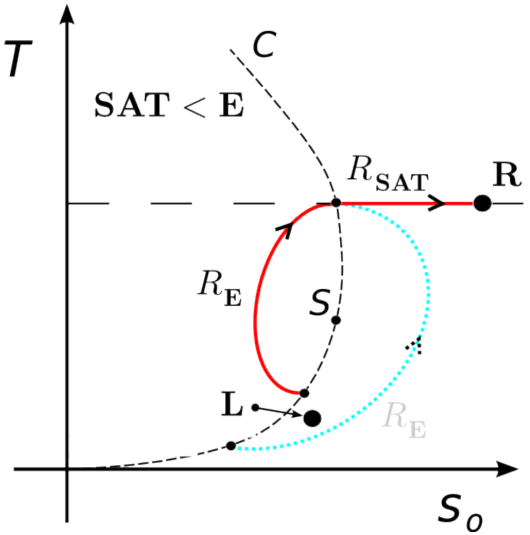
A Riemann solution without intermediate states

Fast wave curve reaching state **R**:



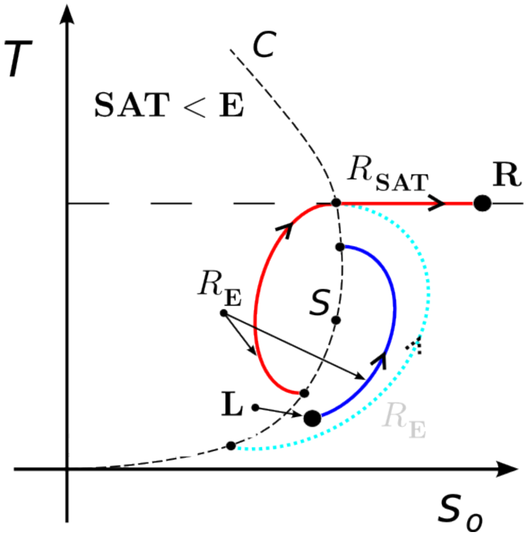
A Riemann solution without intermediate states

State **L**:



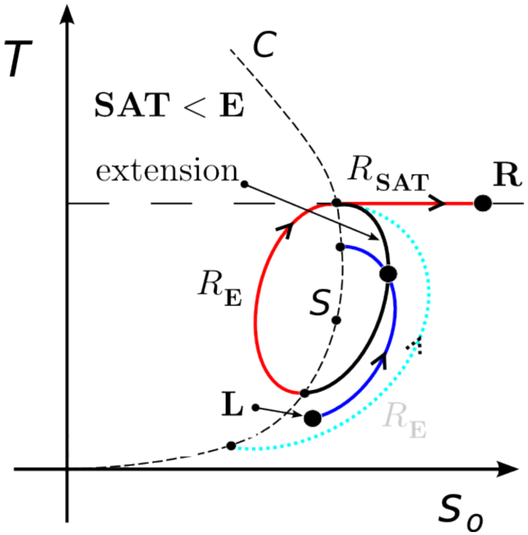
A Riemann solution without intermediate states

Slow wave curve emanating from state **L**:



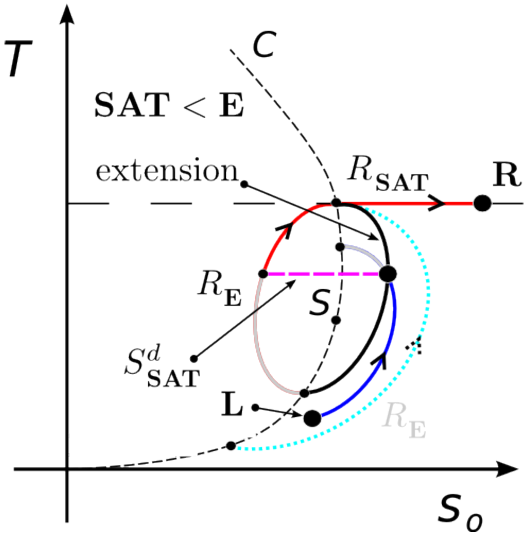
A Riemann solution without intermediate states

Characteristic extension of $R_{\mathbf{E}}$:

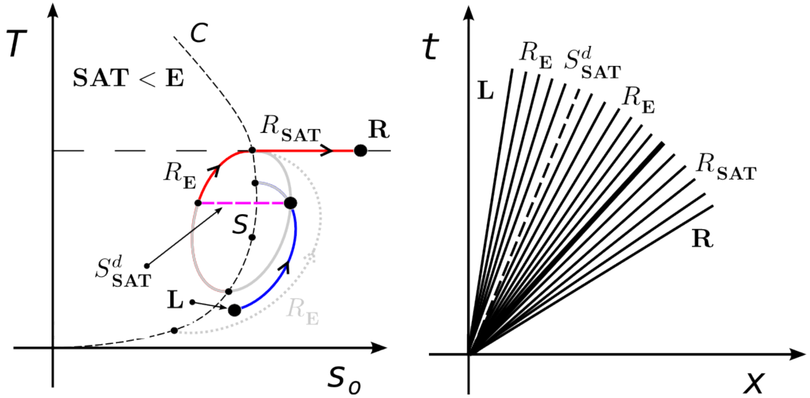


A Riemann solution without intermediate states

SAT doubly characteristic shock:



A Riemann solution without intermediate states

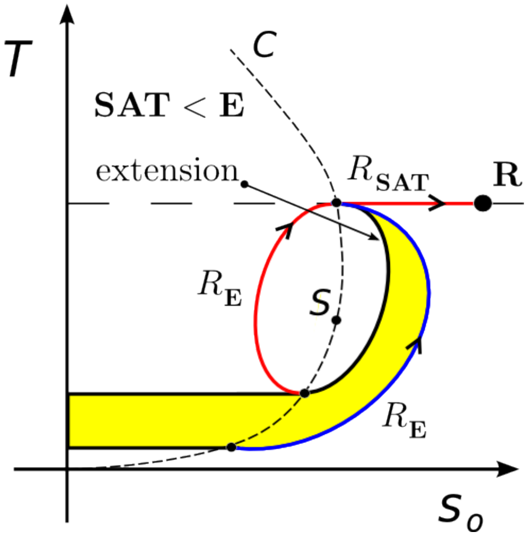


Riemann solution:

$$\mathbf{L} \xrightarrow{R_{\mathsf{E}} \cdot S^d_{\mathsf{SAT}} \cdot R_{\mathsf{E}} \cdot R_{\mathsf{SAT}}} \mathbf{R}.$$

Bifurcation structure near the singular point

The R. solution for **L** $\in$ Yellow, **R** possesses a single wave group.



Open double contact locus I

The secondary bifurcation set:

States $\mathbf{w}^+ \neq \mathbf{w}^-$, such that $\mathbf{w}^+$ is contained in the Hugoniot locus of $\mathbf{w}^-$ and the Jacobian of the Hugoniot function:

$$\begin{aligned} d\mathcal{H}_{\mathbf{w}^-}(\mathbf{w}^+, u^+, \sigma) = &\left(u^+ \mathrm{D}F(\mathbf{w}^+) - \sigma \mathrm{D}G(\mathbf{w}^+)\right) d\mathbf{w}^+ \\ &+ F(\mathbf{w}^+)\, du^+ - \left(G(\mathbf{w}^+) - G(\mathbf{w}^-)\right) d\sigma \end{aligned}$$

is singular.

To this end the following identities must hold:

$$\sigma = \lambda(\mathbf{w}^+, u^+) \quad \text{and} \quad \vec{l}(\mathbf{w}^+)(G(\mathbf{w}^+) - G(\mathbf{w}^-)) = 0.$$

Open double contact locus II

Lemma

Away from the coincidence locus, the **E** *family eigenvector can be written as a function of the temperature alone:* $\vec{l}_e = \vec{l}_e(T)$.

Lemma

In a isotherm, if $\sigma(P^-; P^+) = \lambda_e(\mathbf{w}^+, u^+)$ *holds then* $\sigma(P^-; P^+) = \lambda_e(\mathbf{w}^-, u^-)$.

Theorem

Assume that the Hugoniot locus in $\mathbf{TP}$ *only bifurcates at intersections of the* **SAT** *branch with the* **E** *branch. Then the* **E** *self-intersection locus is a two dimensional manifold.*

Conclusion

The underlying structure:

- The "singular point family" **E** is genuinely nonlinear almost everywhere.
- The projection of the **E**-double contact manifold in state space is open.

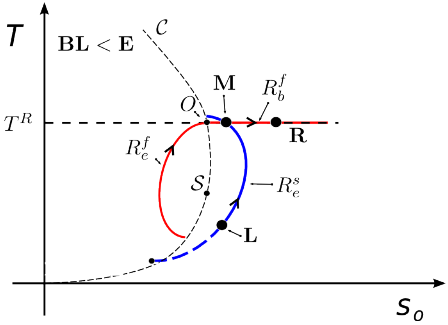
The previous results are a direct consequence of the form of equations:

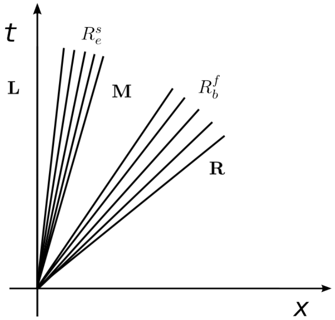
$$\frac{\partial}{\partial t}\Big(\alpha(T)s_o + \beta(T)\Big) + \frac{\partial}{\partial x}\Big[u\big(\alpha(T)f_o(s_o, T) + \gamma(T)\big)\Big] = 0.$$

dictated by physical principles!

Thank you!

Adjacent Riemann problem I

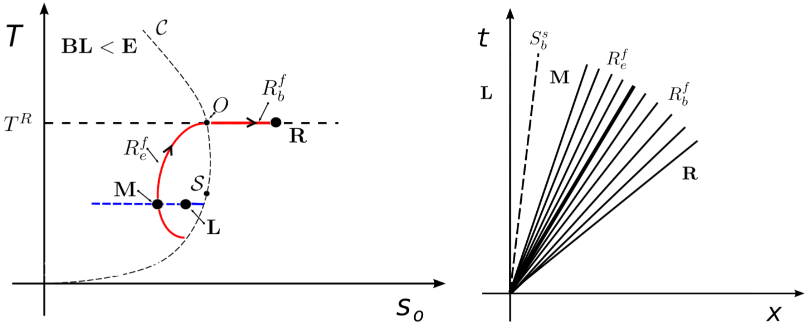




Riemann solution:

$$\mathbf{L} \xrightarrow{R_e^s} \mathbf{M} \xrightarrow{R_b^f} \mathbf{R}.$$

Adjacent Riemann problem II



Riemann solution:

$$\mathbf{L} \xrightarrow{S_b^s} \mathbf{M} \xrightarrow{R_e^f} \widehat{\mathbf{O}} \xrightarrow{R_b^f} \mathbf{R}.$$

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