Solving the Equations of Radiation-Magnetohydrodynamics

Andrew McMurry, Franz Fuchs, Siddhartha Mishra
University of Oslo, Norway

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Magnetohydrodynamics

$$
\begin{aligned}
\rho_t + \nabla \cdot (\rho \mathbf{u}) &= 0 \\
(\rho \mathbf{u})_t + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} + \tilde{p}\mathbf{I} - \mathbf{B} \otimes \mathbf{B}) &= -\mathbf{B}(\nabla \cdot \mathbf{B}) \\
E_t + \nabla \cdot ((E + \tilde{p})\,\mathbf{u} - (\mathbf{u} \cdot \mathbf{B})\,\mathbf{B}) &= -(\mathbf{B} \cdot \mathbf{u})(\nabla \cdot \mathbf{B}) \\
\mathbf{B}_t + \nabla \cdot (\mathbf{u} \otimes \mathbf{B} - \mathbf{B} \otimes \mathbf{u}) &= -\mathbf{u}(\nabla \cdot \mathbf{B}) \\
\nabla \cdot \mathbf{B} &= 0
\end{aligned}
$$

$$
\begin{aligned}
\tilde{p} &= p + \frac{1}{2}\mathbf{B}^2 \\
E &= \frac{p}{\gamma - 1} + \frac{1}{2}\rho \mathbf{u}^2 + \frac{1}{2}\mathbf{B}^2
\end{aligned}
$$

Energy Transfer Mechanisms

- Convection - handled by MHD

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- Radiation ...

Radiative Transfer

$$\mathcal{I}_t + c\hat{\mathbf{n}} \cdot \nabla \mathcal{I} = \eta - \xi \mathcal{I}$$

- $\mathcal{I}$ is the radiative intensity
- c is the speed of propogation (speed of light)
- $\hat{\mathbf{n}}$ is the direction of propogation
- η is the emission function
- ξ is the absorption function

Radiative Intensity

$$\mathcal{I} : (\mathbb{R}^3, \mathbb{R}^+, \mathbb{S}^2, \mathbb{R}^+) \to \mathbb{R}$$

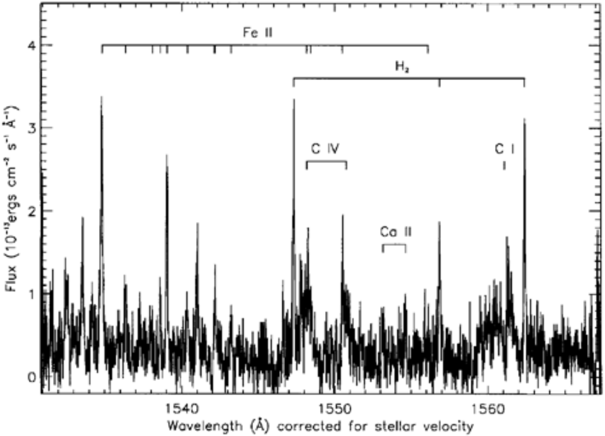
- Space
- Time
- Direction: $\hat{\mathbf{n}}$
- Frequency

Speed of Light

$$c = 2.993 \times 10^8 ms^{-1}$$

$\sim 10^3 - 10^4$ times the fastest MHD wave speed in our models.

Interaction with Material



M1 model of Radiation

$$\mathcal{E} = \frac{1}{c}\oint_{\mathbb{S}^2} \mathcal{I}\, d\Omega$$
$$\mathcal{F} = \frac{1}{c}\oint_{\mathbb{S}^2} \hat{\mathbf{n}}\mathcal{I}\, d\Omega$$
$$\mathcal{P} = \frac{1}{c}\oint_{\mathbb{S}^2} \hat{\mathbf{n}} \otimes \hat{\mathbf{n}}\mathcal{I}\, d\Omega$$

M1 model of Radiation

$$
\begin{aligned}
\mathcal{E}_t + \nabla \cdot (c\mathcal{F}) &= c\sigma(aT^4 - \mathcal{E}) \\
\mathcal{F}_t + \nabla \cdot (c\mathcal{P}) &= -c\sigma\mathcal{F} \\
E_t &= -c\sigma(aT^4 - \mathcal{E})
\end{aligned}
$$

Closing the M1 model

$$
\begin{aligned}
\mathcal{P} &= D\mathcal{E} \\
D &= \frac{1-\chi}{2}\mathbf{I} + \frac{3\chi-1}{2}\frac{\mathcal{F}\otimes\mathcal{F}}{\|\mathcal{F}\|^2} \\
\chi &= \frac{3+4f^2}{5+2\sqrt{4-3f^2}} \\
f &= \left\|\frac{\mathcal{F}}{\mathcal{E}}\right\|
\end{aligned}
$$

Our Approach

- M1 Moment model

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- Grey radiation, or small number of frequency cells

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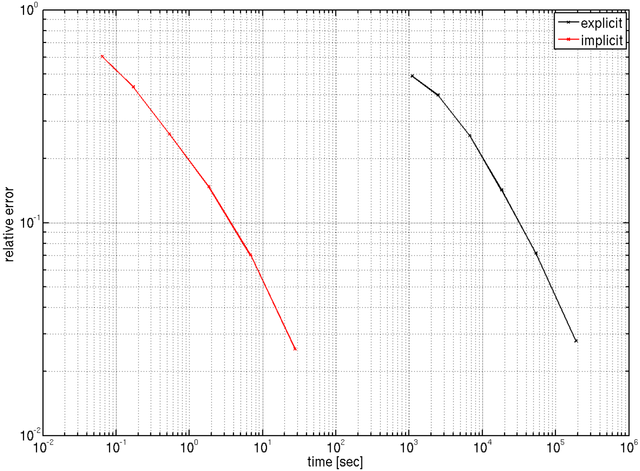
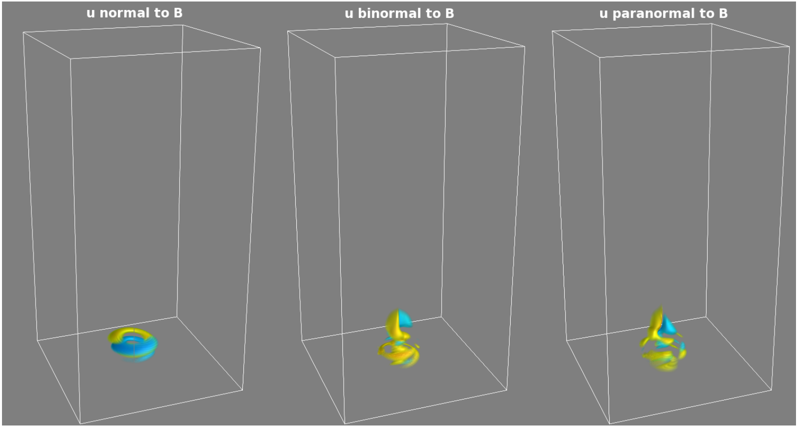
- M1 Moment model
- Grey radiation, or small number of frequency cells
- Semi-implicit solver, with implicit radiation

Semi-implicit Efficiency



Using gaussian-elimination for implicit solver.

Waves in the Solar Atmosphere



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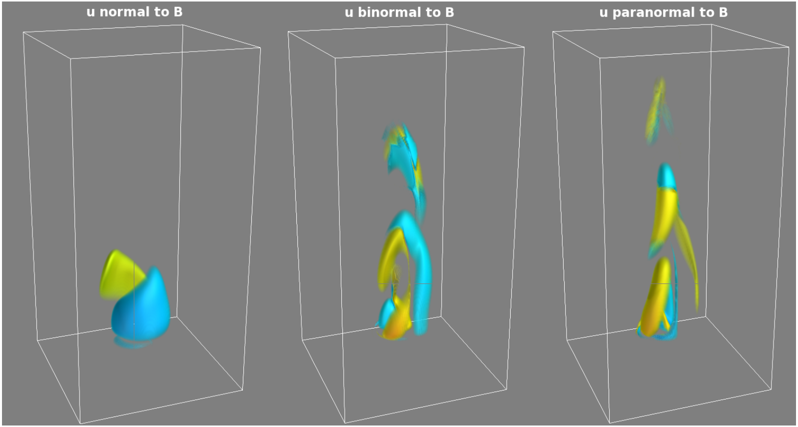

Waves in the Solar Atmosphere