New Central-Upwind Schemes for Euler Equations of Gas Dynamics

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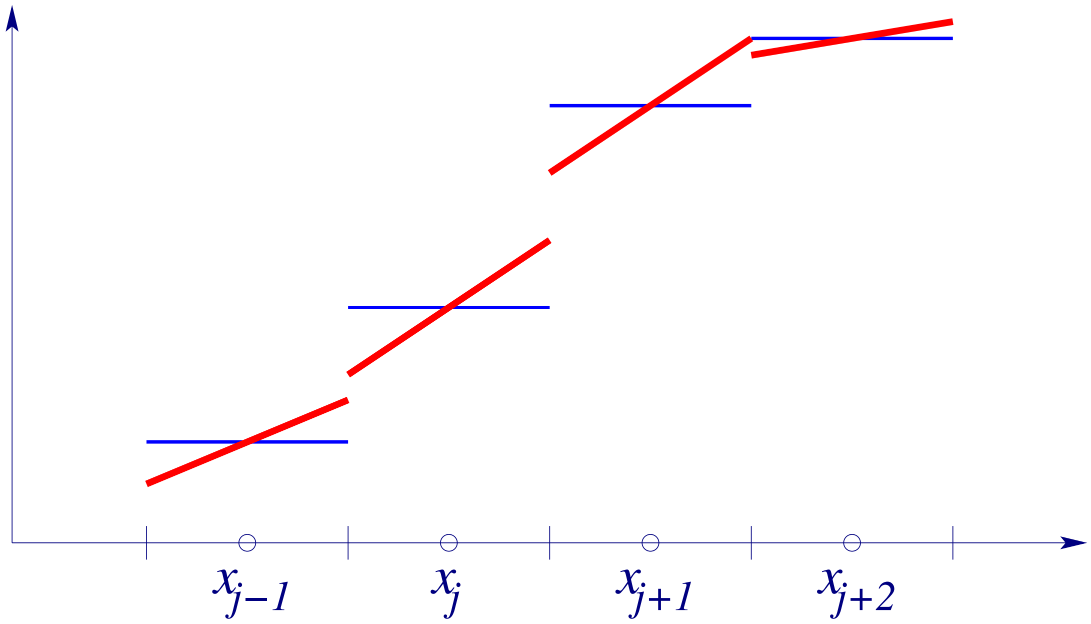
Finite-Volume Schemes

1-D System: $\mathbf{q}_t + \mathbf{f}(\mathbf{q})_x = \mathbf{0}$

$\bar{\mathbf{q}}_j^n \approx \frac{1}{\Delta x} \int\limits_{C_j} \mathbf{q}(x, t^n)\, dx$: cell averages over $C_j := (x_{j-\frac{1}{2}}, x_{j+\frac{1}{2}})$

This solution is approximated by a piecewise linear (conservative, second-order accurate, non-oscillatory) reconstruction:

$\widetilde{\mathbf{q}}^n(x) = \bar{\mathbf{q}}_j^n + (\mathbf{q}_x)_j^n (x - x_j)$ for $x \in C_j$



For example,

$$\boxed{(\mathbf{q}_x)_j^n = \text{minmod}\left(\theta\frac{\bar{\mathbf{q}}_j^n - \bar{\mathbf{q}}_{j-1}^n}{\Delta x}, \frac{\bar{\mathbf{q}}_{j+1}^n - \bar{\mathbf{q}}_{j-1}^n}{2\Delta x}, \theta\frac{\bar{\mathbf{q}}_{j+1}^n - \bar{\mathbf{q}}_j^n}{\Delta x}\right)} \quad \theta \in [1,2]$$

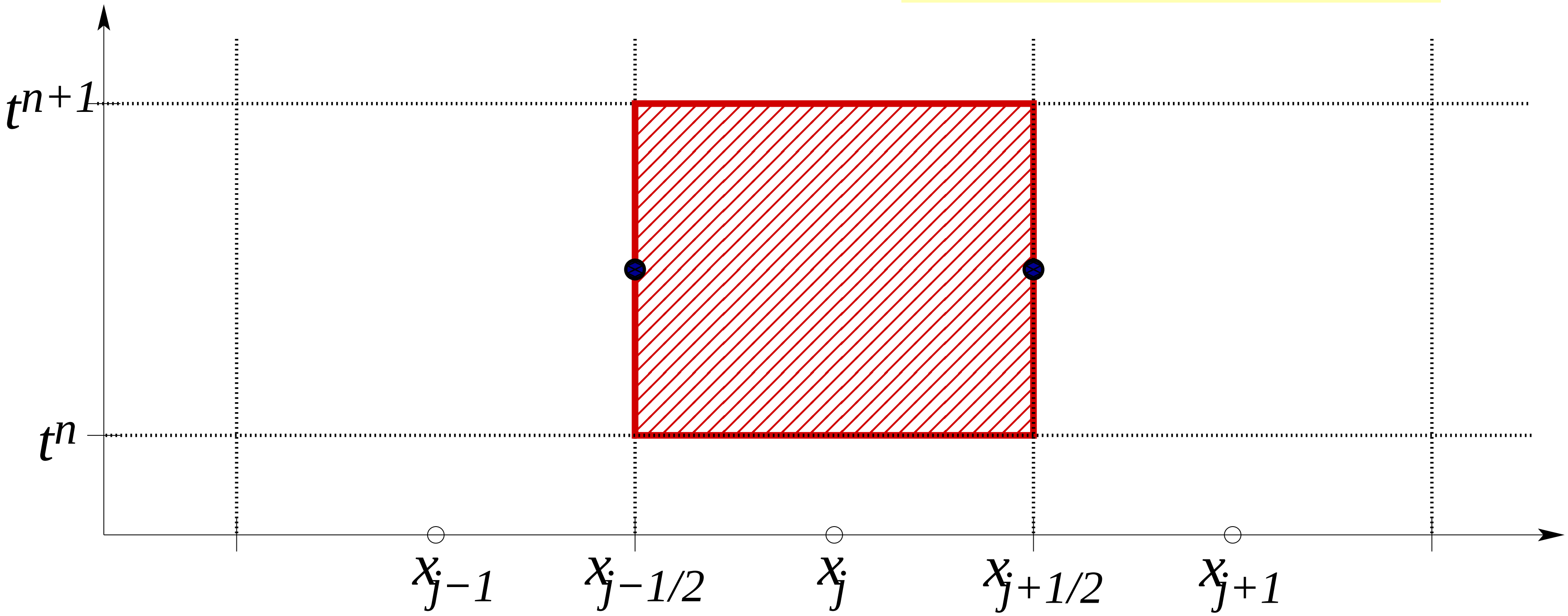
where the minmod function is defined as:

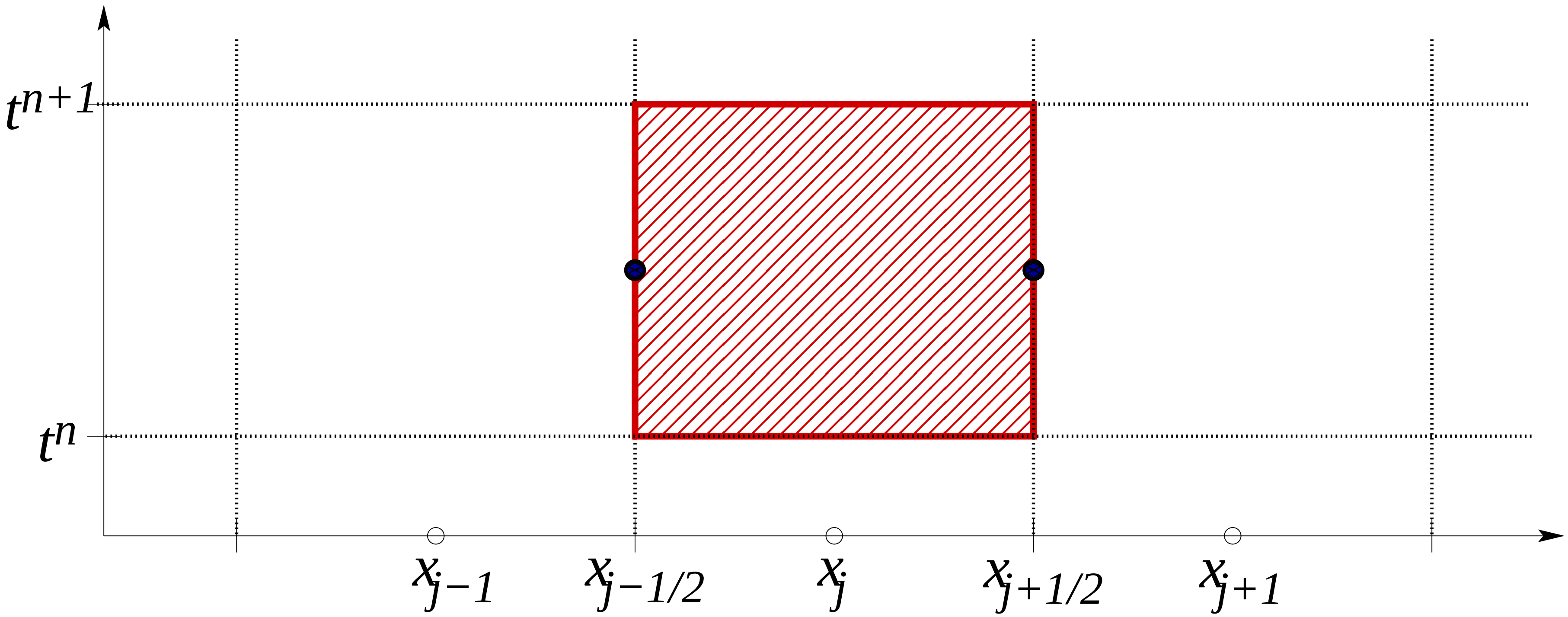
$$\boxed{\text{minmod}(z_1, z_2, ...) := \begin{cases} \min_j\{z_j\}, & \text{if} \quad z_j > 0 \quad \forall j, \\ \max_j\{z_j\}, & \text{if} \quad z_j < 0 \quad \forall j, \\ 0, & \text{otherwise.} \end{cases}}$$

Godunov-type upwind schemes are designed by integrating

$$\mathbf{q}_t + \mathbf{f}(\mathbf{q})_x = \mathbf{0}$$

over the space-time control volumes $[x_{j-\frac{1}{2}}, x_{j+\frac{1}{2}}] \times [t^n, t^{n+1}]$





$$\bar{\mathbf{q}}_j^{n+1} = \bar{\mathbf{q}}_j^n - \frac{1}{\Delta x} \int_{t^n}^{t^{n+1}} \left[\mathbf{f}(\mathbf{q}(x_{j+\frac{1}{2}}, t)) - \mathbf{f}(\mathbf{q}(x_{j-\frac{1}{2}}, t)) \right] dt$$

In order to evaluate the flux integrals on the RHS, one has to (approximately) solve the generalized Riemann problem.

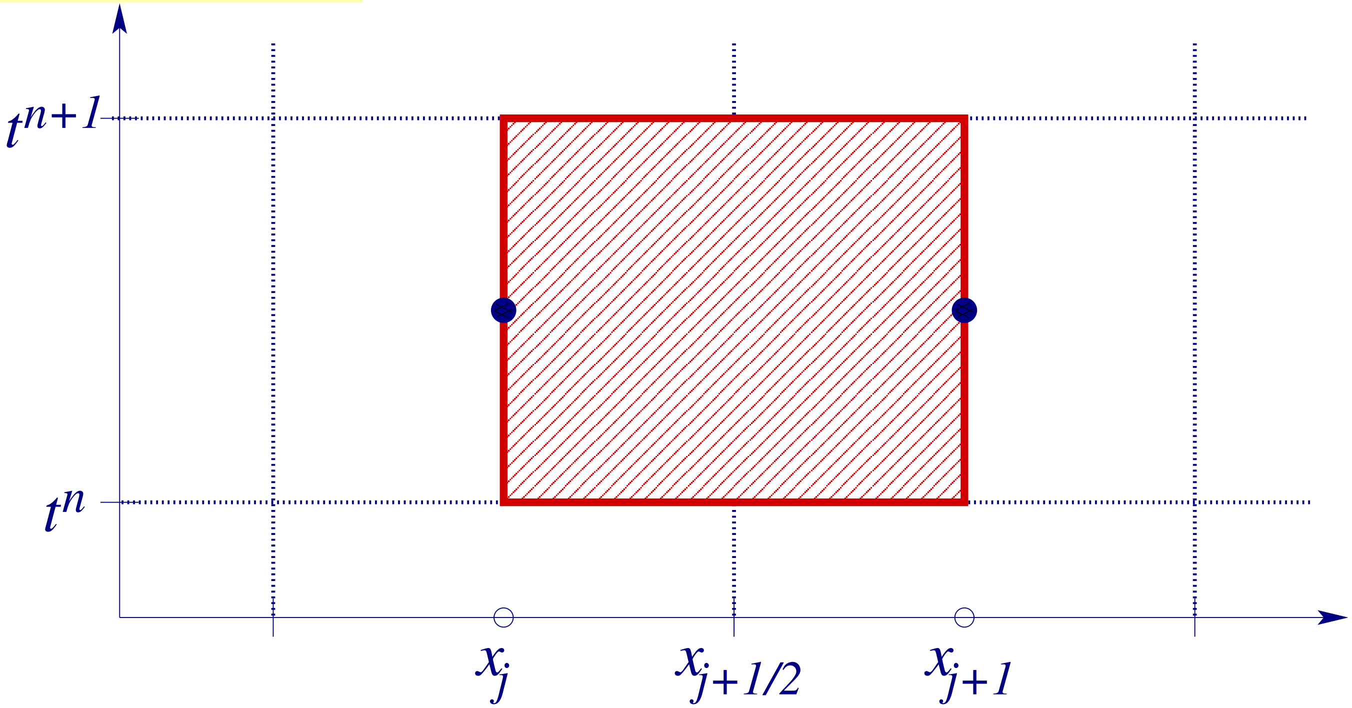
This may be hard or even impossible...

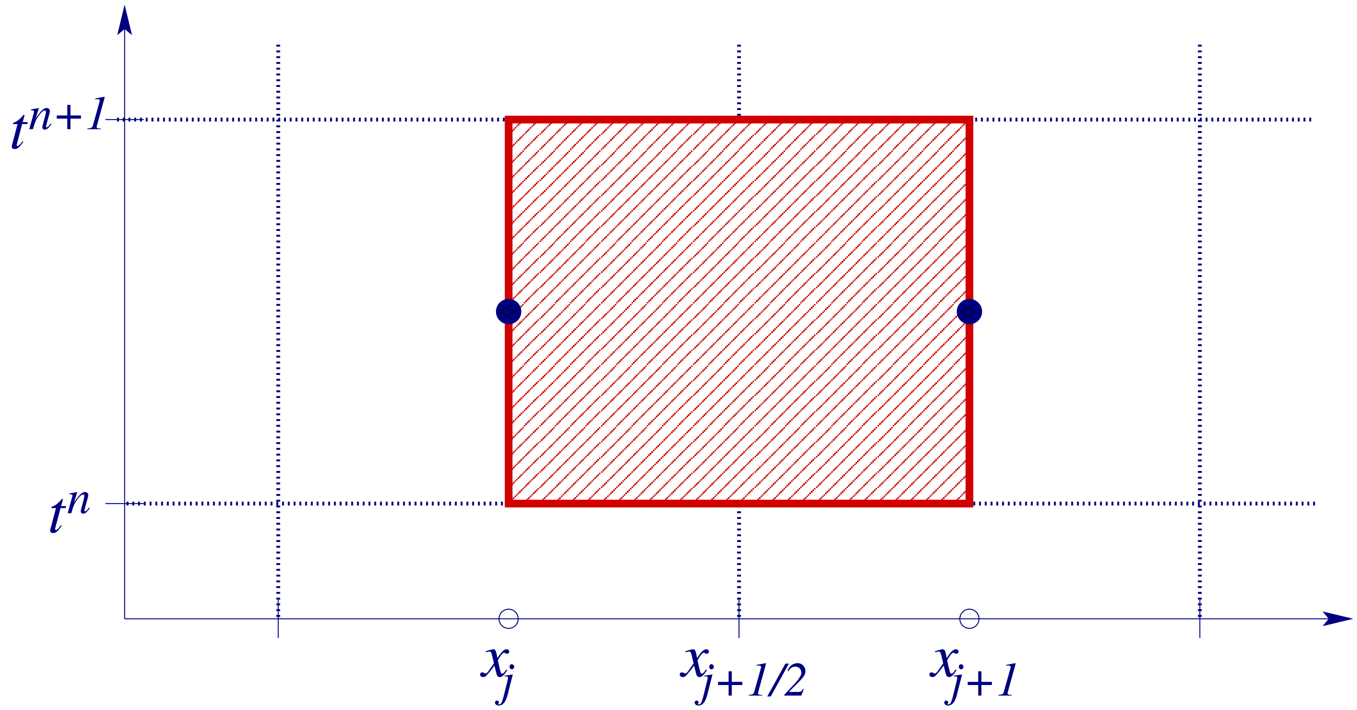
Nessyahu-Tadmor Scheme

The Nessyahu-Tadmor [Nessyahu, Tadmor; 1990] scheme is a central Godunov-type scheme. It is designed by integrating

$$\mathbf{q}_t + \mathbf{f}(\mathbf{q})_x = \mathbf{0}$$

over the different set of staggered space-time control volumes $[x_j, x_{j+1}] \times [t^n, t^{n+1}]$ containing the Riemann fans





$$\bar{\mathbf{q}}^{n+1}_{j+\frac{1}{2}} = \frac{1}{\Delta x} \int_{x_j}^{x_{j+1}} \tilde{\mathbf{q}}^n(x)\,dx - \frac{1}{\Delta x} \int_{t^n}^{t^{n+1}} \Big[\mathbf{f}(\mathbf{q}(x_{j+1},t)) - \mathbf{f}(\mathbf{q}(x_j,t))\Big] dt$$

Due to the finite speed of propagation, this can be reduced to:

$$\bar{\mathbf{q}}^{n+1}_{j+\frac{1}{2}} = \frac{\bar{\mathbf{q}}^n_j + \bar{\mathbf{q}}^n_{j+1}}{2} + \frac{\Delta x}{8}\Big((\mathbf{q}_x)^n_j - (\mathbf{q}_x)^n_{j+1}\Big) - \frac{\Delta t}{\Delta x}\Big[\mathbf{f}(\mathbf{q}^{n+\frac{1}{2}}_{j+1}) - \mathbf{f}(\mathbf{q}^{n+\frac{1}{2}}_j)\Big]$$

Values of $\mathbf{q}$ at $t = t^{n+\frac{1}{2}}$ are approximated using the Taylor expansion:

$$\mathbf{q}_j^{n+\frac{1}{2}} \approx \tilde{\mathbf{q}}^n(x_j) + \frac{\Delta t}{2}\mathbf{q}_t(x_j, t^n)$$

- $\tilde{\mathbf{q}}^n(x) = \bar{\mathbf{q}}_j^n + (\mathbf{q}_x)_j^n(x - x_j) \Longrightarrow \boxed{\tilde{\mathbf{q}}^n(x_j) = \bar{\mathbf{q}}_j^n}$

- $\boxed{\mathbf{q}_t(x_j, t^n) = -\mathbf{f}(\bar{\mathbf{q}}_j^n)_x}$

The space derivatives $\mathbf{f}_x$ are computed using the (minmod) limiter:

$$\mathbf{f}(\bar{\mathbf{q}}_j^n)_x = \text{minmod}\left(\theta\frac{\mathbf{f}(\bar{\mathbf{q}}_j^n) - \mathbf{f}(\bar{\mathbf{q}}_{j-1}^n)}{\Delta x}, \frac{\mathbf{f}(\bar{\mathbf{q}}_{j+1}^n) - \mathbf{f}(\bar{\mathbf{q}}_{j-1}^n)}{2\Delta x}, \theta\frac{\mathbf{f}(\bar{\mathbf{q}}_{j+1}^n) - \mathbf{f}(\bar{\mathbf{q}}_j^n)}{\Delta x}\right)$$

Higher-Order and Multidimensional Staggered Central Schemes

[Arminjon, Viallon, Madrane; 1997]

[Jiang, Tadmor; 1998]

[Liu, Tadmor; 1998]

[Bianco, Puppo, Russo; 1999]

[Levy, Puppo, Russo; 1999, 2000, 2002]

[Lie, Noelle; 2000]

Central-Upwind Schemes

[A.K., Tadmor; 2000]

[A.K., Petrova; 2000]

[A.K., Petrova; 2001]

[A.K., Noelle, Petrova; 2001]

[A.K., Lin; 2007]

Goal: **to reduce numerical dissipation of central schemes**

Example — Numerical Dissipation of the Staggered LxF Scheme

$$q_{j+\frac{1}{2}}^{n+1} = \frac{q_{j+1}^n + q_j^n}{2} - \frac{\Delta t}{\Delta x}\left[f(q_{j+1}^n) - f(q_j^n)\right]$$

$$q_{j+\frac{1}{2}}^{n+1} - q_{j+\frac{1}{2}}^n + \frac{\Delta t}{\Delta x}\left[f(q_{j+1}^n) - f(q_j^n)\right] = \frac{q_{j+1}^n - 2q_{j+\frac{1}{2}}^n + q_j^n}{2}$$

$$\frac{q_{j+\frac{1}{2}}^{n+1} - q_{j+\frac{1}{2}}^n}{\Delta t} + \frac{f(q_{j+1}^n) - f(q_j^n)}{\Delta x} = \boxed{\frac{(\Delta x)^2}{8\Delta t}} \cdot \frac{q_{j+1}^n - 2u_{j+\frac{1}{2}}^n + q_j^n}{(\Delta x/2)^2}$$

- As Δt decreases, the numerical dissipation increases
- As $\Delta t \sim (\Delta x)^2$, the LxF scheme is inconsistent
- As $\Delta t \to 0$, the numerical dissipation blows up

The discontinuities appearing at the reconstruction step at the interface points $\{x_{j+\frac{1}{2}}\}$ propagate at finite speeds estimated by:

$$a^{+}_{j+\frac{1}{2}} := \max\left\{\lambda_N\left(\frac{\partial \mathbf{f}}{\partial \mathbf{q}}(\mathbf{q}^{-}_{j+\frac{1}{2}})\right), \lambda_N\left(\frac{\partial \mathbf{f}}{\partial \mathbf{q}}(\mathbf{q}^{+}_{j+\frac{1}{2}})\right), 0\right\}$$

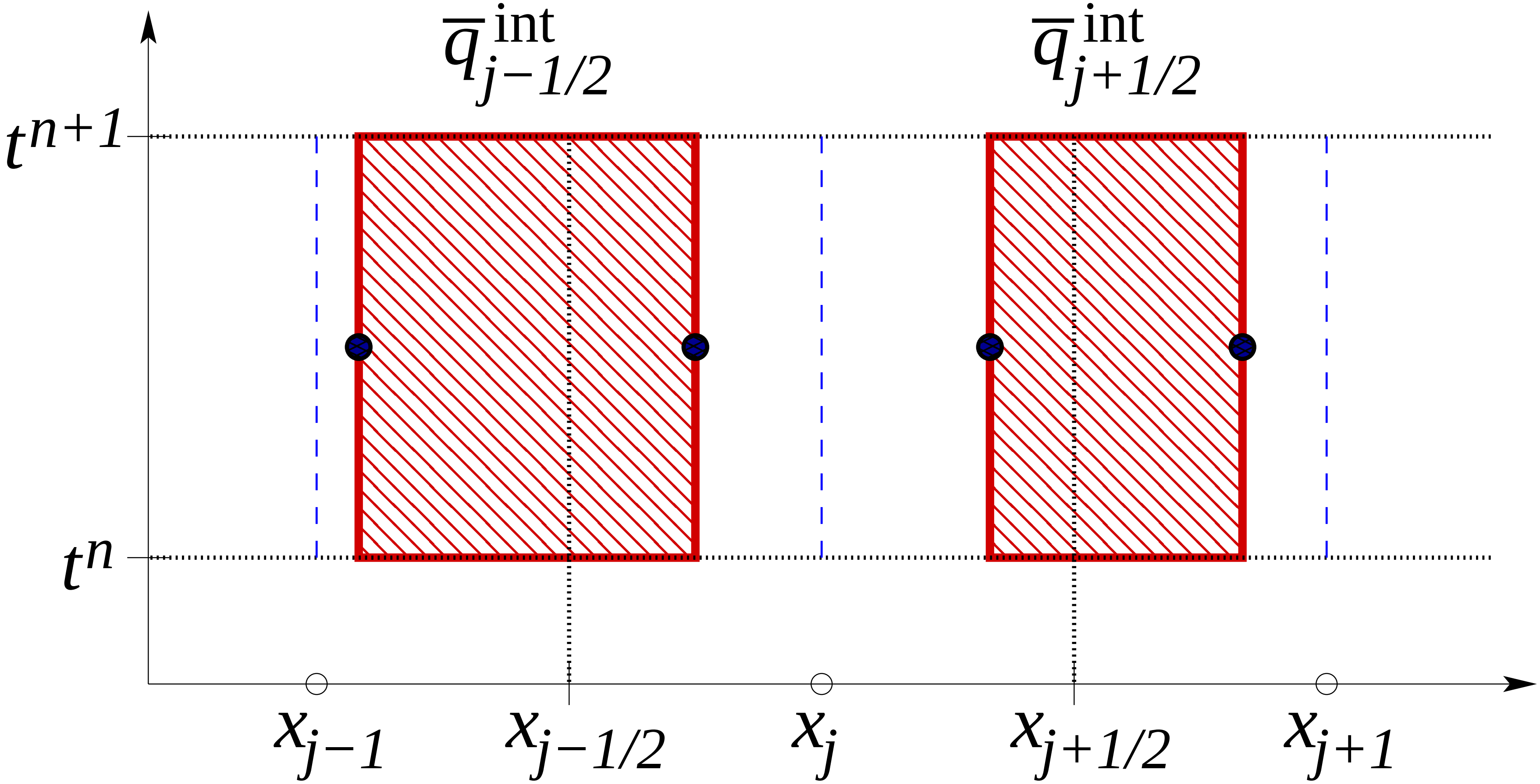
$$a^{-}_{j+\frac{1}{2}} := \min\left\{\lambda_1\left(\frac{\partial \mathbf{f}}{\partial \mathbf{q}}(\mathbf{q}^{-}_{j+\frac{1}{2}})\right), \lambda_1\left(\frac{\partial \mathbf{f}}{\partial \mathbf{q}}(\mathbf{q}^{+}_{j+\frac{1}{2}})\right), 0\right\}$$

$\lambda_1 < \lambda_2 < \ldots < \lambda_N$: N eigenvalues of the Jacobian $\frac{\partial \mathbf{f}}{\partial \mathbf{q}}$

$$\mathbf{q}^{-}_{j+\frac{1}{2}} := \lim_{x \to x_{j+\frac{1}{2}}-} \widetilde{\mathbf{q}}(x, t^n) = \bar{\mathbf{q}}^n_j + \frac{\Delta x}{2}(\mathbf{q}_x)^n_j$$

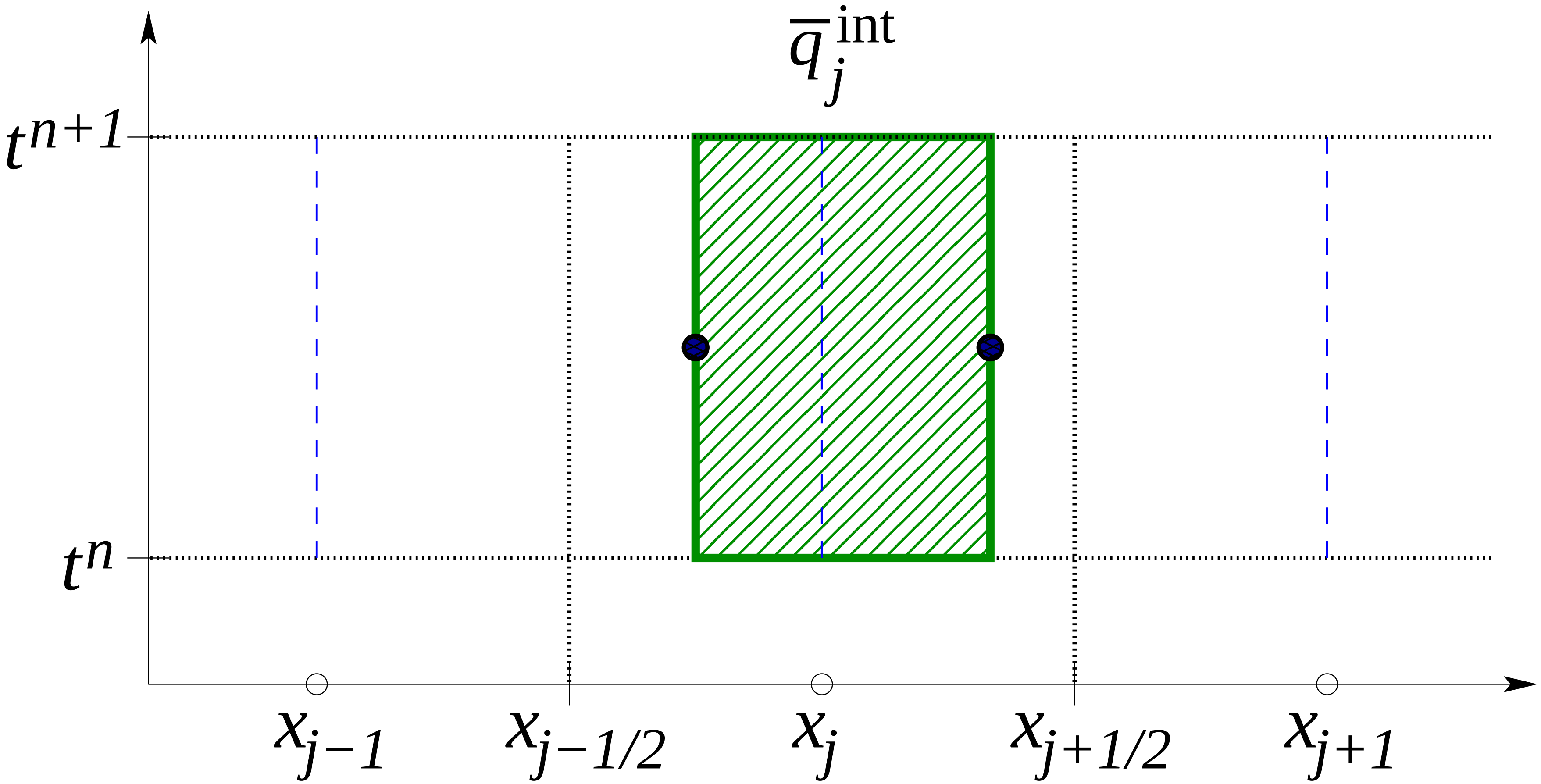
$$\mathbf{q}^{+}_{j+\frac{1}{2}} := \lim_{x \to x_{j+\frac{1}{2}}+} \widetilde{\mathbf{q}}(x, t^n) = \bar{\mathbf{q}}^n_{j+1} - \frac{\Delta x}{2}(\mathbf{q}_x)^n_{j+1}$$

Idea: **Select control volumes according to the size of each Riemann fan**



$$\left[x_{j-\frac{1}{2}} + a^-_{j-\frac{1}{2}}\Delta t, x_{j-\frac{1}{2}} + a^+_{j-\frac{1}{2}}\Delta t\right] \times [t^n, t^{n+1}]$$

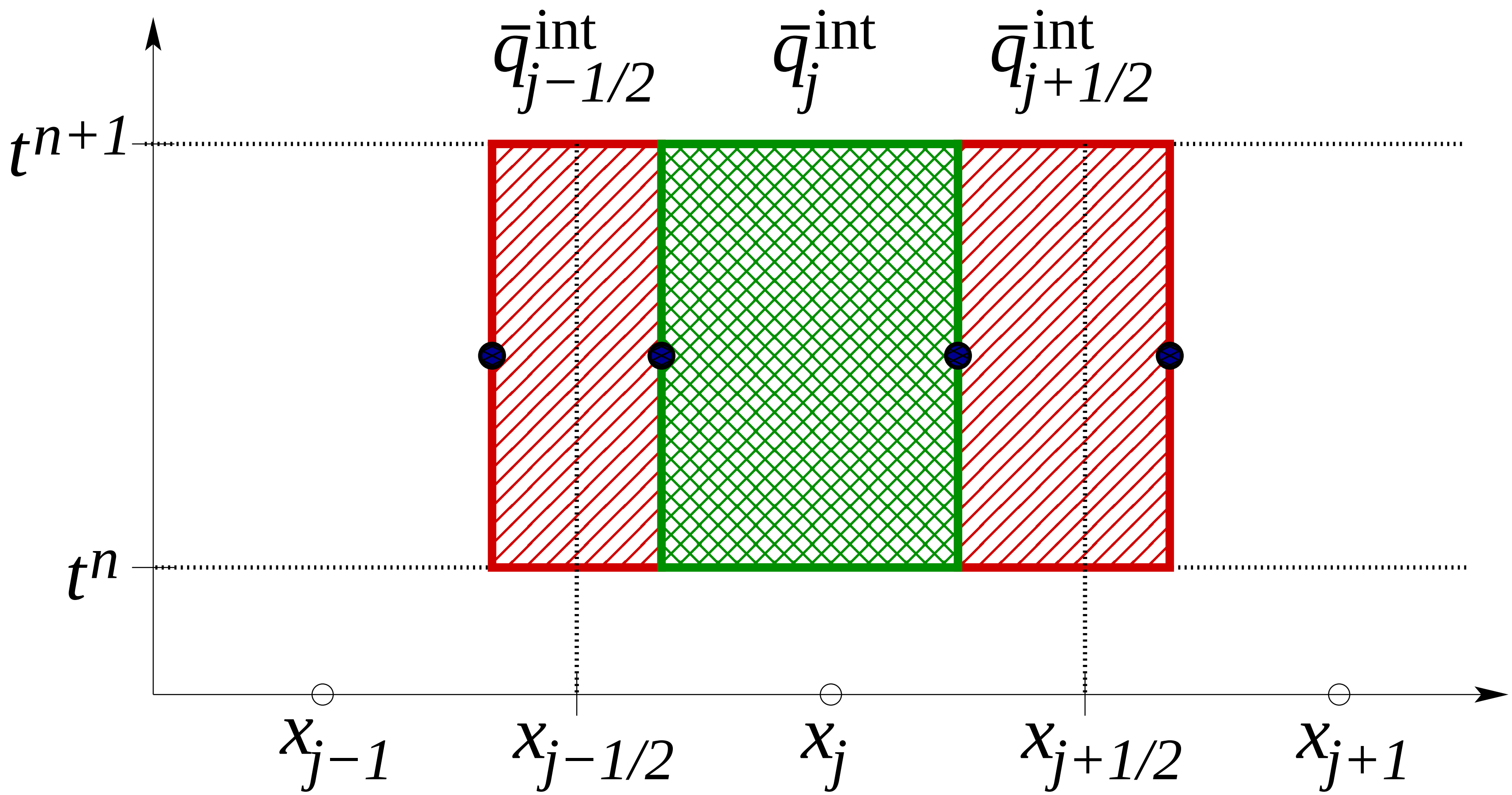
$$\left[x_{j+\frac{1}{2}} + a^-_{j+\frac{1}{2}}\Delta t, x_{j+\frac{1}{2}} + a^+_{j+\frac{1}{2}}\Delta t\right] \times [t^n, t^{n+1}]$$

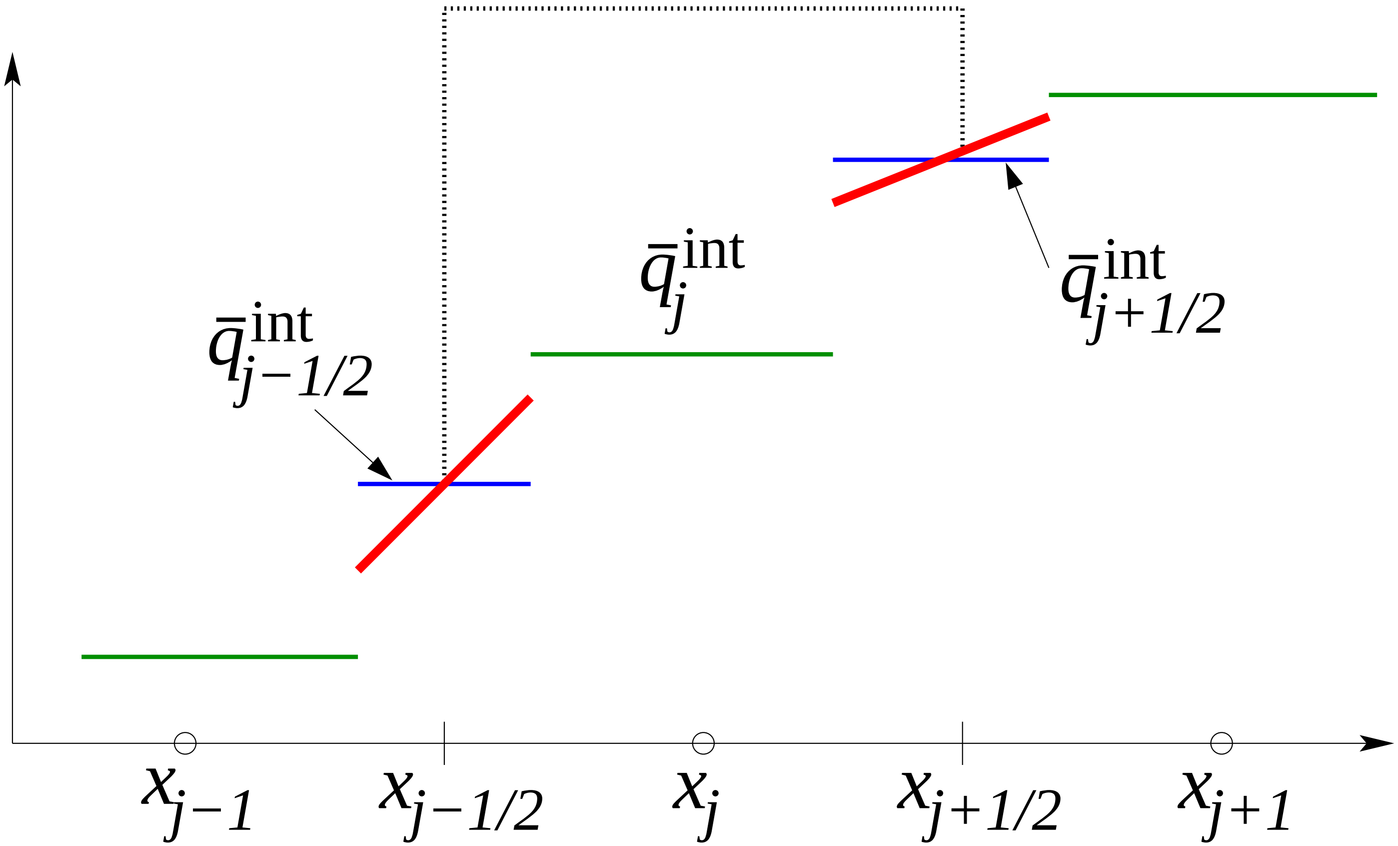


$$\left[x_{j-\frac{1}{2}} + a^+_{j-\frac{1}{2}}\Delta t, x_{j+\frac{1}{2}} + a^-_{j-\frac{1}{2}}\Delta t\right] \times [t^n, t^{n+1}]$$

Final Step: **Projection onto the Original Grid**

A piecewise linear interpolant, $\widetilde{\mathbf{q}}^{\text{int}}(x)$, reconstructed from the evolved intermediate cell averages $\{\bar{\mathbf{q}}_j^{\text{int}}\}$ and $\{\bar{\mathbf{q}}_{j+\frac{1}{2}}^{\text{int}}\}$, is projected back onto the original grid by averaging it over the intervals $[x_{j-\frac{1}{2}}, x_{j+\frac{1}{2}}]$.





New projected cell averages:

$$\bar{\mathbf{q}}_j^{n+1} = \frac{a^+_{j-\frac{1}{2}}\Delta t}{\Delta x}\bar{\mathbf{q}}^{\text{int}}_{j-\frac{1}{2}} + \left[1 + \frac{\left(a^-_{j-\frac{1}{2}} - a^+_{j+\frac{1}{2}}\right)\Delta t}{\Delta x}\right]\bar{\mathbf{q}}^{\text{int}}_j - \frac{a^-_{j+\frac{1}{2}}\Delta t}{\Delta x}\bar{\mathbf{q}}^{\text{int}}_{j+\frac{1}{2}}$$
$$+ \frac{(\Delta t)^2}{2\Delta x}\left[\boxed{\boxed{(\mathbf{q}_x)^{\text{int}}_{j+\frac{1}{2}}}}\, a^+_{j+\frac{1}{2}} a^-_{j+\frac{1}{2}} - \boxed{\boxed{(\mathbf{q}_x)^{\text{int}}_{j-\frac{1}{2}}}}\, a^+_{j-\frac{1}{2}} a^-_{j-\frac{1}{2}}\right]$$

1-D Semi-Discrete Central-Upwind Scheme

$$
\begin{aligned}
\frac{d}{dt}\bar{\mathbf{q}}_j(t^n) = \lim_{\Delta t\to 0}\frac{\bar{\mathbf{q}}_j^{n+1}-\bar{\mathbf{q}}_j^n}{\Delta t} &= \frac{a^+_{j-\frac12}}{\Delta x}\lim_{\Delta t\to 0}\bar{\mathbf{q}}^{\text{int}}_{j-\frac12} - \frac{a^-_{j+\frac12}}{\Delta x}\lim_{\Delta t\to 0}\bar{\mathbf{q}}^{\text{int}}_{j+\frac12} \\
&+\frac{a^-_{j-\frac12}-a^+_{j+\frac12}}{\Delta x}\lim_{\Delta t\to 0}\bar{\mathbf{q}}^{\text{int}}_j + \lim_{\Delta t\to 0}\left\{\frac{\bar{\mathbf{q}}^{\text{int}}_j-\bar{\mathbf{q}}^n_j}{\Delta t}\right\} \\
&+\frac{1}{2\Delta x}\lim_{\Delta t\to 0}\left[\Delta t\left((\mathbf{q}_x)^{\text{int}}_{j+\frac12}a^+_{j+\frac12}a^-_{j+\frac12}-(\mathbf{q}_x)^{\text{int}}_{j-\frac12}a^+_{j-\frac12}a^-_{j-\frac12}\right)\right]
\end{aligned}
$$

We then substitute $\mathbf{q}^{\text{int}}_{j\pm\frac12}$, $\mathbf{q}^{\text{int}}_j$ and $(\mathbf{q}_x)^{\text{int}}_{j\pm\frac12}$ into here to obtain the 1-D semi-discrete central-upwind scheme

(for details see [A.K., Lin; 2007])

$$\frac{d}{dt}\bar{\mathbf{q}}_j(t) = -\frac{\mathbf{H}_{j+\frac{1}{2}}(t) - \mathbf{H}_{j-\frac{1}{2}}(t)}{\Delta x}$$

The central-upwind numerical flux is:

$$\mathbf{H}_{j+\frac{1}{2}} = \frac{a^+_{j+\frac{1}{2}}\mathbf{f}(\mathbf{q}^-_{j+\frac{1}{2}}) - a^-_{j+\frac{1}{2}}\mathbf{f}(\mathbf{q}^+_{j+\frac{1}{2}})}{a^+_{j+\frac{1}{2}} - a^-_{j+\frac{1}{2}}} + \frac{a^+_{j+\frac{1}{2}}a^-_{j+\frac{1}{2}}}{a^+_{j+\frac{1}{2}} - a^-_{j+\frac{1}{2}}}\left(\mathbf{q}^+_{j+\frac{1}{2}} - \mathbf{q}^-_{j+\frac{1}{2}}\right) - \boxed{\mathbf{d}_{j+\frac{1}{2}}}$$

The built-in "anti-diffusion" term is:

$$\mathbf{d}_{j+\frac{1}{2}} = \frac{a^+_{j+\frac{1}{2}}a^-_{j+\frac{1}{2}}}{a^+_{j+\frac{1}{2}} - a^-_{j+\frac{1}{2}}}\ \text{minmod}\left(\mathbf{q}^+_{j+\frac{1}{2}} - \mathbf{q}^*_{j+\frac{1}{2}}, \mathbf{q}^*_{j+\frac{1}{2}} - \mathbf{q}^-_{j+\frac{1}{2}}\right)$$

The intermediate values $\mathbf{q}^*_{j+\frac{1}{2}}$ are:

$$\mathbf{q}^*_{j+\frac{1}{2}} = \frac{a^+_{j+\frac{1}{2}}\mathbf{q}^+_{j+\frac{1}{2}} - a^-_{j+\frac{1}{2}}\mathbf{q}^-_{j+\frac{1}{2}} - \left\{\mathbf{f}(\mathbf{q}^+_{j+\frac{1}{2}}) - \mathbf{f}(\mathbf{q}^-_{j+\frac{1}{2}})\right\}}{a^+_{j+\frac{1}{2}} - a^-_{j+\frac{1}{2}}}$$

Remarks

1. $\mathbf{d}_{j+\frac{1}{2}} \equiv 0$ corresponds to the original central-upwind scheme from [A.K., Noelle, Petrova; 2001]

$\mathbf{d}_{j+\frac{1}{2}} \equiv 0$ and $a^{+}_{j+\frac{1}{2}} \equiv -a^{-}_{j+\frac{1}{2}}$ correspond to the scheme from [A.K., Tadmor; 2000]

2. No additional flux function evaluations are required in computing the anti-diffusion term $\mathbf{d}_{j+\frac{1}{2}}$.

3. In the scalar case, the numerical flux is monotone provided $f \in C^2$ is convex and satisfies two technical assumptions

4. The (formal) order of the scheme is determined only by the order of the piecewise polynomial reconstruction $\widetilde{\mathbf{u}}$, used to compute the values $\mathbf{u}^{\pm}_{j+\frac{1}{2}}$, and the order of the ODE solver

Gas Dynamics

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho u \\ E\end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix}\rho u \\ \rho u^2 + p \\ u(E+p)\end{bmatrix} = 0$$

$p = (\gamma - 1)\left[E - \frac{\rho u^2}{2}\right]$: equation of state

ρ: density

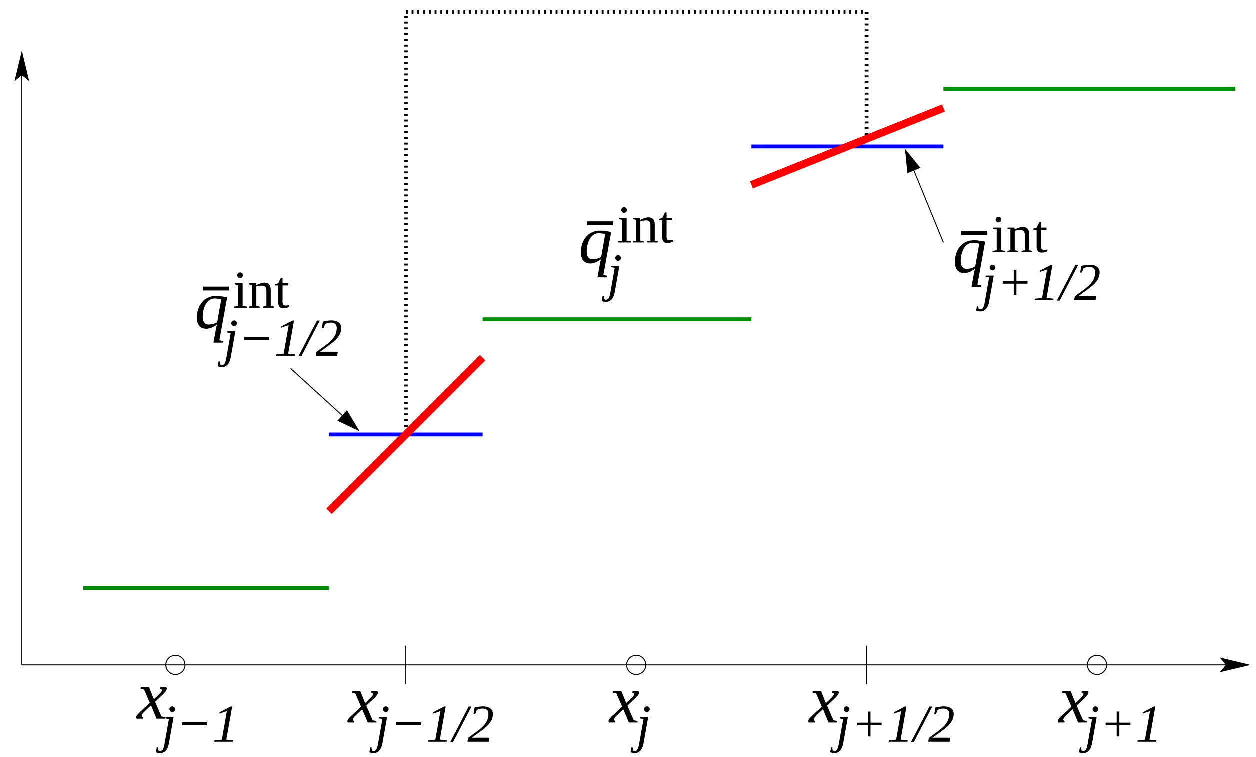
u: velocity

p: pressure

E: total energy

$\gamma = 1.4$

Idea: **Modify the projection step**



- The solution is averaged over the Riemann fans

- All conservative variables remain continuous in the cell $(x^{\ell}_{j+\frac{1}{2}}, x^{r}_{j+\frac{1}{2}})$

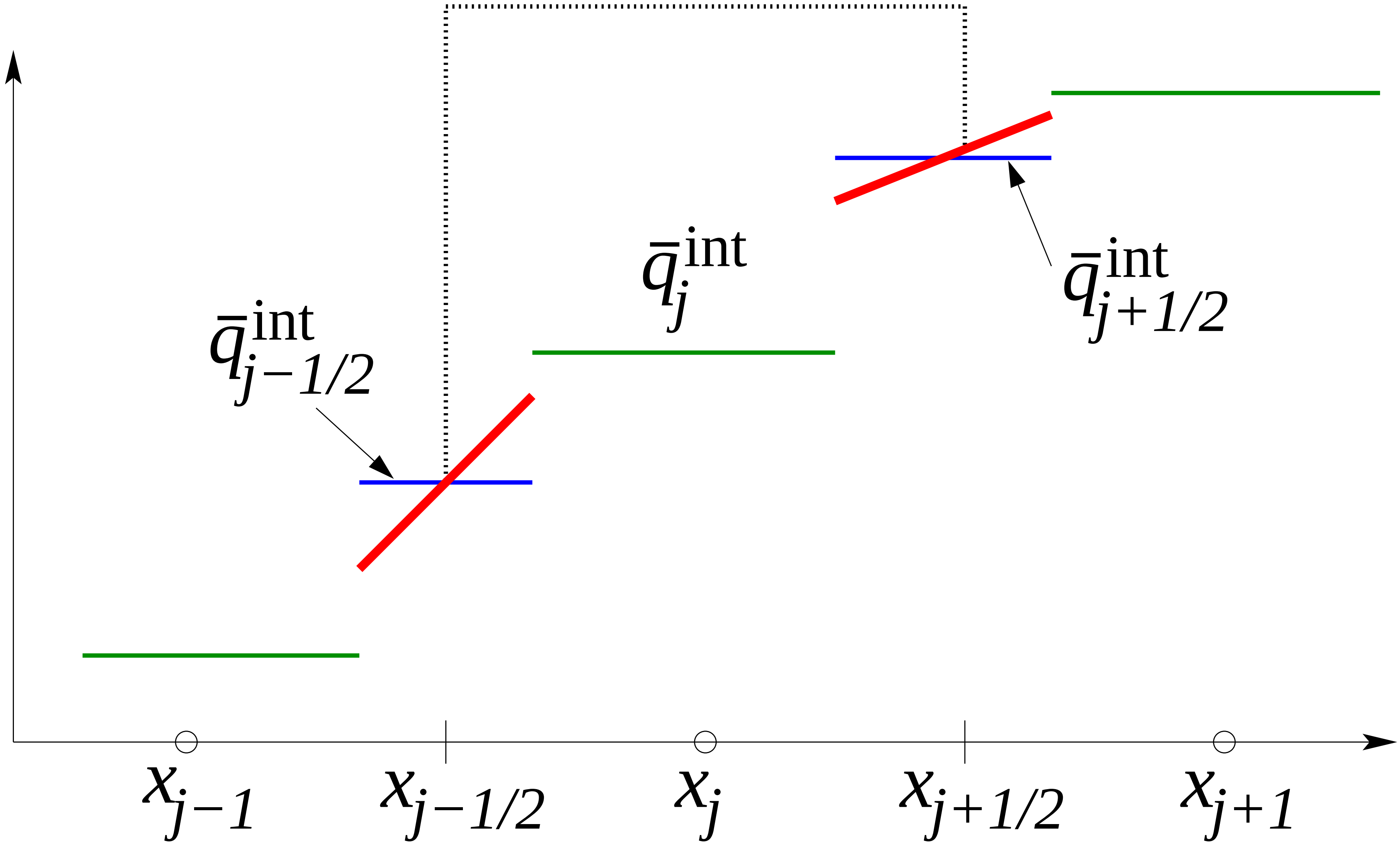
This brings excessive numerical dissipation!

In principle, a better approximation of $\mathbf{q}^{\text{int}}$ in the cell $(x^{\ell}_{j+\frac{1}{2}}, x^{r}_{j+\frac{1}{2}})$ can be obtained by incorporating the wave propagation information into the interpolant. However, this approach will require the solution of the (generalized) Riemann problem.

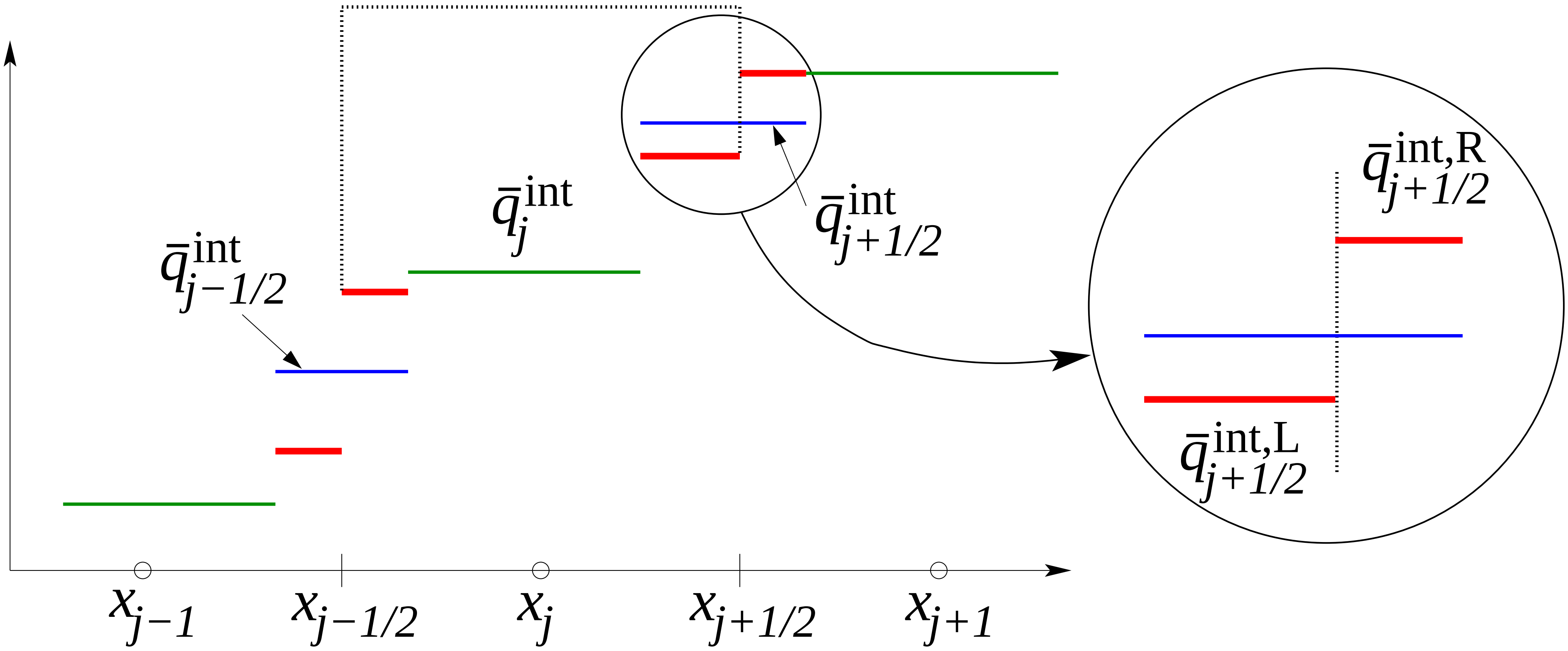
Alternatively, recall that our goal is to derive a semi-discrete scheme, that is, to pass to the $\Delta t \to 0$ limit, in which case the interval $(x^{\ell}_{j+\frac{1}{2}}, x^{r}_{j+\frac{1}{2}})$ shrinks into a point where $\mathbf{q}^{\text{int}}$ may have at most two one-sided values.

We therefore replace the intermediate piece $\bar{\mathbf{q}}^{\text{int}}_{j+\frac{1}{2}}$ with two constant pieces, $\bar{\mathbf{q}}^{\text{int,L}}_{j+\frac{1}{2}}$ and $\bar{\mathbf{q}}^{\text{int,R}}_{j+\frac{1}{2}}$.

That is, instead of



we perform the projection as follows:



For the compressible Euler equations:

$$\bar{\mathbf{q}}^{\text{int,L}}_{j+\frac{1}{2}} = \left(\bar{\rho}^{\text{int,L}}_{j+\frac{1}{2}}, \bar{m}^{\text{int,L}}_{j+\frac{1}{2}}, \bar{E}^{\text{int,L}}_{j+\frac{1}{2}}\right)^T \text{ and } \bar{\mathbf{q}}^{\text{int,R}}_{j+\frac{1}{2}} = \left(\bar{\rho}^{\text{int,R}}_{j+\frac{1}{2}}, \bar{m}^{\text{int,R}}_{j+\frac{1}{2}}, \bar{E}^{\text{int,R}}_{j+\frac{1}{2}}\right)^T$$

represent six pieces of information, which can be used to adjust the projection step.

For instance, one can enforce continuity of the velocity and pressure (which are continuous across contact discontinuities!) by setting

$$u^{\text{int,L}}_{j+\frac{1}{2}} = u^{\text{int,R}}_{j+\frac{1}{2}}, \qquad p^{\text{int,L}}_{j+\frac{1}{2}} = p^{\text{int,R}}_{j+\frac{1}{2}}$$

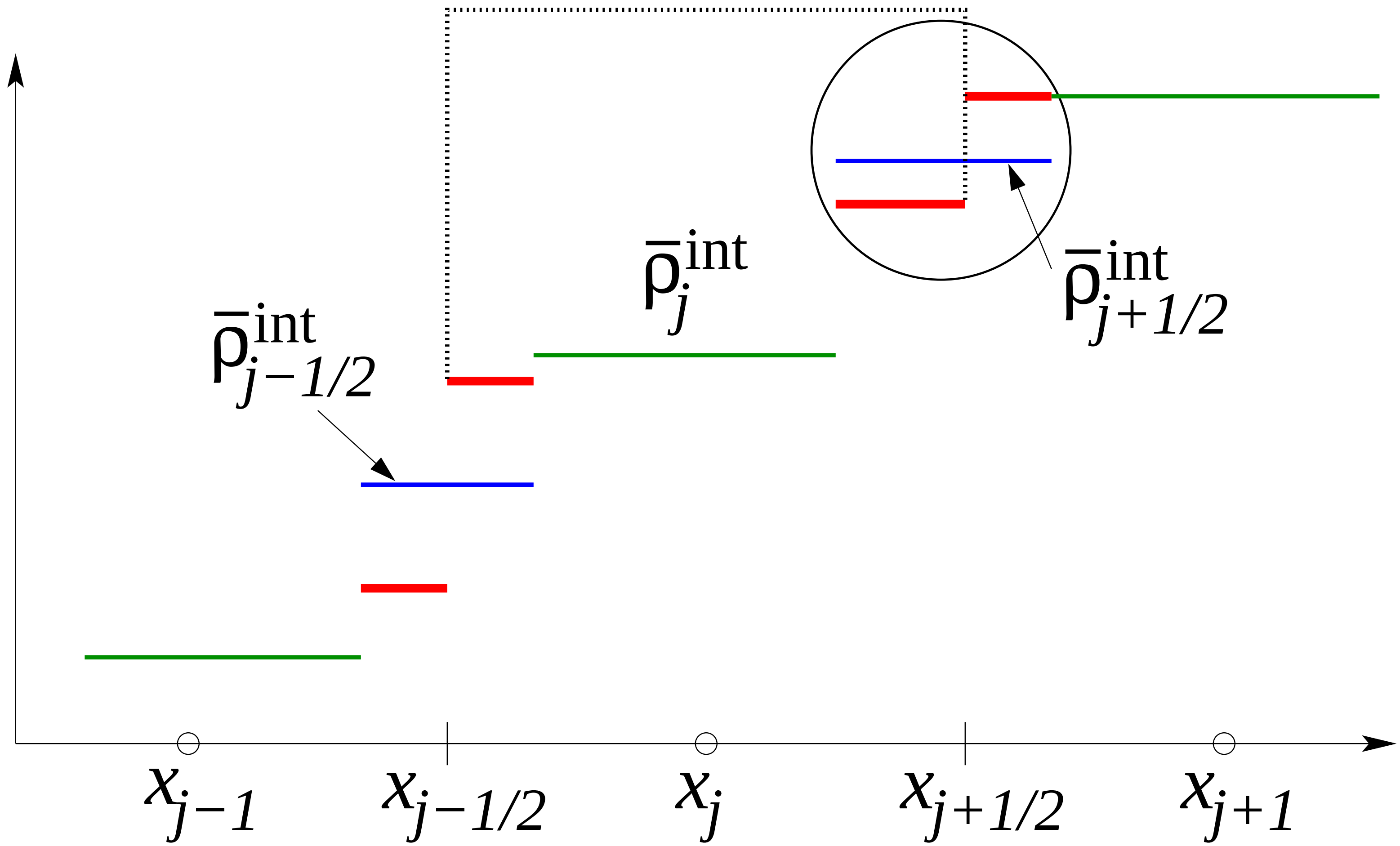
This together with three conservation requirements,

$$\bar{\mathbf{q}}^{\text{int,R}}_{j+\frac{1}{2}} a^{+}_{j+\frac{1}{2}} - \bar{\mathbf{q}}^{\text{int,L}}_{j+\frac{1}{2}} a^{-}_{j+\frac{1}{2}} = \bar{\mathbf{q}}^{\text{int}}_{j+\frac{1}{2}} (a^{+}_{j+\frac{1}{2}} - a^{-}_{j+\frac{1}{2}})$$

result in five equations to be satisfied.

The remaining degree of freedom can be used for obtaining a sharper approximation of $\mathbf{q}^{\text{int}}$.

For example, one can make the value of $\bar{\rho}^{\text{int,R}}_{j+\frac{1}{2}} - \bar{\rho}^{\text{int,L}}_{j+\frac{1}{2}}$ as close as possible to $\bar{\rho}^{\text{int}}_{j+1} - \bar{\rho}^{\text{int}}_{j}$ without sacrificing the monotonicity of ρ:



The new projection procedure leads to the same semi-discrete central-upwind scheme

$$\frac{d}{dt}\bar{\mathbf{q}}_j(t) = -\frac{\mathbf{H}_{j+\frac{1}{2}}(t) - \mathbf{H}_{j-\frac{1}{2}}(t)}{\Delta x}$$

$$\mathbf{H}_{j+\frac{1}{2}} = \frac{a^+_{j+\frac{1}{2}}\mathbf{f}(\mathbf{q}^-_{j+\frac{1}{2}}) - a^-_{j+\frac{1}{2}}\mathbf{f}(\mathbf{q}^+_{j+\frac{1}{2}})}{a^+_{j+\frac{1}{2}} - a^-_{j+\frac{1}{2}}} + \frac{a^+_{j+\frac{1}{2}}a^-_{j+\frac{1}{2}}}{a^+_{j+\frac{1}{2}} - a^-_{j+\frac{1}{2}}}\left(\mathbf{q}^+_{j+\frac{1}{2}} - \mathbf{q}^-_{j+\frac{1}{2}}\right) - \boxed{\mathbf{d}_{j+\frac{1}{2}}}$$

but with the modified "anti-diffusion" term:

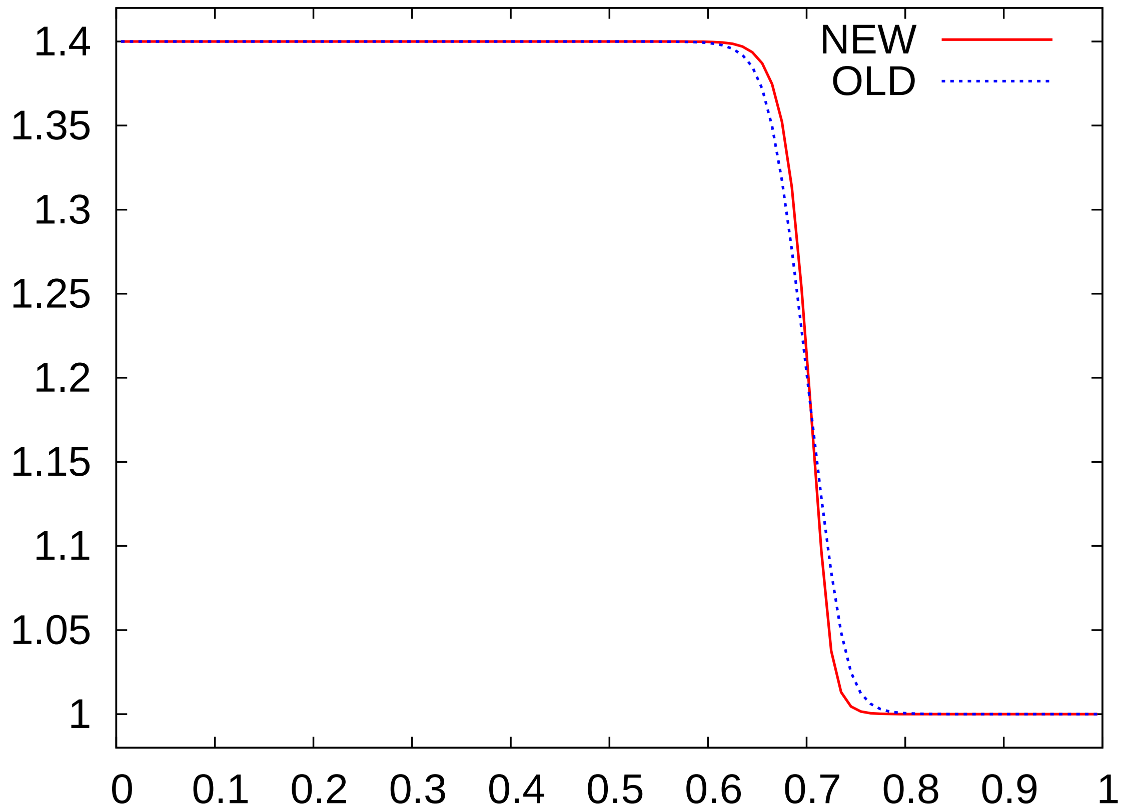
$$\mathbf{d}_{j+\frac{1}{2}} = -\text{minmod}\left(a^+_{j+\frac{1}{2}}(\rho^+_{j+\frac{1}{2}} - \rho^*_{j+\frac{1}{2}}), -a^-_{j+\frac{1}{2}}(\rho^*_{j+\frac{1}{2}} - \rho^-_{j+\frac{1}{2}})\right)\begin{pmatrix} 1 \\ u^*_{j+\frac{1}{2}} \\ \frac{(u^*_{j+\frac{1}{2}})^2}{2} \end{pmatrix}$$

$$\mathbf{q}^*_{j+\frac{1}{2}} = \frac{a^+_{j+\frac{1}{2}}\mathbf{q}^+_{j+\frac{1}{2}} - a^-_{j+\frac{1}{2}}\mathbf{q}^-_{j+\frac{1}{2}} - \left\{\mathbf{f}(\mathbf{q}^+_{j+\frac{1}{2}}) - \mathbf{f}(\mathbf{q}^-_{j+\frac{1}{2}})\right\}}{a^+_{j+\frac{1}{2}} - a^-_{j+\frac{1}{2}}}$$

Example — Moving Contact Wave

Initial data:

$$(\rho, u, p)(x, 0) = \begin{cases} (1.4, 0.1, 1), & x < 0.3 \\ (1.0, 0.1, 1), & x > 0.3 \end{cases}$$



Example — Stationary Contact Wave, Traveling Shock and Rarefaction

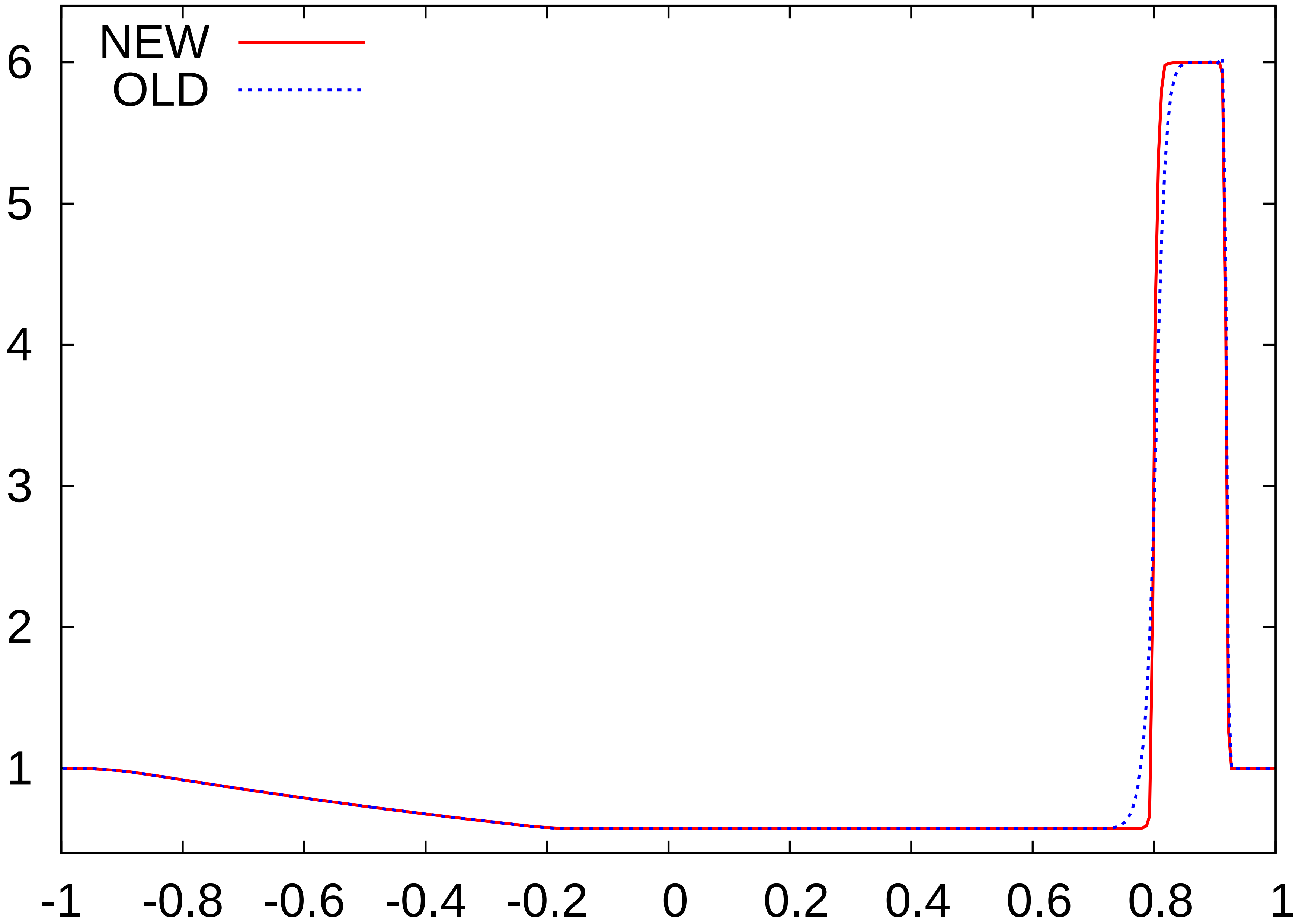
$$\frac{\partial}{\partial t}\begin{bmatrix}\rho\\ \rho u\\ E\end{bmatrix}+\frac{\partial}{\partial x}\begin{bmatrix}\rho u\\ \rho u^2+p\\ u(E+p)\end{bmatrix}=0$$

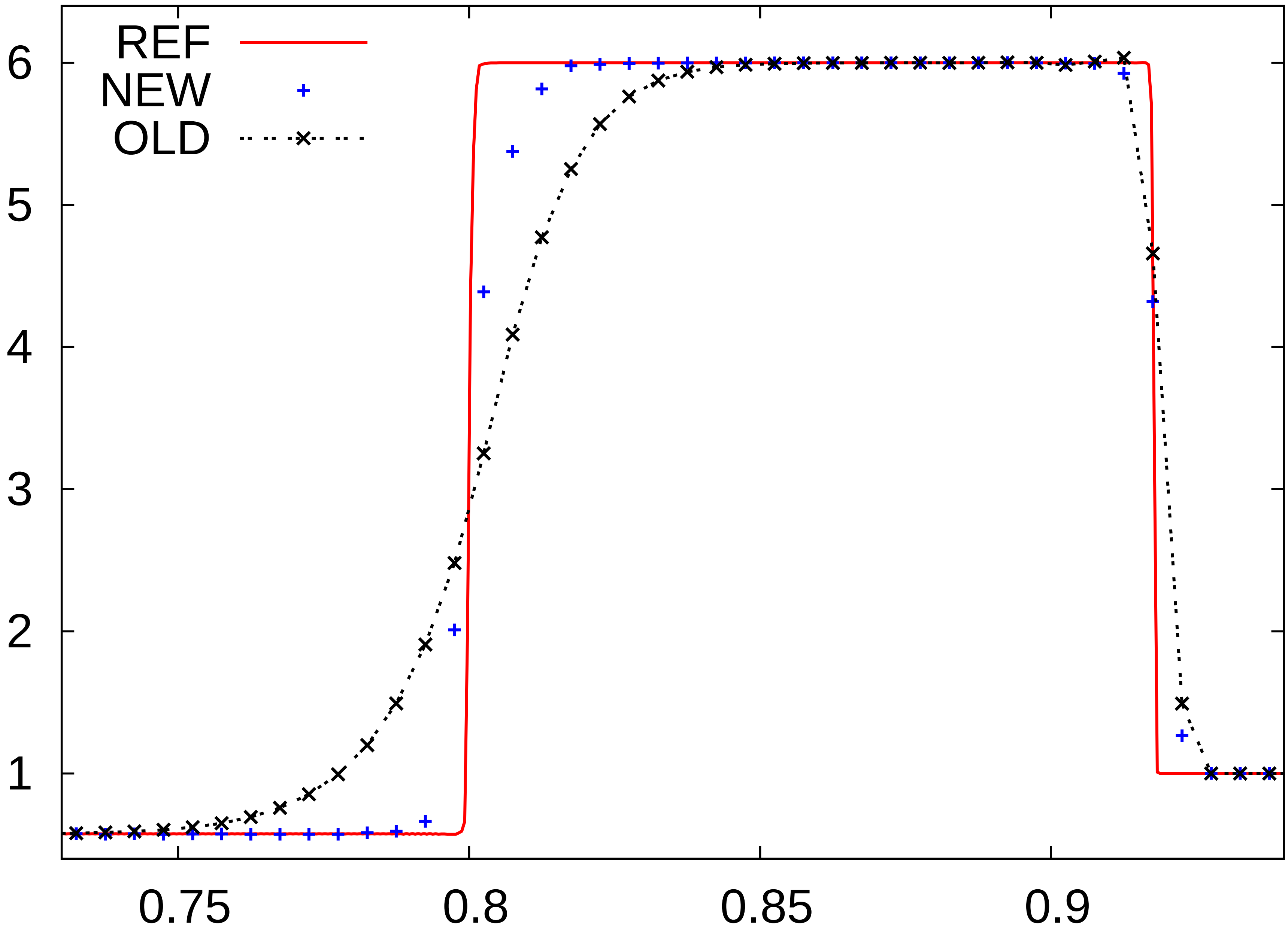
$$p=(\gamma-1)\left[E-\frac{\rho u^2}{2}\right]$$

Initial data:

$$(\rho,u,p)(x,0)=\begin{cases}(1,-19.59745,1000), & x<0.8\\ (1,-19.59745,0.01), & x>0.8\end{cases}$$

Final time: $t=0.03$


NEW
OLD
6
5
4
3
2
1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


REF
NEW
OLD
6
5
4
3
2
1
0.75
0.8
0.85
0.9

Example — "Shock-Bubble" Interaction

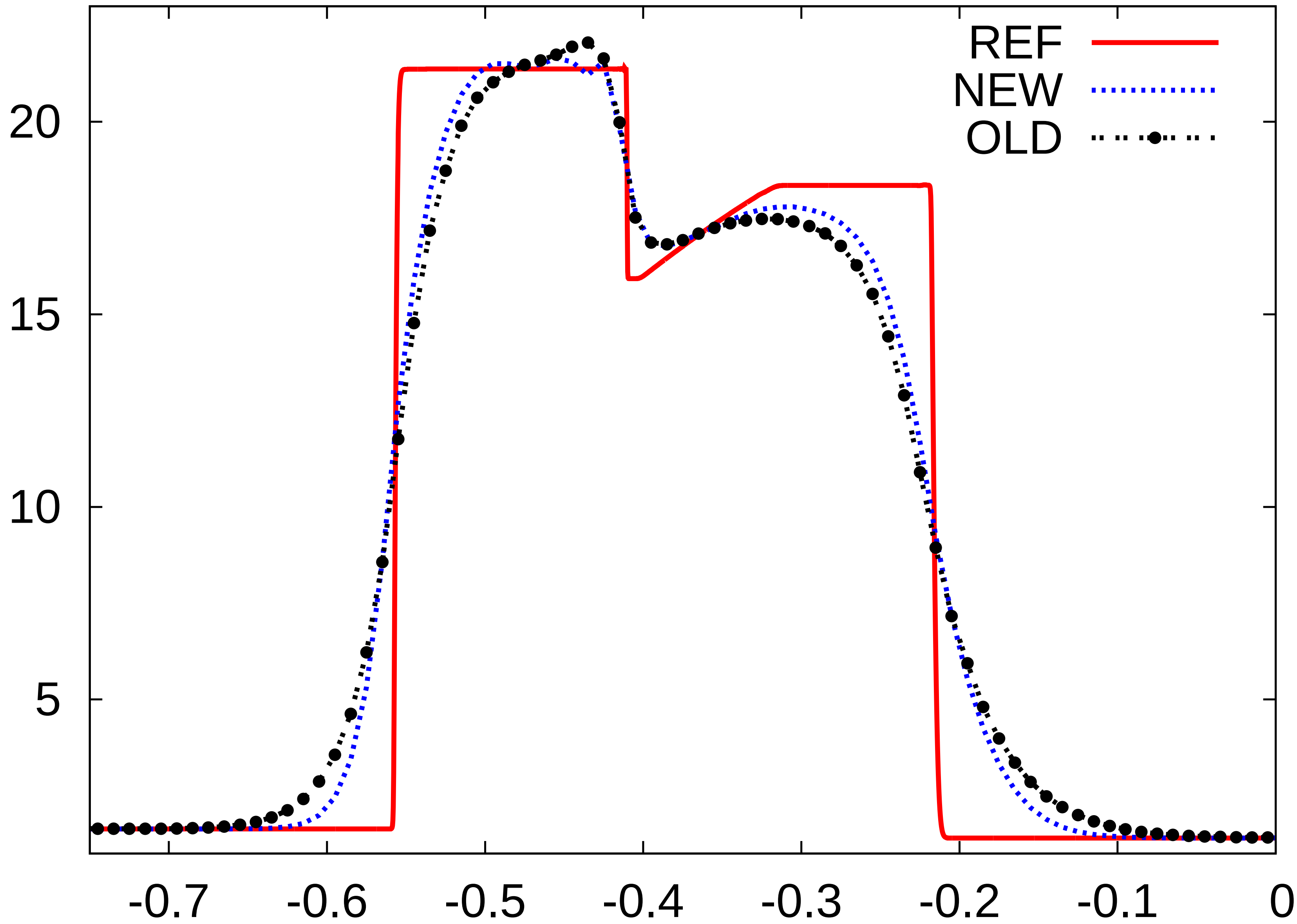
The initial data correspond to a left-moving shock, initially located at $x = 0.75$, and a "bubble" with radius 0.25, initially located at the origin:

$$(\rho, u, p)(x, y, 0) = \begin{cases} (13.1538, 0, 1), & |x| < 0.25 \\ (1.3333, -0.3535, 1.5), & x > 0.75 \\ (1, 0, 1), & \text{otherwise} \end{cases}$$

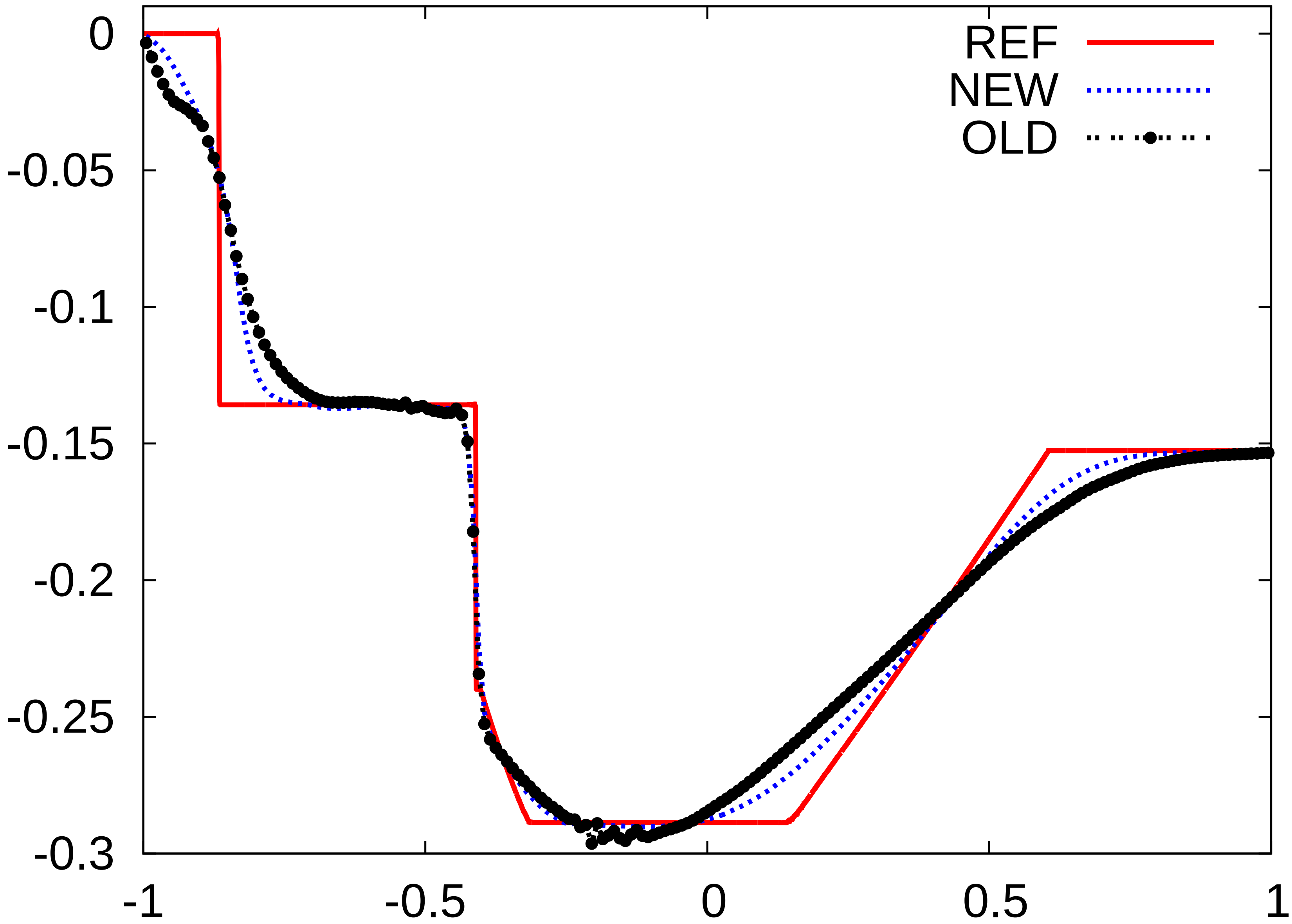
Solid wall boundary condition on the right.

Final time $t = 3$.

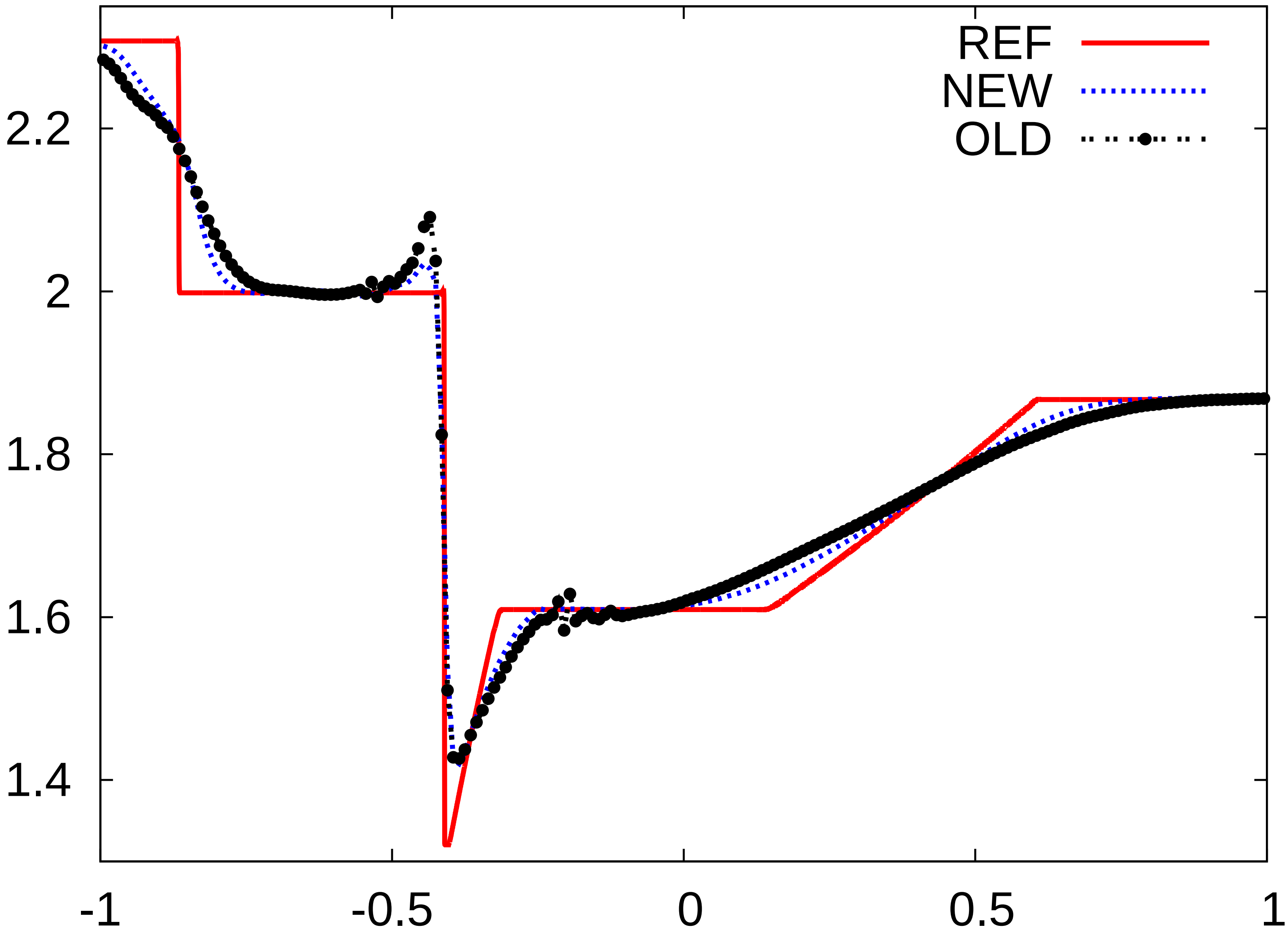
Density, $\Delta x = 1/100$



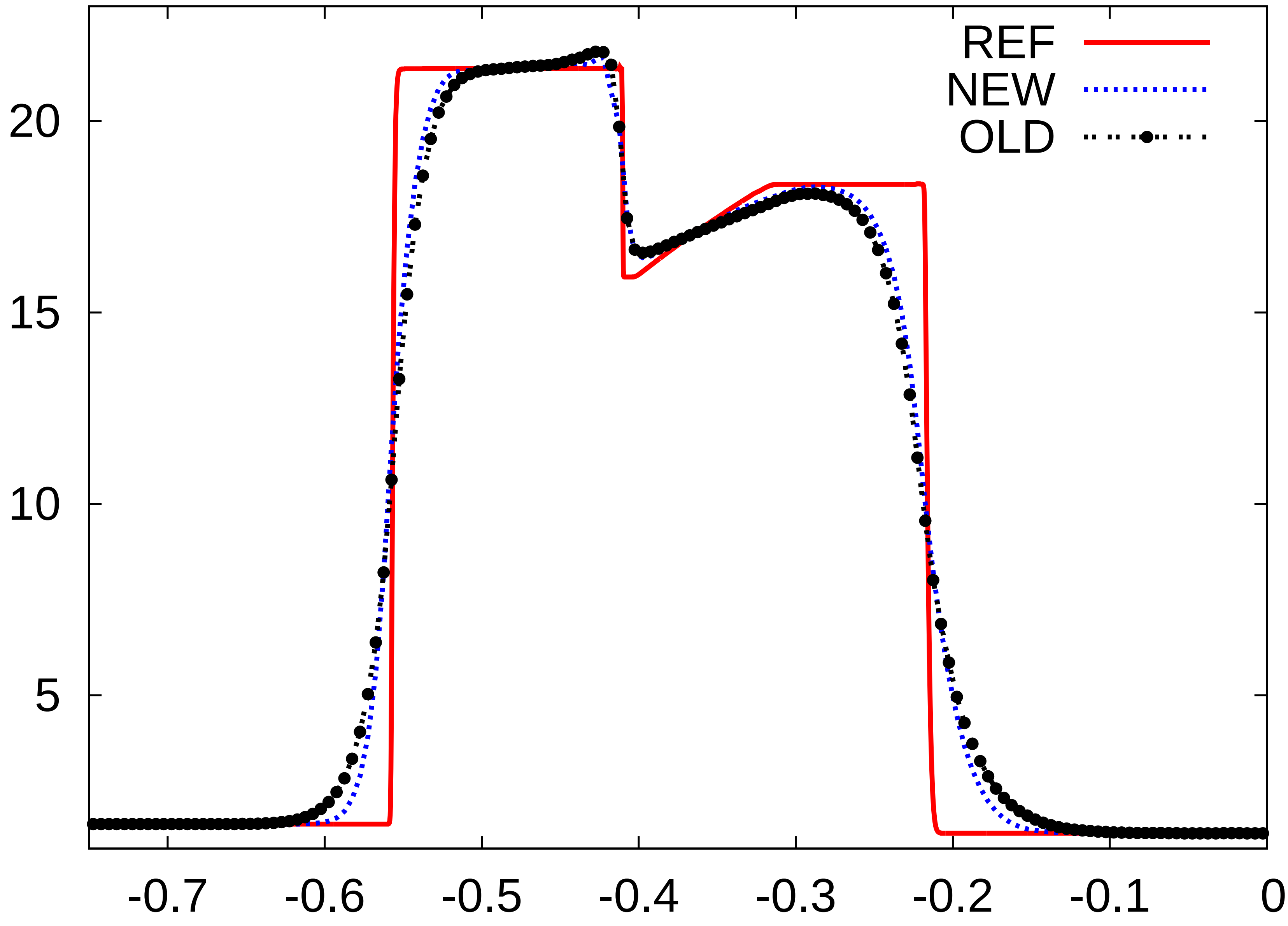
Velocity, $\Delta x = 1/100$



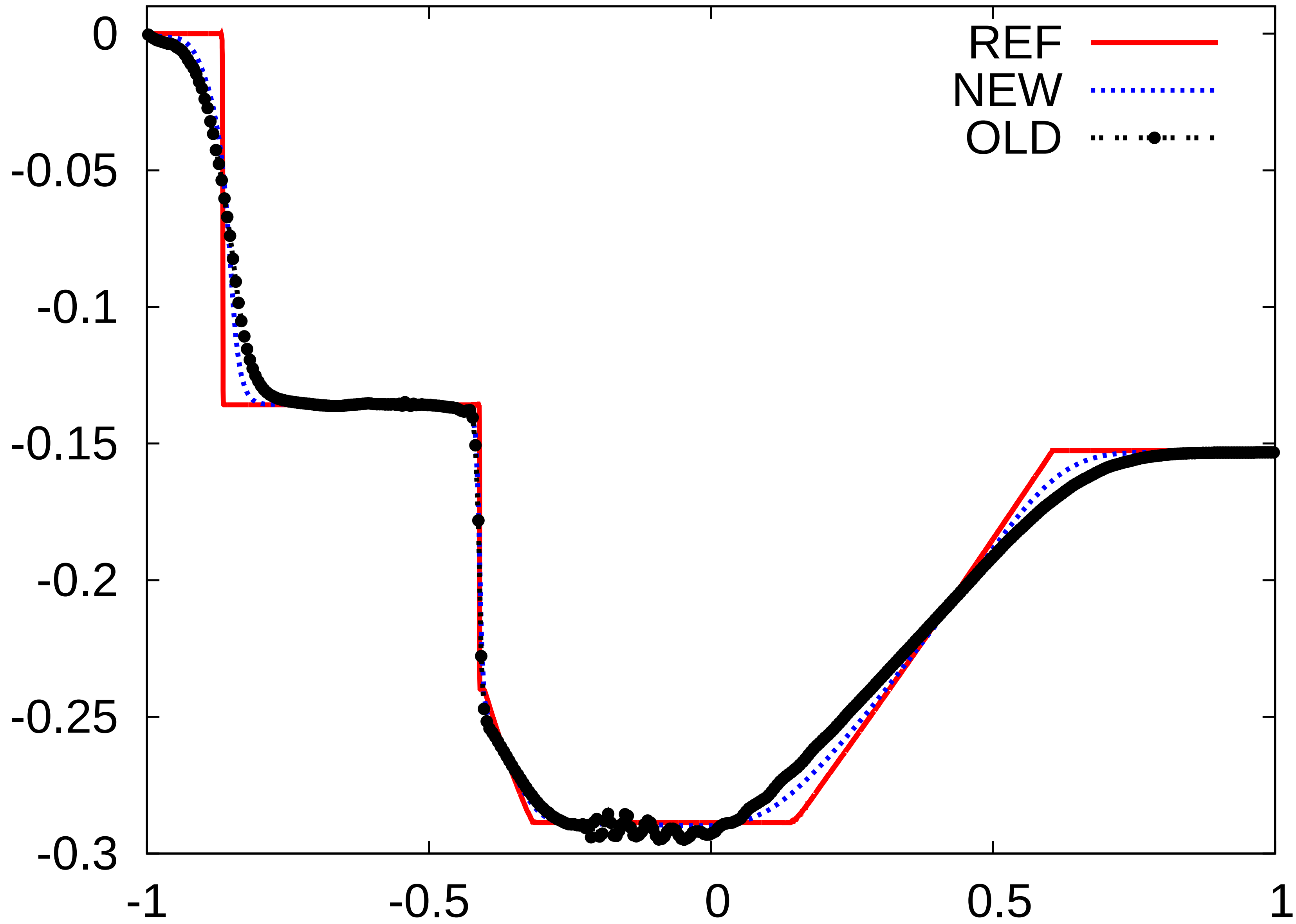
Pressure, $\Delta x = 1/100$



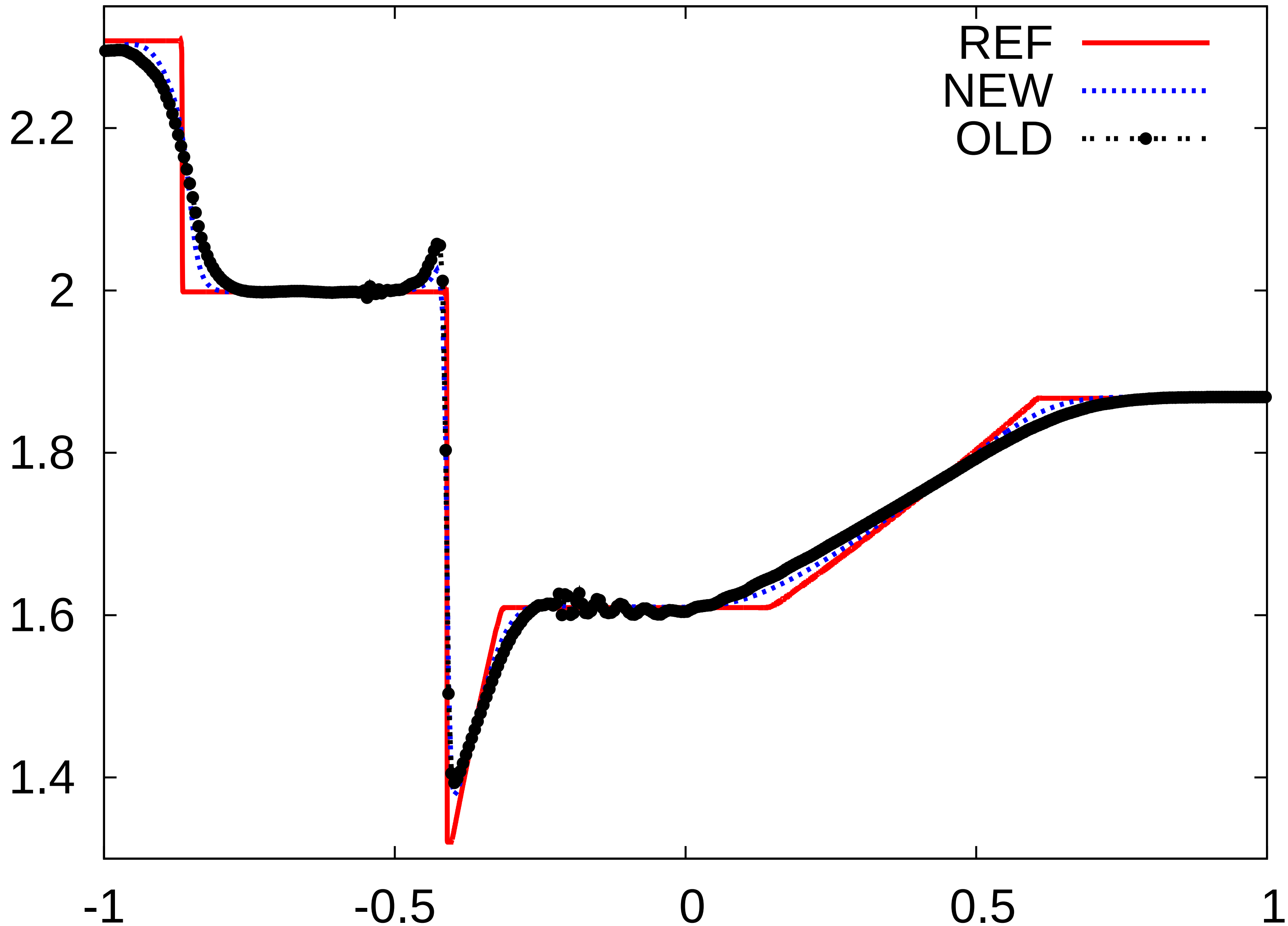
Density, $\Delta x = 1/200$



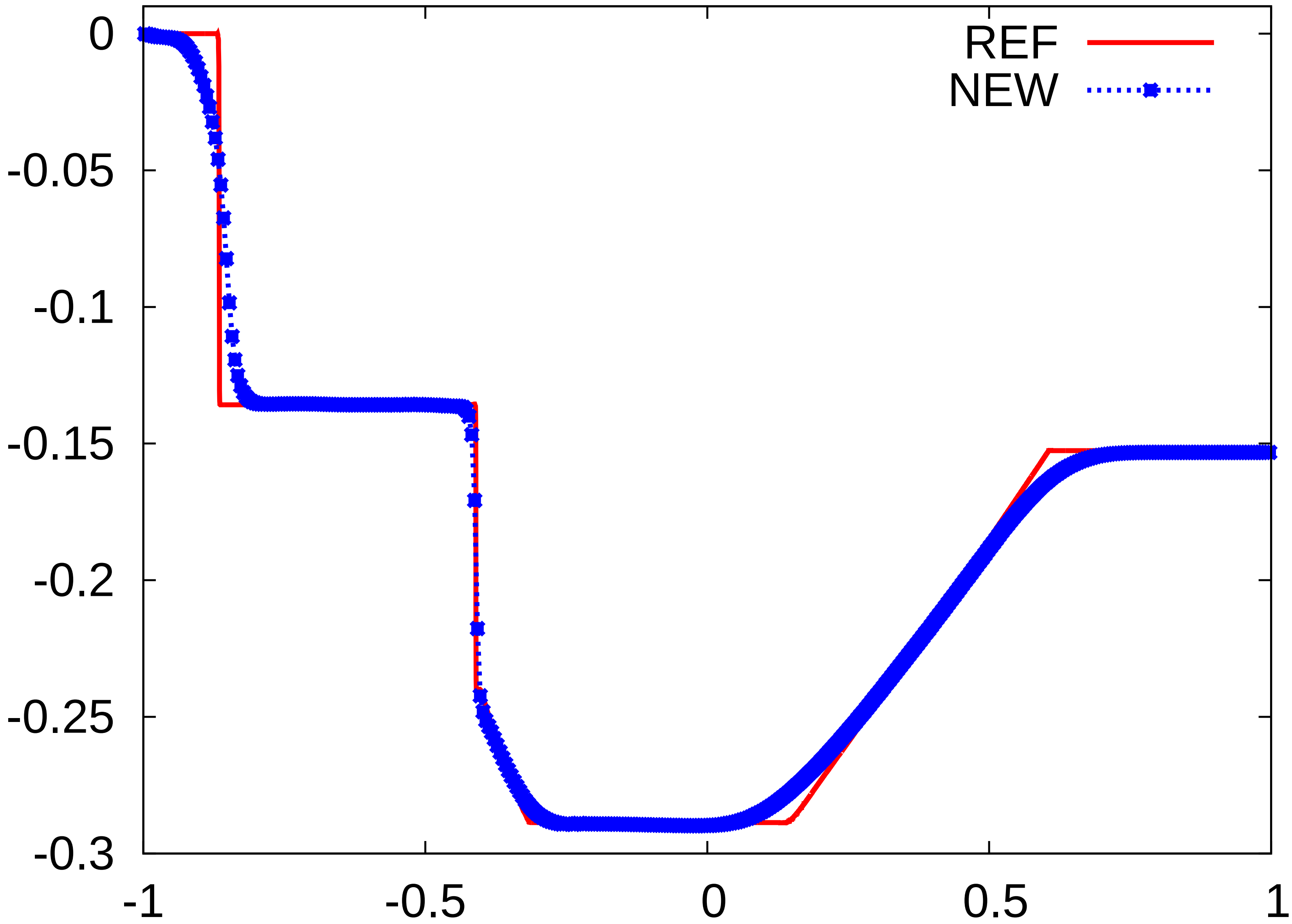
Velocity, $\Delta x = 1/200$



Pressure, $\Delta x = 1/200$



Velocity, $\Delta x = 1/200$



Pressure, $\Delta x = 1/200$