Fourteenth International Conference
on Hyperbolic Problems:
Theory, Numerics and Applications

HYP2012

HYPERBOLIC EXPLICIT-PARABOLIC LINEARLY IMPLICIT FINITE DIFFERENCE METHODS FOR DEGENERATE CONVECTION DIFFUSION EQUATIONS

F. Cavalli

Dipartimento di Matematica, University Statale of Milano

University of Padova
June 25–29, 2012

Nonlinear convection-diffusion equation

$$\partial_t u + \partial_x f(u) = \partial_{xx} p(u), \quad (x,t) \in [a,b] \times [0,T]$$

+ boundary conditions and initial datum

$p(u)$ non linear, Lipschitz continuous, possibly degenerate ($p'(u) = 0$)

Numerical approaches for the parabolic term

Explicit time integration:

- Very accurate, high order schemes, non linear reconstructions
- Computationally expensive: $\Delta t \leq ch^2$

Implicit time integration:

- Parabolic equation is unconditionally stable
- Require non linear iterative solvers, converge for "small" Δt

Nonlinear convection-diffusion equation

$$\partial_t u + \partial_x f(u) = \partial_{xx} p(u), \quad (x,t) \in [a,b] \times [0,T]$$

+ boundary conditions and initial datum
$p(u)$ non linear, Lipschitz continuous, possibly degenerate ($p'(u) = 0$)

Numerical approaches for the parabolic term

Explicit time integration:

- Very accurate, high order schemes, non linear reconstructions
- Computationally expensive: $\Delta t \leq ch^2$

Implicit time integration:

- Parabolic equation is unconditionally stable
- Require non linear iterative solvers, converge for "small" Δt

Nonlinear convection-diffusion equation

$$\partial_t u + \partial_x f(u) = \partial_{xx} p(u), \quad (x,t) \in [a,b] \times [0,T]$$

+ boundary conditions and initial datum
$p(u)$ non linear, Lipschitz continuous, possibly degenerate ($p'(u) = 0$)

Numerical approaches for the parabolic term

Explicit time integration:

- Very accurate, high order schemes, non linear reconstructions
- Computationally expensive: $\Delta t \leq ch^2$

Implicit time integration:

- Parabolic equation is unconditionally stable
- Require non linear iterative solvers, converge for "small" Δt

Nonlinear convection-diffusion equation

$$\partial_t u + \partial_x f(u) = \partial_{xx} p(u), \quad (x,t) \in [a,b] \times [0,T]$$

+ boundary conditions and initial datum

$p(u)$ non linear, Lipschitz continuous, possibly degenerate ($p'(u) = 0$)

Numerical approaches for the parabolic term

Explicit time integration:

- Very accurate, high order schemes, non linear reconstructions
- Computationally expensive: $\Delta t \leq ch^2$

Implicit time integration:

- Parabolic equation is unconditionally stable
- Require non linear iterative solvers, converge for “small” Δt

Parabolic non linear equation

Semi discrete scheme

Goals

- Avoid parabolic stability constraints, only $\Delta t \leq ch$
- Avoid to solve non linear implicit problems
- Develop high order schemes for smooth solutions
- Be accurate where solution is non smooth

Linear implicit

Non linear Chernoff formula based schemes [1]

$$\begin{cases} q^n = p(u^n)/\xi \\ q^{n+1} = q^n + \Delta t \partial_{xx} \xi q^{n+1} \\ u^{n+1} = u^n + q^{n+1} - q^n \end{cases}$$

Stability is proved under condition $\xi \geq L_p$
Poor accuracy, first order scheme

[1] Berger, A.; Brezis, H. Rogers, J. A numerical method for solving the problem $u_t - \Delta f(u) = 0$ RAIRO numerical analysis, 1979, 13, 297-312

Parabolic non linear equation

Semi discrete scheme

Goals

- Avoid parabolic stability constraints, only $\Delta t \leq ch$
- Avoid to solve non linear implicit problems
- Develop high order schemes for smooth solutions
- Be accurate where solution is non smooth

Linear implicit

Non linear Chernoff formula based schemes [1]

$$\begin{cases} q^n = p(u^n)/\xi \\ q^{n+1} = q^n + \Delta t \partial_{xx} \xi q^{n+1} \\ u^{n+1} = u^n + q^{n+1} - q^n \end{cases}$$

Stability is proved under condition $\xi \geq L_p$
Poor accuracy, first order scheme

[1] Berger, A.; Brezis, H. Rogers, J. A numerical method for solving the problem $u_t - \Delta f(u) = 0$ RAIRO numerical analysis, 1979, 13, 297-312

Parabolic non linear equation

Semi discrete scheme

Goals

- Avoid parabolic stability constraints, only $\Delta t \le ch$
- Avoid to solve non linear implicit problems
- Develop high order schemes for smooth solutions
- Be accurate where solution is non smooth

Linear implicit

Non linear Chernoff formula based schemes [1]

$$\begin{cases} q^n = p(u^n)/\xi \\ q^{n+1} = q^n + \Delta t \partial_{xx} \xi q^{n+1} \\ u^{n+1} = u^n + q^{n+1} - q^n \end{cases}$$

Stability is proved under condition $\xi \ge L_p$
Poor accuracy, first order scheme

[1] Berger, A.; Brezis, H. Rogers, J. A numerical method for solving the problem $u_t - \Delta f(u) = 0$ RAIRO numerical analysis, 1979, 13, 297-312

Parabolic non linear equation

Semi discrete scheme

Goals

- Avoid parabolic stability constraints, only $\Delta t \leq ch$
- Avoid to solve non linear implicit problems
- Develop high order schemes for smooth solutions
- Be accurate where solution is non smooth

Linear implicit

Non linear Chernoff formula based schemes [1]

$$\begin{cases} q^n = p(u^n)/\xi \\ q^{n+1} = q^n + \Delta t \partial_{xx} \xi q^{n+1} \\ u^{n+1} = u^n + q^{n+1} - q^n \end{cases}$$

Stability is proved under condition $\xi \geq L_p$
Poor accuracy, first order scheme

[1] Berger, A.; Brezis, H. Rogers, J. A numerical method for solving the problem $u_t - \Delta f(u) = 0$ RAIRO numerical analysis, 1979, 13, 297-312

Parabolic non linear equation

Semi discrete scheme

Goals

- Avoid parabolic stability constraints, only $\Delta t \leq ch$
- Avoid to solve non linear implicit problems
- Develop high order schemes for smooth solutions
- Be accurate where solution is non smooth

Linear implicit

Non linear Chernoff formula based schemes [1]

$$\begin{cases} q^n = p(u^n)/\xi \\ q^{n+1} = q^n + \Delta t \partial_{xx} \xi q^{n+1} \\ u^{n+1} = u^n + q^{n+1} - q^n \end{cases}$$

Stability is proved under condition $\xi \geq L_p$
Poor accuracy, first order scheme

[1] Berger, A.; Brezis, H. Rogers, J. A numerical method for solving the problem $u_t - \Delta f(u) = 0$ RAIRO numerical analysis, 1979, 13, 297-312

Parabolic non linear equation

Semi discrete scheme

Goals

- Avoid parabolic stability constraints, only $\Delta t \leq ch$
- Avoid to solve non linear implicit problems
- Develop high order schemes for smooth solutions
- Be accurate where solution is non smooth

Linear implicit

Non linear Chernoff formula based schemes [1]

$$\begin{cases} q^n = p(u^n)/\xi \\ q^{n+1} = q^n + \Delta t \partial_{xx} \xi q^{n+1} \\ u^{n+1} = u^n + q^{n+1} - q^n \end{cases}$$

Stability is proved under condition $\xi \geq L_p$

Poor accuracy, first order scheme

[1] Berger, A.; Brezis, H. Rogers, J. A numerical method for solving the problem $u_t - \Delta f(u) = 0$ RAIRO numerical analysis, 1979, 13, 297-312

Parabolic non linear equation

Goals

- Avoid parabolic stability constraints, only $\Delta t \leq ch$
- Avoid to solve non linear implicit problems
- Develop high order schemes for smooth solutions
- Be accurate where solution is non smooth

Linear implicit

Non linear Chernoff formula based schemes [1]

$$\begin{cases} q^n = p(u^n)/\xi \\ q^{n+1} = q^n + \Delta t \partial_{xx} \xi q^{n+1} \\ u^{n+1} = u^n + q^{n+1} - q^n \end{cases}$$

Stability is proved under condition $\xi \geq L_p$
Poor accuracy, first order scheme

[1] Berger, A.; Brezis, H. Rogers, J. A numerical method for solving the problem $u_t - \Delta f(u) = 0$ RAIRO numerical analysis, 1979, 13, 297-312

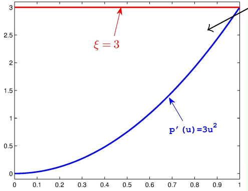
Non linearity

Brezis scheme

Example $p(u) = u^3$

inaccurate

very inaccurate
near degeneracy



Solution: more local[1] [2] [3] form of ξ, in particular near degeneracy

[1] Jäger, W.; Kaĉur J., Solution of porous medium type systems by linear approximation schemes, Numer. Math. (1991) 60: 407–427

[2] Pop, I.S.; Yong, W. A., A numerical approach to degenerate parabolic equations Numer. Math. (2002) 92: 357–381

[3] Slodiĉka, M., Approximation of a nonlinear degenerate parabolic equation via a linear relaxation scheme, Numerical Methods for Partial Differential Equations (2005) Vol 21 Issue 2 191–212

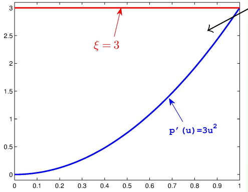
Non linearity

Brezis scheme

Example $p(u) = u^3$

inaccurate

very inaccurate
near degeneracy



Solution: more local[1] [2] [3] form of ξ, in particular near degeneracy

[1] Jäger, W.; Kaĉur J., Solution of porous medium type systems by linear approximation schemes, Numer. Math. (1991) 60: 407–427

[2] Pop, I.S.; Yong, W. A., A numerical approach to degenerate parabolic equations Numer. Math. (2002) 92: 357–381

[3] Slodiĉka, M., Approximation of a nonlinear degenerate parabolic equation via a linear relaxation scheme, Numerical Methods for Partial Differential Equations (2005) Vol 21 Issue 2 191–212

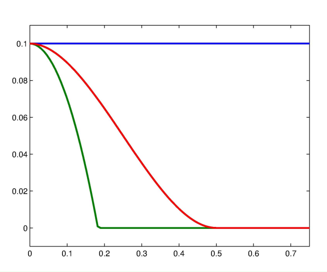
Accuracy

$\xi = p'(u^n)$: in general we do not have a stable scheme
Correction: we consider

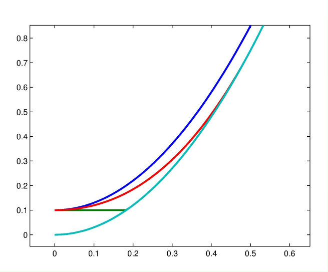
$$\xi(u_h^n) = \min\left(p'(u_h^n) + \alpha(u_h^n), L_p\right)$$

with $\alpha(u_h^n) \geq 0$, for example

α



ξ



Convection Diffusion problem

First order modified

IMEX(1,1,1) scheme

Explicit convection Linear implicit diffusion

$$\begin{cases} q^n = \dfrac{p(u^n)}{\xi(u^n)} \\ q^{n+1} + \Delta t \partial_x f(u^n) = q^n + \Delta t \partial_{xx}(\xi(u^n) q^{n+1}) \\ u^{n+1} = u^n + q^{n+1} - q^n \end{cases}$$

- Linearly implicit: no iterative methods for non linear problems
- Accurate, generalizable to higher order IMEX schemes[1]

[1] Ascher U.; Ruuth S.; Spiteri R., Implicit-explicit Runge-Kutta methods for time-dependent partial differential equations, Applied Numerical Mathematics (1997) Vol 25 Issue 2–3 151–167

Convection Diffusion problem

First order modified

IMEX(1,1,1) scheme

Explicit convection Linear implicit diffusion

$$\begin{cases} q^n = \dfrac{p(u^n)}{\xi(u^n)} \\ q^{n+1} + \Delta t \partial_x f(u^n) = q^n + \Delta t \partial_{xx}(\xi(u^n) q^{n+1}) \\ u^{n+1} = u^n + q^{n+1} - q^n \end{cases}$$

- Linearly implicit: no iterative methods for non linear problems
- Accurate, generalizable to higher order IMEX schemes[1]

[1] Ascher U.; Ruuth S.; Spiteri R., Implicit-explicit Runge-Kutta methods for time-dependent partial differential equations, Applied Numerical Mathematics (1997) Vol 25 Issue 2–3 151–167

Convection Diffusion problem

First order modified

IMEX(1,1,1) scheme

Explicit convection Linear implicit diffusion

$$\begin{cases} q^n = \dfrac{p(u^n)}{\xi(u^n)} \\ q^{n+1} + \Delta t \partial_x f(u^n) = q^n + \Delta t \partial_{xx}(\xi(u^n) q^{n+1}) \\ u^{n+1} = u^n + q^{n+1} - q^n \end{cases}$$

- Linearly implicit: no iterative methods for non linear problems
- Accurate, generalizable to higher order IMEX schemes[1]

[1] Ascher U.; Ruuth S.; Spiteri R., Implicit-explicit Runge-Kutta methods for time-dependent partial differential equations, Applied Numerical Mathematics (1997) Vol 25 Issue 2–3 151–167

Space discretization

Fully discrete

$$\begin{cases} q_i^n = \dfrac{p(u_i^n)}{\xi(u_i^n)} \\ q_i^{n+1} + \Delta t(\mathcal{D}_h u_h^n)_i = q_i^n + \Delta t(\mathcal{L}_h \xi(u_h^n) q_h^{n+1})_i \\ u_i^{n+1} = u_i^n + q_i^{n+1} - q_i^n \end{cases}$$

Finite difference operators

$\mathcal{L}_h$: discrete operator approximating ∂_{xx}
$\mathcal{D}_h$: discrete operator approximating $\partial_x f$, for example

$$(\mathcal{D}_h u_h)_i \approx (\partial_x(f(u)))_i \approx \frac{\hat{F}_{i+1/2}(u_h) - \hat{F}_{i-1/2}(u_h)}{h}$$

where $\hat{F}$ is a numerical flux, we can use non linear reconstructions (ENO scheme)

Space discretization

Fully discrete

$$\begin{cases} q_i^n = \dfrac{p(u_i^n)}{\xi(u_i^n)} \\ q_i^{n+1} + \Delta t(\mathcal{D}_h u_h^n)_i = q_i^n + \Delta t(\mathcal{L}_h \xi(u_h^n) q_h^{n+1})_i \\ u_i^{n+1} = u_i^n + q_i^{n+1} - q_i^n \end{cases}$$

Finite difference operators

$\mathcal{L}_h$: discrete operator approximating ∂_{xx}
$\mathcal{D}_h$: discrete operator approximating $\partial_x f$, for example

$$(\mathcal{D}_h u_h)_i \approx (\partial_x(f(u)))_i \approx \frac{\hat{F}_{i+1/2}(u_h) - \hat{F}_{i-1/2}(u_h)}{h}$$

where $\hat{F}$ is a numerical flux, we can use non linear reconstructions (ENO scheme)

Analysis of first order scheme

We have the following results for convex $p(u)$

Consistency

- The method is consistent with the problem
- Each system involved is non-singular for $\alpha \geq 0$

Stability

- we can find $\alpha = C$ for which the scheme is stable if the hyperbolic problem is stable (parabolic problem is unconditionally stable in maximum norm, i.e. if $\|u^n\|_\infty \leq M$ then $\|u^{n+1}\|_\infty \leq M$).
- If the analytical solution is sufficiently smooth on $[a, b] \times [t^n, t^{n+1}]$ then $C = c\Delta t$;
- C is bounded from above uniformly for $\Delta t, h \to 0$

Analysis of first order scheme

We have the following results for convex $p(u)$

Consistency

- The method is consistent with the problem
- Each system involved is non-singular for $\alpha \geq 0$

Stability

- we can find $\alpha = C$ for which the scheme is stable if the hyperbolic problem is stable (parabolic problem is unconditionally stable in maximum norm, i.e. if $\|u^n\|_\infty \leq M$ then $\|u^{n+1}\|_\infty \leq M$).
- If the analytical solution is sufficiently smooth on $[a, b] \times [t^n, t^{n+1}]$ then $C = c\Delta t$;
- C is bounded from above uniformly for $\Delta t, h \to 0$

Analysis of first order scheme

We have the following results for convex $p(u)$

Consistency

- The method is consistent with the problem
- Each system involved is non-singular for $\alpha \geq 0$

Stability

- we can find $\alpha = C$ for which the scheme is stable if the hyperbolic problem is stable (parabolic problem is unconditionally stable in maximum norm, i.e. if $\|u^n\|_\infty \leq M$ then $\|u^{n+1}\|_\infty \leq M$).
- If the analytical solution is sufficiently smooth on $[a, b] \times [t^n, t^{n+1}]$ then $C = c\Delta t$;
- C is bounded from above uniformly for $\Delta t, h \to 0$

Analysis of first order scheme

We have the following results for convex $p(u)$

Consistency

- The method is consistent with the problem
- Each system involved is non-singular for $\alpha \geq 0$

Stability

- we can find $\alpha = C$ for which the scheme is stable if the hyperbolic problem is stable (parabolic problem is unconditionally stable in maximum norm, i.e. if $\|u^n\|_\infty \leq M$ then $\|u^{n+1}\|_\infty \leq M$).
- If the analytical solution is sufficiently smooth on $[a, b] \times [t^n, t^{n+1}]$ then $C = c\Delta t$;
- C is bounded from above uniformly for $\Delta t, h \to 0$

IMEX(s,$s+1$,p) scheme

s stage implicit $s+1$ stage explicit

$$
\begin{array}{c|cccc}
0 & 0 & 0 & \cdots & 0 \\
c_1 & \tilde{a}_{2,1} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
c_{s-1} & \tilde{a}_{s,1} & \tilde{a}_{s,2} & \cdots & 0 \\
\hline
 & \tilde{b}_1 & \tilde{b}_2 & \cdots & \tilde{b}_s
\end{array}
\qquad
\begin{array}{c|cccc}
c_1 & a_{1,1} & 0 & \cdots & 0 \\
c_2 & a_{2,1} & a_{2,2} & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
c_s & a_{s,1} & a_{2,s} & \cdots & a_{s,s} \\
\hline
 & b_1 & b_2 & \cdots & b_s
\end{array}
$$

High order scheme

$$
\begin{cases}
q_h^n = \dfrac{p(u_h^n)}{\xi(u_h^n)}, \quad q_h^{(0)} = q_h^n, \qquad u_h^{(0)} = u_h^n \\
\text{For i=1,\ldots,s} \\
\quad q_h^{(i)} = q_h^n + \Delta t \sum_{k=1}^{i} a_{i,j} \mathcal{L}_h(\xi(u_h^n) q_h^{(j)}) - \Delta t \sum_{j=0}^{i-1} \tilde{a}_{i+1,j+1} \mathcal{D}_h(u_h^{(j)}) \\
\quad u_h^{(i)} = u_h^n + q_h^{(i)} - q_h^n \\
q_h^{n+1} = q_h^n + \Delta t \sum^{s} b_i \mathcal{L}_h(\xi(u_h^n) q_h^{(i)}) - \Delta t \sum^{s} \tilde{b}_{i+1} \mathcal{D}_h(u_h^{(i)})
\end{cases}
$$

High Order accuracy

IMEX scheme

IMEX(s,$s+1$,p) scheme

s stage implicit $s+1$ stage explicit

$$\begin{array}{c|c} c & \tilde{A} \\ \hline 0 & \tilde{b} \end{array} \quad \begin{array}{c|c} c & A \\ \hline 0 & b \end{array}$$

High order scheme

$$\begin{cases} q_h^n = \dfrac{p(u_h^n)}{\xi(u_h^n)}, \quad q_h^{(0)} = q_h^n, \qquad u_h^{(0)} = u_h^n \\ \text{For i=1,...,s} \\ \quad q_h^{(i)} = q_h^n + \Delta t \displaystyle\sum_{k=1}^{i} a_{i,j} \mathcal{L}_h(\xi(u_h^n) q_h^{(j)}) - \Delta t \sum_{j=0}^{i-1} \tilde{a}_{i+1,j+1} \mathcal{D}_h(u_h^{(j)}) \\ \quad u_h^{(i)} = u_h^n + q_h^{(i)} - q_h^n \\ q_h^{n+1} = q_h^n + \Delta t \displaystyle\sum_{i=1}^{s} b_i \mathcal{L}_h(\xi(u_h^n) q_h^{(i)}) - \Delta t \sum_{i=0}^{s} \tilde{b}_{i+1} \mathcal{D}_h(u_h^{(i)}) \\ u_h^{n+1} = u_h^n + q_h^{n+1} - q_h^n \end{cases}$$

Analysis of high order schemes

Accuracy

If we use an IMEX method of order $r \geq 2$, if the solution is sufficiently smooth, then the consistency error of the scheme is 2 provided that $\alpha(u_h^n) = O(\Delta t)$.
Stability in the general case has yet to be proved
However, for both regular and non regular solutions the estimates given in the first order case seem reliable for higher order cases: for smooth solutions we can find $C = O(\Delta t)$ and obtain a “stable” and second order accurate scheme.

Burger + Porous media

$$\begin{cases} \partial_t u + \partial_x(u^2) = \partial_{xx} u^3 & (x,t) \in [-3/2, 3/2] \times [0, 0.01] \\ u(x,0) = \cos^2(\frac{\pi}{2}x)\chi_{[-1,1]} & x \in [-3/2, 3/2] \\ u(\pm 1, t) = 0 & t \in [0, 0.01]; \end{cases}$$

$u(x, 0.01) \in C^2(-3/2, 3/2)$

Error and rates

	IMEX(1,1,1)		IMEX(2,3,2)		IMEX(3,4,3)	
N	E_1	r	E_1	r	E_1	r
10	1.86e-01		1.86e-01		1.86e-01	
30	2.25e-02	1.92	8.25e-03	2.84	6.14e-03	3.11
90	6.98e-03	1.07	6.61e-04	2.30	2.72e-04	2.84
270	2.04e-03	1.12	5.89e-05	2.20	1.15e-05	2.88
810	6.80e-04	1.00	5.90e-06	2.09	1.16e-06	2.09
2430	2.27e-04	0.99	6.32e-07	2.03	1.34e-07	1.96

Burger + Porous media

$$\begin{cases} \partial_t u + \partial_x(u^2) = \partial_{xx} u^3 & (x,t) \in [-3/2, 3/2] \times [0, 0.01] \\ u(x,0) = \cos^2(\frac{\pi}{2} x)\chi_{[-1,1]} & x \in [-3/2, 3/2] \\ u(\pm 1, t) = 0 & t \in [0, 0.01]; \end{cases}$$

$u(x, 0.01) \in C^2(-3/2, 3/2)$

Error and rates

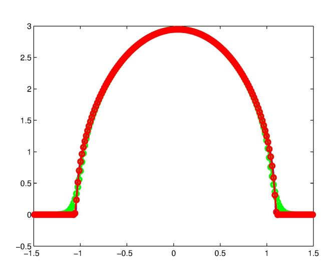
	IMEX(1,1,1)		IMEX(2,3,2)		IMEX(3,4,3)	
N	E_1	r	E_1	r	E_1	r
10	1.86e-01		1.86e-01		1.86e-01	
30	2.25e-02	1.92	8.25e-03	2.84	6.14e-03	3.11
90	6.98e-03	1.07	6.61e-04	2.30	2.72e-04	2.84
270	2.04e-03	1.12	5.89e-05	2.20	1.15e-05	2.88
810	6.80e-04	1.00	5.90e-06	2.09	1.16e-06	2.09
2430	2.27e-04	0.99	6.32e-07	2.03	1.34e-07	1.96

Numerical tests

Initial datum: step function

$$u(x,0) = 5\chi_{[-1/2,1/2]}$$

IMEX(3,4,3) + third order spatial accuracy, $N = 200$

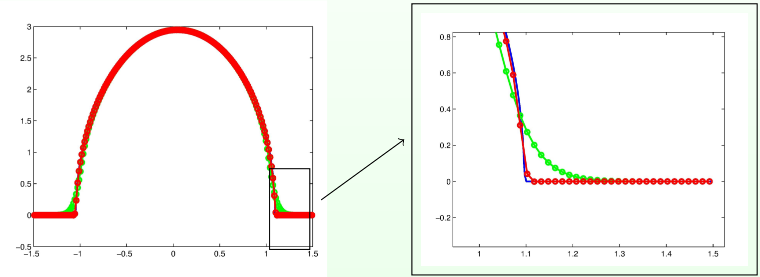


Comparison : "Exact solution", numerical solution with non constant ξ, numerical solution with constant ξ

Initial datum: step function

$$u(x,0) = 5\chi_{[-1/2,1/2]}$$

IMEX(3,4,3) + third order spatial accuracy, $N = 200$

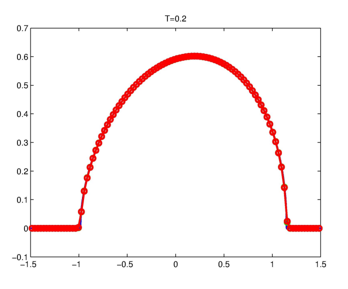


Comparison : “Exact solution”, numerical solution with non constant ξ, numerical solution with constant ξ

Numerical tests

Accuracy

Approximation: IMEX(2,3,2) + 2nd order spatial operators, $N = 100$

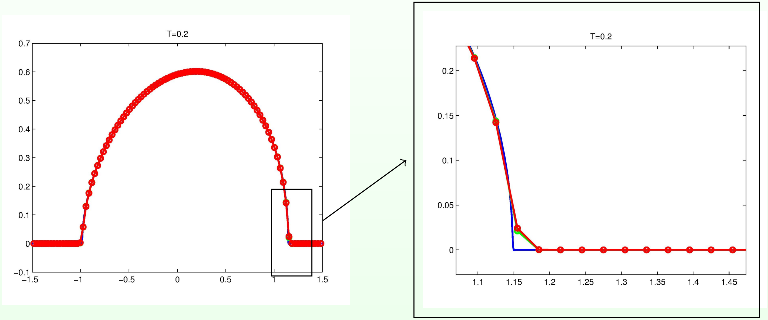


Comparison : "Exact solution"(-) and Numerical solution with Chernoff modified scheme (-o), Numerical solution of non linear scheme+Newton (-o)

Numerical tests

Accuracy

Approximation: IMEX(2,3,2) + 2nd order spatial operators, $N = 100$

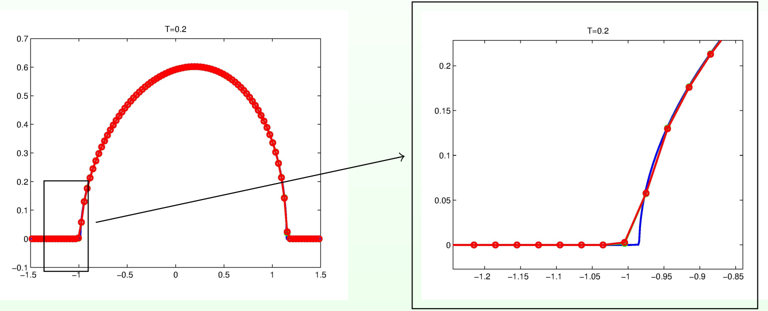


Comparison : “Exact solution”(-) and Numerical solution with Chernoff modified scheme (-o), Numerical solution of non linear scheme+Newton (-o)

Numerical tests

Accuracy

Approximation: IMEX(2,3,2) + 2[nd] order spatial operators, $N = 100$

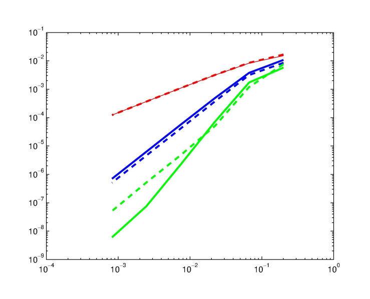


Comparison : "Exact solution"(-) and Numerical solution with Chernoff modified scheme (-o), Numerical solution of non linear scheme+Newton (-o)

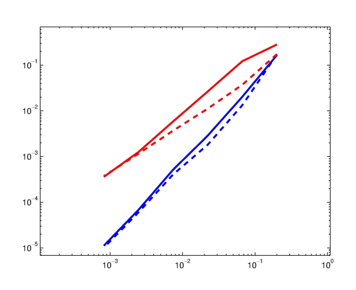
Numerical tests

Linear vs. non linear

Smooth solution



Non-smooth solution



Corrected Chernoff scheme (-) and Non-linear scheme (- -),
IMEX(1,1,1), IMEX(2,3,2), IMEX(3,4,3)

Problem

Non convex convection $f(u) = \dfrac{u^2}{u^2 + (1-u)^2}$

Non convex diffusion $p(u) = 10^{-2}(2u^2 - \dfrac{4}{3}u^3)$

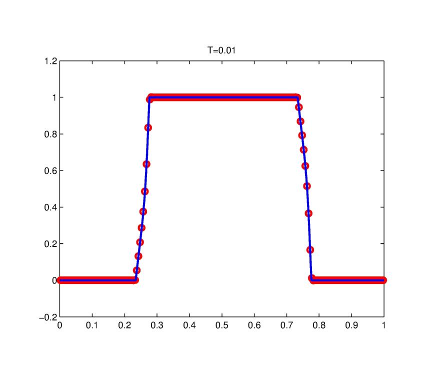
Initial datum $u(x,0) = \chi_{[1/2,3/4]}$

Problem

Non convex convection $f(u) = \dfrac{u^2}{u^2 + (1-u)^2}$

Non convex diffusion $p(u) = 10^{-2}(2u^2 - \frac{4}{3}u^3)$

Initial datum $u(x,0) = \chi_{[1/2,3/4]}$

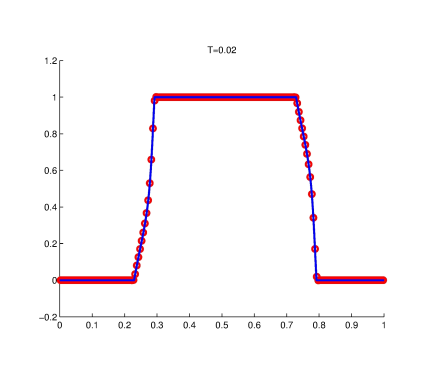


Problem

Non convex convection $f(u) = \dfrac{u^2}{u^2 + (1-u)^2}$

Non convex diffusion $p(u) = 10^{-2}(2u^2 - \frac{4}{3}u^3)$

Initial datum $u(x,0) = \chi_{[1/2,3/4]}$



Numerical tests

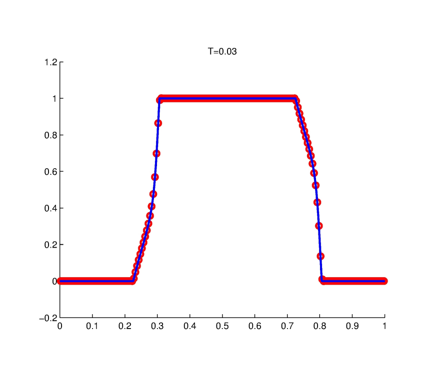
Non convex diffusion

Problem

Non convex convection $f(u) = \dfrac{u^2}{u^2 + (1-u)^2}$

Non convex diffusion $p(u) = 10^{-2}(2u^2 - \frac{4}{3}u^3)$

Initial datum $u(x,0) = \chi_{[1/2,3/4]}$



Numerical tests

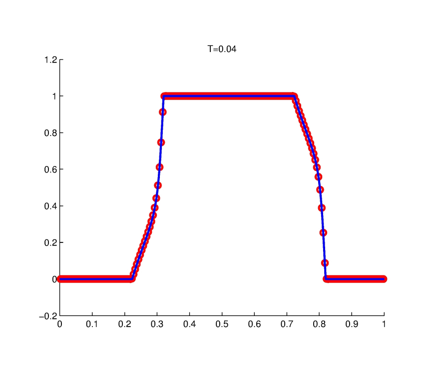
Non convex diffusion

Problem

Non convex convection $f(u) = \dfrac{u^2}{u^2 + (1-u)^2}$

Non convex diffusion $p(u) = 10^{-2}(2u^2 - \frac{4}{3}u^3)$

Initial datum $u(x,0) = \chi_{[1/2,3/4]}$

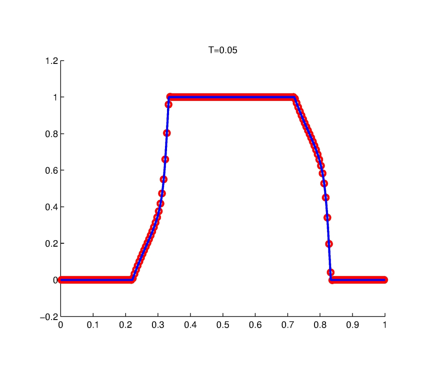


Problem

Non convex convection $f(u) = \dfrac{u^2}{u^2 + (1-u)^2}$

Non convex diffusion $p(u) = 10^{-2}(2u^2 - \frac{4}{3}u^3)$

Initial datum $u(x,0) = \chi_{[1/2,3/4]}$

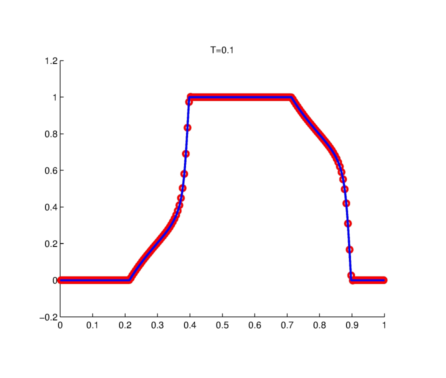


Problem

Non convex convection $f(u) = \dfrac{u^2}{u^2 + (1-u)^2}$

Non convex diffusion $p(u) = 10^{-2}(2u^2 - \dfrac{4}{3}u^3)$

Initial datum $u(x,0) = \chi_{[1/2,3/4]}$



Numerical tests

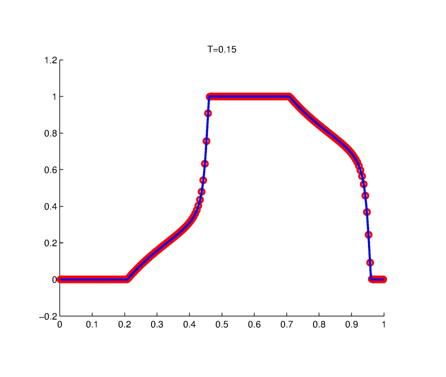
Non convex diffusion

Problem

Non convex convection $f(u) = \dfrac{u^2}{u^2 + (1-u)^2}$

Non convex diffusion $p(u) = 10^{-2}(2u^2 - \dfrac{4}{3}u^3)$

Initial datum $u(x,0) = \chi_{[1/2,3/4]}$



Conclusions and perspectives

Conclusions

- Developed high order schemes
- Linearly implicit
- Second order Chernoff correction
- Hyperbolic stability constraint
- Estimate of correction term α

Perspectives

- Study high order schemes
- Extend the analysis to the non convex case and refine the estimate for α
- Study the strongly degenerate case
- Improve the choice of the time integration method
- Study mesh and method adaptivity
- Consider system of PDEs

Conclusions and perspectives

Conclusions

- Developed high order schemes
- Linearly implicit
- Second order Chernoff correction
- Hyperbolic stability constraint
- Estimate of correction term α

Perspectives

- Study high order schemes
- Extend the analysis to the non convex case and refine the estimate for α
- Study the strongly degenerate case
- Improve the choice of the time integration method
- Study mesh and method adaptivity
- Consider system of PDEs

Conclusions and perspectives

Conclusions

- Developed high order schemes
- Linearly implicit
- Second order Chernoff correction
- Hyperbolic stability constraint
- Estimate of correction term α

Perspectives

- Study high order schemes
- Extend the analysis to the non convex case and refine the estimate for α
- Study the strongly degenerate case
- Improve the choice of the time integration method
- Study mesh and method adaptivity
- Consider system of PDEs

Conclusions and perspectives

Conclusions

- Developed high order schemes
- Linearly implicit
- Second order Chernoff correction
- Hyperbolic stability constraint
- Estimate of correction term α

Perspectives

- Study high order schemes
- Extend the analysis to the non convex case and refine the estimate for α
- Study the strongly degenerate case
- Improve the choice of the time integration method
- Study mesh and method adaptivity
- Consider system of PDEs

Conclusions and perspectives

Conclusions

- Developed high order schemes
- Linearly implicit
- Second order Chernoff correction
- Hyperbolic stability constraint
- Estimate of correction term α

Perspectives

- Study high order schemes
- Extend the analysis to the non convex case and refine the estimate for α
- Study the strongly degenerate case
- Improve the choice of the time integration method
- Study mesh and method adaptivity
- Consider system of PDEs

Conclusions and perspectives

Conclusions

- Developed high order schemes
- Linearly implicit
- Second order Chernoff correction
- Hyperbolic stability constraint
- Estimate of correction term α

Perspectives

- Study high order schemes
- Extend the analysis to the non convex case and refine the estimate for α
- Study the strongly degenerate case
- Improve the choice of the time integration method
- Study mesh and method adaptivity
- Consider system of PDEs

Conclusions and perspectives

Conclusions

- Developed high order schemes
- Linearly implicit
- Second order Chernoff correction
- Hyperbolic stability constraint
- Estimate of correction term α

Perspectives

- Study high order schemes
- Extend the analysis to the non convex case and refine the estimate for α
- Study the strongly degenerate case
- Improve the choice of the time integration method
- Study mesh and method adaptivity
- Consider system of PDEs

Thank you!