Decay property for symmetric hyperbolic systems with non-symmetric relaxation

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In this talk, we consider the Cauchy problem for the first-order linear symmetric hyperbolic system of equations with relaxation:

$$A^0 u_t + \sum_{j=1}^{n} A^j u_{x_j} + Lu = 0 \tag{1}$$

with $u|_{t=0} = u_0$. Here $u = u(t,x) \in \mathbb{R}^m$ over $t > 0$, $x \in \mathbb{R}^n$ is an unknown function, $u_0 = u_0(x) \in \mathbb{R}^m$ over $x \in \mathbb{R}^n$ is a given function, and A^j ($j = 0, 1, \cdots, n$) and L are $m \times m$ real constant matrices, where integers $m \geq 1$, $n \geq 1$ denote dimensions. Throughout this talk, it is assumed that all A^j ($j = 0, 1, \cdots, n$) are symmetric, A^0 is positive definite and L is nonnegative definite with a nontrivial kernel. Notice that L is not necessarily symmetric. For this general linear degenerately dissipative system it is interesting to study its decay structure under additional conditions on the coefficient matrices and further investigate the corresponding time-decay property of solutions to the Cauchy problem.

When the degenerate relaxation matrix L is symmetric, Umeda-Kawashima-Shizuta [5] proved the large-time asymptotic stability of solutions for a class of equations of hyperbolic-parabolic type with applications to both electro-magneto-fluid dynamics and magnetohydrodynamics. The key idea in [5] and the later generalized work [2] that first introduced the so-called Kawashima-Shizuta condition is to design the compensating matrix to capture the dissipation of systems over the degenerate kernel space of L. The typical feature of the time-decay property of solutions established in those work is that the high frequency part decays exponentially while the low frequency part decays polynomially with the rate of the heat kernel.

Unfortunately, when the degenerate relaxation matrix L is not symmetric, the theorems derived in [2,5] can not be applied any longer. In fact, this is the case for some concrete systems, for example, the Timoshenko system [1] and the Euler-Maxwell system [3,4], where the linearized relaxation matrix L indeed has a nonzero skew-symmetric part while it was still proved that solutions decay in time in some different way. Therefore, our purpose of this talk is to formulate some new structural conditions in order to extend the previous works to the general system (1) when L is not symmetric, which can include both the Timoshenko system and the Euler-Maxwell system.

References

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