Numerical scheme for a viscous Shallow Water system including new friction laws of second order

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Consider a 1D viscous Shallow Water model

$$\begin{cases} \dfrac{\partial h}{\partial t} + \dfrac{\partial (hu)}{\partial x} = 0, \\ \dfrac{\partial (hu)}{\partial t} + \dfrac{\partial (hu^2)}{\partial x} + \dfrac{g}{2}\dfrac{\partial h^2}{\partial x} = S_f - gh\dfrac{\partial Z}{\partial x} + 4\mu \dfrac{\partial}{\partial x}\left(h \dfrac{\partial u}{\partial x}\right) \end{cases} \tag{1}$$

where h is the flow depth, u the flow velocity, Z topography variations, g the gravity acceleration, μ the viscosity of the fluid.
In (1), the novelty lives in the friction term S_f. A new model of second order friction term based on Darcy-Weisbach's or Manning's formula is proposed. It can be written into the form

$$S_f = -\frac{kh^{-\alpha}|u|u}{(1 + \frac{k}{3\mu}|u|h^{1-\alpha})^2}\,. \tag{2}$$

If $\alpha = 0$ or $\alpha = \frac{1}{3}$, then a Darcy-Weisbach type formula or a Manning type formula is obtained respectively.
The derivation of (1)–(2) originating from the free surface Navier-Stokes equations follows the same lines as in [1] and [3]. The key point is to prescribe at the bottom, stresses with a Darcy-Weisbach's or a Manning's formula.
In order to solve numerically system (1), a scheme based on finite volume method for hyperbolic system of conservation laws with source terms is suggested.
Following the same lines of [2], analytic solutions for (1)–(2) are proposed. These solutions provide a numerical validation of the scheme.

References

[1] J.-F Gerbeau and B. Perthame, Derivation of viscous Saint-Venant system for laminar shallow water; numerical validation, *Discrete Contin. Dyn. Syst. Ser.*, **1(1)** (2001), pp. 89-102.

[2] I. MacDonald, *Analysis and computation of steady open channel flow*, PhD thesis, University of Reading, (1996).

[3] F. Marche, Derivation of a new two-dimensional viscous shallow water model with varying topography, bottom friction and capillary effects, *Eur. J. Mech. B Fluids*, **26(1)** (2007), pp. 49-63.

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