# The Closest Point Method for Surfaces PDEs and Applications to Thin Film Flow

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Partial differential equations (PDEs) are essential for modeling and understanding processes in all areas of science. The specialty of the PDEs considered here is that the differential operators involved are intrinsic to a curved surface. Such differential operators are used to model for example the flow of thin liquid films on a curved substrate (e.g., in industrial coating) [2].

The Closest Point Method [1,3] is a set of mathematical principles and associated numerical techniques for solving partial differential equations (PDEs) posed on surfaces. In this talk we present a calculus on surfaces based on closest point functions [4]. This calculus then forms the theoretical basis of the Closest Point Method and we show how to use it to set up a numerical method. Finally, we demonstrate the performance of the method on the hyperbolic thin film model [2]

$$\partial_t h + \operatorname{div}_S \left( \frac{h^3}{3} \nabla_S \kappa \right) = 0$$

which applies in situations where the mean curvature $\kappa$ of the substrate is not negligible.

## References

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[3] Macdonald C., Ruuth S., Level set equations on surfaces via the Closest Point Method, *Journal of Scientific Computing*, **35** (2008), pp. 219-240.

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