Bryan's effect and nonlinear damping

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Modern vibratory gyroscopes [1] are designed using the fact that when a near-perfectly manufactured vibrating structure is subjected to a rotation in three-dimensional space (referred to as inertial rotation), the vibrating pattern rotates (within the structure) at a rate proportional to the inertial angular rate. This effect, known as "Bryan's effect", was first observed by G.H. Bryan in 1890 [2]. For the constant of proportionality, Bryan made the following calculation for a body consisting of a ring or cylinder

$$BF = \frac{\text{Rate of rotation of the vibrating pattern}}{\text{Inertial rate of rotation of the vibrating structure}}$$

for various modes of vibration. This constant of proportionality BF has come to be known as "Bryan's factor". In 2011 a slowly rotating, fluid-filled sphere undergoing light anisotropic viscously damped vibrations was considered by Joubert, Shatalov and Coetzee [3]. Some structures have particles that vibrate at a high frequency and hence particle velocity is expected to be high at various points in time and consequently, a viscous damping model for Bryan's effect might not be ideal. Indeed, Sir Isaac Newton stated in Principia: Vol. 1: The Motion of Bodies (1687), that viscous damping "is more a mathematical hypothesis than a physical one." In this paper we tentatively introduce nonlinear damping into the equations of motion of a vibrating, slowly rotating ring or shell and show that the rate of rotation of the vibrating pattern is affected strongly by such damping.

References

[1] E.J. Loper Jr. and D.D. Lynch, Hemispherical resonator gyroscope, *U.S. Patent No.* **4,951,508** (1990).

[2] G.H. Bryan, On the beats in the vibrations of a revolving cylinder or bel, *Proceedings of the Cambridge Philosophical Society* **7** (1890), pp. 101-111.

[3] S.V. Joubert, M.Y. Shatalov and C.E. Coetzee, Analysing manufacturing imperfections in a spherical vibratory gyroscope, in *Proceedings of the 4th IEEE International Workshop on Advances in Sensors and Interfaces*, IEEE Catalog Number: **CFP11IWI-USB** (2011), pp. 165-170.

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