1D hemodynamic simulations thanks to numerical methods for Shallow Water system

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We are interested in blood flow simulation with variable elasticity arteries thanks to a one dimensional conservative model (mass and momentum conservations):

$$\begin{cases} \partial_t A + \partial_x Q = 0 \\ \partial_t Q + \partial_x \left(\dfrac{Q^2}{A} + \dfrac{1}{3\sqrt{\pi}\rho} k A^{3/2} \right) = \dfrac{A}{\sqrt{\pi}\rho} \left(\partial_x \mathcal{A}_0 - \dfrac{2}{3}\sqrt{A}\partial_x k \right) - C_f \dfrac{Q}{A} \,, \end{cases}$$

where $A(x,t)$ is the cross-section area ($A = \pi R^2$ with R the radius of the arteria), $Q(x,t) = A(x,t)u(x,t)$ the discharge, $u(t,x)$ the mean flow velocity, ρ the blood density, $k(x)$ the stiffness of the artery and $\mathcal{A}_0 = k\sqrt{A_0}$ where $A_0(x)$ is the cross section at rest.

We present here a well-balance finite volume scheme based on recent developments in shallow water equations context. We thus get a mass conservative scheme which also preserves the man at "eternal rest equilibrium" (*i.e.* $Q = 0$). This numerical method is validated on analytical tests.

References

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