Indirect internal stabilization of the thermoelastic Bresse system

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In this work, we study the energy decay rate for a thermoelastic Bresse system. The system consists of three wave equations and two heat equations coupled in certain pattern. The two wave equations about the longitudinal displacement and shear angle displacement are effectively damped by the dissipation from the two heat equations, however. The system is governed by the following differential partial equations:

$$\begin{aligned}
\rho h\varphi_{tt} - Gh(\varphi_x + \psi + k\omega)_x - kEh(\omega_x - k\varphi) &= 0, && (1)\\
\rho I\psi_{tt} - EI\psi_{xx} + Gh(\varphi_x + \psi + k\omega) + \alpha\xi_x &= 0, && (2)\\
\rho h\omega_{tt} - Eh(\omega_x - k\varphi)_x + kGh(\varphi_x + \psi + k\omega) + \alpha\theta_x &= 0, && (3)\\
\rho c\theta_t - \theta_{xx} + \alpha T_0\omega_{tx} &= 0, && (4)\\
\rho c\xi_t - \xi_{xx} + \alpha T_0\psi_{tx} &= 0, && (5)
\end{aligned}$$

where φ, ψ, ω are the vertical, rotation angle and longitudinal displacements; θ and ξ are the temperature deviations from the reference temperature T_0 along the longitudinal and vertical directions; E, G, ρ, I, h, k, c, are positive constants for the elastic and thermal material properties. The notation u_t (respectively u_x) indicate the partial derivatives with respect to time $t \geq 0$ (respectively with respect to spatial location $x \in [0, L]$). In this thesis, we study the energy decay rate for the thermoelastic Bresse system (1)-(5) with the boundary conditions

$$\omega_x(t,x) = \varphi(t,x) = \psi_x(t,x) = \theta(t,x) = \xi(t,x) = 0,\ for\ x = 0, L, \qquad (6)$$

or

$$\omega(t,x) = \varphi(t,x) = \psi(t,x) = \theta(t,x) = \xi(t,x) = 0,\ for\ x = 0, L, \qquad (7)$$

and initial conditions

$$\omega(0,x) = \omega_0(x),\ \ \omega_t(0,x) = \omega_1(x),\ \ \psi(0,x) = \psi_0(x),\ \ \psi_t(0,x) = \psi_1(x),$$

$$\varphi(0,x) = \varphi_0(x),\ \varphi_t(0,x) = \varphi_1(x),\ \ \theta(0,x) = \theta_0(x),\ \ \xi(0,x) = \xi_0(x). \qquad (8)$$

There are number of publications concerning the stabilization of the Bresse system [3], [4], [2] and [1]. In particular, in [3], Liu and Rao studied the stabilization of the Bresse system with two different temperature dissipation law effective on the equations about the longitudinal displacement and shear angle displacement. Under the equal speed wave propagation condition, they established an exponential energy decay rate. Otherwise, they showed that the smooth solution decays polynomially to zero with rates $\frac{1}{t^{1/2}}$ or $\frac{1}{t^{1/4}}$ provided the boundary conditions is Dirichlet-Neumann-Neumann or Dirichlet-Dirichlet-Dirichlet type respectively. In [2], Fatori and Rivera consider the Bresse system with one globally temperature dissipation law effective on the equation about the shear angle displacement. They established the same exponential energy decay rate in the case of equal speed wave propagation condition. Otherwise, they showed that the smooth solution decays polynomially to zero with rates $\frac{1}{t^{1/3}}$.

In this work, we consider the thermoelastic Bresse system damped by two locally internal distributed temperature dissipation laws with Dirichlet-Neumann-Neumann or Dirichlet-Dirichlet-Dirichlet boundary conditions type. Under the equal speed wave propagation condition, $E = G$, we establish the same exponential energy decay rate for usual initial data. On the contrary, when $E \neq G$, we first prove the non-exponential decay rate for the Bresse system with Dirichlet-Neumann-Neumann condition type. Therefore, we establish a new polynomial energy decay rate of type $\frac{1}{t}$ for the smooth solution.

References

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