On hyperbolic equations describing longitudinal vibration of accreting rods

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In this paper we analyse longitudinal vibration of a thin rod which is fixed at the left end and free at the right end. It is assumed that the rod is growing at its right end, i.e. its length is increasing according to a special law and hence it is a known function of time. This problem is described it terms of the linear classical, Rayleigh-Love and Rayleigh-Bishop models. For solution of this problem we make a special change of variables which transforms the original equations into new non-autonomous equations. It is shown that these equations are hyperbolic and possess several interesting and important properties. First of all, the amplitudes of vibration of the rod are growing with time. For example, if the rod length is increasing proportionally to time the amplitudes are also growing proportionally to time. Secondly, if a particular mode is excited it excites other modes. In this case the mechanism of the modes excitation is asymmetric, which means that the low frequency modes possess higher amplitudes compared to the higher frequency modes. A physical explanation of the above mentioned phenomena is proposed and a simplified model describing these effects is analysed.

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