Optimal control of level set dynamics via a finite-dimensional approximation scheme

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Optimal control of systems that describe the dynamics of level sets is investigated by using a methodology for approximate feedback control design. The design of the proposed regulators is achieved by resorting to an approximation scheme based on the extended Ritz method [1]. Such a scheme, which we first investigated in [2] and [3], consists in constraining the manipulable terms of the model equation to take on a fixed structure, where a finite number of free parameters can be suitably chosen. The original infinite-dimensional optimization problem is then reduced to a mathematical nonlinear programming one, in which the parameters have to be optimized. The proposed methodology is general since it allows one to deal with problems with different types of control (distributed or boundary control, control in the coefficients) within the same approximation framework.

In [2] and [3], we successfully applied the above-described method to a distributed optimal control problem for the Burgers' equation. As another application of the proposed approach, we present here an example based on the normal flow equation. The goal is to control the velocity term of the equation in order to track a desired closed curve on $\mathbb{R}^2$ associated with the zero level set of the solution of the normal flow equation. The exact solution of such a problem is very difficult to find, and thus the need of searching for approximate solutions arises. The velocity term is then replaced by a parametrized control law that depends on the values of the zero level set of the unknown function, thus turning out to be a feedback control law. The parametrization is based on radial basis functions with variable centers and widths. By substituting such a structure into the model equation and cost functional to minimize (given by the integral squared difference between the reference and actual zero level sets), we obtain a nonlinear programming problem, which is solved by using a sequential quadratic programming algorithm. At each iteration of such an algorithm we have to numerically solve the model equation. Simulation results are presented to show the effectiveness of the proposed approach as to both accuracy of suboptimal solutions and required computational effort.

References

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