Relaxing the CFL Number of the Discontinuous Galerkin Method

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Discontinuous Galerkin methods (DGM) have a Courant-Friedrichs-Lewy (CFL) number decreasing with the increase of the order of approximation p for convection dominated problems. This makes them computationally more expensive when compared with finite volume or finite difference methods. We propose a modification of the scheme that results in a family of high order methods which have a less restrictive CFL number. We show that in the standard DG method the dispersion and dissipation errors and the spectrum of the semi-discrete scheme are related to the $[p/p+1]$ Pade approximation of $exp(z)$ and $exp(-z)$. This Pade approximant is responsible for both the superconvergent error in diffusion and dispersion ($O(h^{2p+2})$ and $O(h^{2p+3})$, respectively) and the small CFL number. We propose to modify the DGM so that the resulting rational approximation of the exponent corresponds to a spatial discretization operator with a smaller spectrum, i.e. a less restrictive CFL number. This is achieved by scaling the amount of the numerical flux contribution to the equations evolving solution coefficients in time. For the considered orders of approximation, the improvement in the CFL number ranges between two and five fold depending on how much modification is brought into the scheme. The interesting aspect of the new schemes is that the $(p+1)$st rate of convergence in the L^2 norm as well as the compact stencil of the traditional DGM are preserved. We show that for the same amount of work the new schemes are more efficient for smooth problems and considerably more accurate for problems with discontinuities.

References

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