An entropy-satisfying fast and slow waves splitting method for the Baer & Nunziato two-phase flow model

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In the present work, we consider a PDE model formulated in Eulerian coordinates where balance equations account for the evolution of mass, momentum and energy of each phase. For compressible one-dimensional flows, there are seven unknowns that describe the evolution of the two-phase flow: the velocities of each phase u_k (where $k \in \{1, 2\}$), the phasic densities ρ_k, the total energy of each phase E_k and finally the phase fractions α_k (knowing that $\alpha_1 + \alpha_2 = 1$). The model, which was first introduced by Baer & Nunziato in [1], reads

$$\left\{\begin{array}{l} \partial_t \alpha_1 + u_2 \partial_x \alpha_1 = 0, \\ \partial_t(\alpha_1\rho_1) + \partial_x(\alpha_1\rho_1 u_1) = 0, \\ \partial_t(\alpha_1\rho_1 u_1) + \partial_x(\alpha_1\rho_1 u_1^2 + \alpha_1 p_1) - p_1 \partial_x \alpha_1 = 0, \\ \partial_t(\alpha_1\rho_1 E_1) + \partial_x(\alpha_1\rho_1 E_1 u_1 + \alpha_1 p_1 u_1) - p_1 u_2 \partial_x \alpha_1 = 0, \\ \partial_t(\alpha_2\rho_2) + \partial_x(\alpha_2\rho_2 u_2) = 0, \\ \partial_t(\alpha_2\rho_2 u_2) + \partial_x(\alpha_2\rho_2 u_2^2 + \alpha_2 p_2) - p_1 \partial_x \alpha_2 = 0, \\ \partial_t(\alpha_2\rho_2 E_2) + \partial_x(\alpha_2\rho_2 E_2 u_2 + \alpha_2 p_2 u_2) - p_1 u_2 \partial_x \alpha_2 = 0. \end{array}\right. \tag{1}$$

The phasic total energies are given by $E_k = e_k(\rho_k, p_k) + \dfrac{u_k^2}{2},\ k \in \{1, 2\}$, where $e_k(\rho_k, p_k)$ is the internal energy of phase k, assuming some equation of state.

In this work, we propose a fractional step method for computing approximate solutions of the Baer-Nunziato two-phase flow model. The scheme relies on an operator splitting method corresponding to a separate treatment of fast propagation phenomena due to the acoustic waves on the one hand, and slow propagation phenomena due to the fluid motion on the other. For each step of the splitting method, we provide a very simple and robust numerical treatment.

In addition to the preservation of positive values of the statistical phase fractions and densities, the scheme is proved to satisfy a numerical entropy inequality. We also provide some test-cases that assess the convergence of the method.

References

[1] M.R. Baer and J.W. Nunziato, A two-phase mixture theory for the deflagration -to-detonation transition (ddt) in reactive granular materials, *International Journal of Multiphase Flow*, **12** (1986), pp. 861-889

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