Compressible modeling of a cloudy atmosphere using a general pressure evolution equation

ASAM
All Scale Atmospheric Model

Michael Jähn, Oswald Knoth
Leibniz Institute for Tropospheric Research, Leipzig
jaehn@tropos.de, knoth@tropos.de

Introduction

An accurate formulation of a pressure evolution equation that is valid for simulating atmospheric cloud processes is presented by using an approach of Fedkiw et al. [2002]. This equation is coupled with a conservative prognostic total energy equation. Although the description of dissipative heating and moist processes will yield higher complexity, it is a desired criterion to ensure global energy conservation. Simulation results can differ significantly if traditional prognostic equations with simplifications are used (e.g. dry potential temperature, cf. Bryan and Fritsch [2002] and Marquet [2011]). For numerical reasons, it could be more effective to use an additional pressure tendency equation than a diagnostic relation, where the pressure is a function of the thermodynamical variable and the additional moisture variables. Spatial discretization is realized by standard finite-volume methods. The used numerical solver is ASAM (**A**ll **S**cale **A**tmospheric **M**odel). In this model the Euler equations which are written in flux form are solved in an Eulerian framework (Knoth [2012]). For the time integration we use an implicit procedure by Rosenbrock time integrators with an approximate Jacobian. To compare the different model setups, simulation results of an idealized 2D test case are shown.

Conservative Euler equations for compressible flow

The Euler equations consist of the continuity equation for the total mass density, the momentum equation and an energy equation (which is represented by a thermodynamical variable). To simulate cloud processes, the equation system has to be extended by continuity equations for the water substances. In our case, these are the specific densities of water vapor $q_v = \rho_v/\rho$ and cloud condensate $q_c = \rho_c/\rho$ for the sake of simplicity. Naturally, even more substances could be added to the system, like rain water, snow, ice or number concentrations. To ensure global mass conservation, flux quantities are used in the prognostic equations, e.g. velocity $\mathbf{V} \equiv \rho\mathbf{v}$ or potential temperature $\Theta \equiv \rho\theta$.

(I) "Classical" formulation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0\,, \quad (1)$$

$$\frac{\partial(\rho\mathbf{v})}{\partial t} + \nabla(\rho\mathbf{v}\cdot\mathbf{v}) = -\nabla p - \rho\mathbf{g}\,, \quad (2)$$

$$\frac{\partial(\rho\theta_\rho)}{\partial t} + \nabla\cdot(\rho\theta_\rho\mathbf{v}) = Q_\theta\,, \quad (3)$$

$$\frac{\partial(\rho q_v)}{\partial t} + \nabla\cdot(\rho q_v\mathbf{v}) = Q_v\,, \quad (4)$$

$$\frac{\partial(\rho q_c)}{\partial t} + \nabla\cdot(\rho q_c\mathbf{v}) = Q_c\,. \quad (5)$$

Here, ρ is the density, p the pressure, $\mathbf{g} = (0,0,g)$ is the vector of gravitational acceleration and the source terms Q_v and Q_c at the right-hand sides of (4) and (5) represent microphysical conversion processes. Diffusion and Coriolis force are not considered. The quantity θ_ρ is the so-called density potential temperature:

$$\theta_\rho \equiv \tilde{\theta}\frac{1 + r_v/\epsilon}{1 + r_v + r_c}\,, \quad (6)$$

with $\epsilon = R_d/R_v$, the potential temperature for cloudy air $\tilde{\theta} = T/\tilde{\pi}$ where T is the absolute Temperature, $\tilde{\pi} = (p/p_0)^\kappa$ is the Exner function with $\kappa = (R_d + r_v R_v)/(c_{pd} + r_v c_{pv} + r_c c_{pl})$, the reference pressure $p_0 = 1000$ hPa and the mixing ratios for water vapor $r_v = \rho_v/\rho_d$ and cloud water $r_c = \rho_c/\rho_d$. The constants c_{pd}, c_{pv}, c_{pl} are the specific heats of dry air, water vapor and liquid water at constant pressure, respectively. Note that the dry air density can be computed via $\rho_d = \rho - \rho_v - \rho_c$. In this system, the air pressure is a diagnostic function:

$$p = \left(\frac{\rho R_d \theta_\rho}{p_0^\kappa}\right)^{1/(1-\kappa)}\,. \quad (7)$$

(II) Energy formulation

In (3), the thermodynamical variable θ_ρ is replaced by the total energy $E = \rho(e + \phi + K)$, where $e = (q_d c_{vd} + q_v c_{vv} + q_c c_{pl})T + q_v L_{00}$ is the internal energy, $\phi = gz$ is the potential energy and $K = (\mathbf{v}\cdot\mathbf{v})/2$ is the kinetic energy. This leads to the following tendency equation:

$$\frac{\partial E}{\partial t} + \nabla\cdot([E + p]\mathbf{v}) = 0\,, \quad (8)$$

assuming absence of energy source terms, like radiative forcing, for example. This kind of formulation is used in the Nonhydrostatic Icosahedral Atmospheric Model (NICAM), cf. Satoh et al. [2008].

(III) Combined system

Contrary to formulation (I), we now have one additional prognostic equation for the pressure p:

$$\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p = -\rho c^2\nabla\cdot\mathbf{v} + Q_p(Q_v, Q_c)\,, \quad (9)$$

where

$$c = \sqrt{\frac{\partial p}{\partial\varrho} + \frac{p}{\varrho^2}\frac{\partial p}{\partial e} + \frac{\varrho_v}{\varrho}\frac{\partial p}{\partial\varrho_v} + \frac{\varrho_c}{\varrho}\frac{\partial p}{\partial\varrho_c}} \quad (10)$$

is the speed of sound and

$$Q_p = \frac{\partial p}{\partial\varrho_v}Q_v + \frac{\partial p}{\partial\varrho_c}Q_c \quad (11)$$

is a pressure source term. Eqs. (1), (2), (4) and (5) from equation system (I) remain invariant. Note that both terms on the RHS of (9) are dependent on the additional water substances and an equation of state is still mandatory for the partial derivatives.

Pressure update method

Kwatra et al. [2009] present a method for alleviating the stringent CFL condition for low-Mach compressible flows that is based on the pressure evolution equation (9). They focused on highly non-linear flows with shocks, contacts and rarefactions, whereas here the application of the method for atmospheric flows is in the center of attention.

Considering the onedimensional energy formulation of the compressible Euler equations (1), (2) and (8) in matrix form:

$$\begin{pmatrix}\rho\\ \rho u\\ E\end{pmatrix}_t + \begin{pmatrix}\rho u\\ \rho u^2 + p\\ Eu + pu\end{pmatrix}_x = 0 \quad\text{and} \quad (12)$$

$$p_t + u p_x + \rho c^2 u_x = Q_p\,. \quad (13)$$

Separation of the flux term into an advection part and a non-advection part that contains the pressure terms:

$$\mathbf{F_1(U)} = \underbrace{\begin{pmatrix}\rho u\\ \rho u^2\\ Eu\end{pmatrix}}_{\text{advection part}}\,, \qquad \mathbf{F_2(U)} = \underbrace{\begin{pmatrix}0\\ p\\ pu\end{pmatrix}}_{\text{non-advection part}}\,. \quad (14)$$

We use this partition for time integration with Rosenbrock methods where the Jacobian J is splitted corresponding to $\mathbf{F_1(U)}$ and $\mathbf{F_2(U)}$:

$$(I - \Delta t\gamma\mathrm{J}) \approx (I - \Delta t\gamma\mathrm{J}_1)(I - \Delta t\gamma\mathrm{J}_2) \quad\text{and}\quad \mathrm{J} = \mathrm{J}_1 + \mathrm{J}_2\,. \quad (15)$$

Furthermore, J_1 can become more simplified so that all its diagonal elements are equal to $-u$ (cf. Jebens et al. [2011]). The essential procedure of this scheme is the implicit pressure update for the non-advection parts of momentum (2) and energy (8). At the end of each time step the pressure is recalculated via an equation of state of the form:

$$p(\rho, E, \rho_v, \rho_c) = \frac{(\rho_d R_d + \rho_v R_v)(E - \rho_v L_{00} - \rho g z - \rho\frac{\mathbf{v}\cdot\mathbf{v}}{2})}{\rho_d c_{vd} + \rho_v c_{vv} + \rho_c c_{pl}}\,. \quad (16)$$

Test cases

Bryan and Fritsch [2002] created a moist benchmark 2D test case for nonhydrostatic numerical models. To define a thermally neutral, moist atmosphere, the saturation equivalent potential temperature θ_{es} is used for the background profile:

$$\theta_{es} = T\left(\frac{p_0}{p_d}\right)^{\frac{R_d}{c_{pd} + r_t c_{pl}}}\exp\left[\frac{L_v r_v}{(c_{pd} + r_t c_{pl})T}\right]\,. \quad (17)$$

The background profile is given by $\theta_{es,0} = 320$ K. Moreover, the initial sounding is saturated with a constant total water content $q_t = q_v + q_c = 0.02$ and reversible phase changes are assumed (i.e. $Q_v = -Q_c$). In the center of the domain, a warm perturbation is placed and specified by

$$\theta' = 2\cos^2\left(\frac{\pi L}{2}\right)\,, \quad\text{with}\quad L = \sqrt{\left(\frac{x - x_c}{x_r}\right)^2 + \left(\frac{z - z_c}{z_r}\right)^2}\,. \quad (18)$$

with The parameters $x_c = 10$ km, $z_c = 2$ km und $x_r = z_r = 2$ km characterize the position and the radius of the thermal bubble. The total integration time is 1000 s.

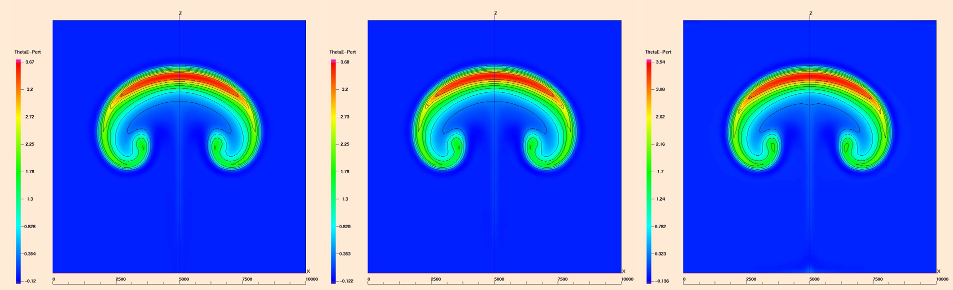

Figure 1:
Results for the moist benchmark simulation with a linear implicit Rosenbrock time integration method. Perturbation equivalent potential temperature θ_e' (8 contour levels) at t = 1000 s. Left: "classical" formulation (I) with θ_ρ as prognostic variable. Middle: energy formulation (II) with E as prognostic variable. Right: Combined system (III) with E and p as prognostic variables.

All three model setups show nearly similar results. Minor differences appear in min/max values of θ_e' and other variables like vertical velocity w (not shown in the picture).

Conclusions and outlook

The implicit pressure update method was successfully applied to ASAM using a Rosenbrock time integration scheme with approximate Jacobian. In this scheme, the pressure is recomputed from an equation of state (16) after each large time step. Contrary to the standard approach there is no need to recompute the pressure on each small time step. Investigations of a more suitable discretization of the pressure evolution equation (9) have to be done in future works.

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