# HYPERBOLIC PROBLEMS IN THE THEORY OF LONGITUDINAL VIBRATIONS OF NON-THIN RODS

Igor Fedotov
*Tshwane University of Technology, Department of Mathematics and Statistics*
fedotovi@tut.ac.za

The longitudinal vibration of rods is normally considered in mathematical physics in terms of the classical model described by the wave equation under assumptions that the rod is thin and relatively long. More general theories were formulated by taking into consideration the effect of the lateral motion effects of shear stress of relatively thick rod and was considered by Rayleigh in 1894 and Bishop in 1952. The Rayleigh-Bishop model is described by a fourth order partial differential equation not containing the fourth time derivative. This model was generalized by Mindlin and Hermann. They considered the lateral displacement proportional to an independent function of time and the longitudinal coordinate. This result can be formulated as an equation of forth order resolved with respect to the highest order time derivative. To obtain more general class of equations, the displacements of rods are expressed in the form of a power series expansion in the lateral coordinate. We wish to classify all of the above mentioned equations within the frame of the general theory of hyperbolic equations (are they strictly hyperbolic, hyperbolic or pseudohyperbolic). The Study of General hyperbolic equations was launched in 1937 by I.G. Petrovsky in his paper on Cauchy problems where he gave a general definition of the hyperbolicity. The initial Petrovsky's results are complete. Further development of the theory was concerned not with obtaining new profound results but rather with the improvement of methods of proofs and the application of modern tools such as Distribution Theory. The Monograph of Leray (1952) can be considered as the next step in this direction. Further substantial progress was made by Garding (1957). In 1938 Petrovsky extended his theory to general systems of partial differential equation not resolved with respect to the highest time derivative. The interest in such problems returned after S.L. Sobolev's paper appeared in 1954. Following Sobolev's investigations S.A. Galpern (1960 and 1963) considered differential operators not resolved with respect to highest time-derivative. A detailed survey of such problems can be found in the monograph of Demidenko and Uspensky. We use the approach on the theory of Hyperbolic equations developed by L.R. Volevich. and S.G. Gindikin (1967,1996 and 1999). They obtained deep results concerning mixed problems for general hyperbolic equations. This talk is about a comment on recent findings of I. Fedotov and L.R. Volevich (2006) which should provide a thorough understanding of the hyperbolic and pseudohyperbolic operator arising in the theory of longitudinal vibrations of elastic bars. Invertibility of some hyperbolic problems is discussed.

---

Joint work with: Herve Michel Tenkam (*Tshwane University of Technology, Department of Mathematics and Statistics*), Micheal Shatalov (*Tshwane University of Technology, Department of Mathematics and Statistics, CSIR, MSM*).