Boundary Controllability of a Hyperbolic PDE with ODE Boundary Conditions Modeling a One-Dimensional Flow

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We study a model that describes the flow of an incompressible fluid in an elastic tube that is connected to two tanks. The model is based on Euler's continuity equation and the law of balance of momentum; it is a system of quasilinear PDEs with nonlinear ODE boundary conditions. The system is linearized about the equilibrium state and semigroup theoretic proofs of well-posedness and stability of this linear system are sketched. Based on this model, a boundary control system is considered. Using some perturbation results for discrete spectral operators and operators in finite-dimensional spaces, it is shown that the normalized eigenvectors of the operator associated to the system form a Riesz basis of the state space, provided that the viscosity is sufficiently small. This induces a proof that the system is exactly controllable. Using a generalized Kadec's one-quarter theorem, a minimal time of controllability is given for single input controls. The control can be obtained by minimizing a cost functional with PDE constraints.

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