Continuum Models of a Limit Order Book

Giancarlo Facchi
The Pennsylvania State University
facchi@math.psu.edu

An external buyer asks for a random amount $X > 0$ of a certain asset. X is distributed according to a probability distribution μ on $\mathbb{R}_+$. We assume that this external buyer will buy the amount X at the lowest available price, as long as this price does not exceed a given upper bound.

Different agents are willing to sell various quantities of the asset at different prices, competing against each other in order to fulfill the random incoming market order. We denote by $u_i(p)$ the density of sell limit orders at price p, placed by agent i. In other words, for any $p > 0$,

$$U_i(p) = \int_0^p u_i(s)\,ds = \left[\begin{array}{c}\text{total amount of asset offered}\\ \text{for sale at price } \leq \ p \text{ by agent } i\end{array}\right]. \tag{1}$$

We consider two types of agents. “Small agents” own a very small amount of assets, compared to the total amount of asset on sale, therefore a single agent cannot influence the probability of execution of the sell orders of the other agents. On the other hand, a “large agent” can significantly influence the probability of execution of the limit orders placed by other competing agents.

Our main goal is to study the existence, uniqueness and stability of a Nash equilibrium, where each agent follows his best strategy, in reply to the actions of all other agents. We seek to describe the optimal strategies for the various agents and estimate their expected payoffs.

We consider several market models where different types of agents interact. The Nash equilibrium is determined as the asymptotic limit for $t \to \infty$ of the solution to a system of conservation laws with nonlocal flux:

$$u_{i,t} + F_i\left(p, u_1, \ldots, u_m, U\right)_p = 0, \quad i = 1, \ldots, m,$$

where $U(p) = \sum_{i=1}^{m} U_i(p)$, and u_i and U_i are as in (1).

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