# Global existence and energy decay of solutions for a nondissipative wave equation with a time varying delay term

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We consider the energy decay for nondissipative wave equation in a bounded domain with a time varying delay term in the internal feedback. We use an approach introduced by Guesmia which leads to decay estimates (known in the dissipative case) when the integral inequalities method due to Haraux-Komornik [4] cannot be applied due to the lack of dissipativity. First we study the stability of a nonlinear wave equation of the form

$$u_{tt}(x,t) - \Delta_x u(x,t) + \mu_1\sigma(t)u_t(x,t) + \mu_2\sigma(t)u_t(x,t-\tau(t)) + \theta(t)h(\nabla_x u) = 0$$

in a bounded domain. We consider the general case with a nonlinear function $h$ satisfying a smallness condition, and obtain the decay of solutions under a relation between the weight of the delay term in the feedback and the weight of the term without delay. We impose no control on the sign of the derivative of the energy related to the above equation.

In the second case we consider the case $\theta \equiv const$ and $h(\nabla u) = -\nabla\Phi\nabla u$. We prove an exponential decay result of the energy without any smallness condition on the function $h$.

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