Wave propagation in discrete heterogeneous media

Aurora Marica
Basque Center for Applied Mathematics, Bilbao, Basque Country, Spain
marica@bcamath.org

In this talk, we describe the *propagation properties* of the one and two-dimensional *wave* and *transport* equations with *variable coefficients* semi-discretized in space by *finite difference* and P_1-finite element schemes on *non-uniform meshes* obtained as *diffeomorphic transformations* of uniform ones. In particular, we introduce and give a rigorous meaning to notions like the *principal symbol* of the discrete wave operator or the corresponding *bi-characteristic rays*. The main mathematical tool we employ is the *discrete Wigner transform*, which, in the limit as the mesh size parameter tends to zero, yields a measure propagating along curves which are solutions of a *Hamiltonian system*. Of course, due to dispersion phenomena, the high frequency dynamics does not coincide with the continuous one. Our analysis holds for $C^{1,1}(\mathbb{R}^d)$-coefficients and $C^{2,1}(\mathbb{R}^d)$-diffeomorphic transformations defining the grid. We also present several numerical simulations that confirm the predicted paths of the space-time projections of the bi-characteristic rays. Based on the theoretical analysis and simulations, we describe some of the *pathological phenomena* that these rays might exhibit as, for example, their *reflection before touching the boundary* of the space domain. This leads, in particular, to the failure of the classical properties of *boundary observability* of continuous waves, arising in *control* and *inverse problems* theory.

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