Viscous Profiles for Shock Waves in Isentropic Magnetohydrodynamics

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Standing planar waves in isentropic magnetohydrodynamics (IMHD) are governed by the autonomous system

$$\begin{aligned} \mu v' &= mv + p(v) + \frac{1}{2}|\mathbf{b}|^2 - j, \\ \nu \mathbf{w}' &= m\mathbf{w} - a\mathbf{b}, \\ \eta \mathbf{b}' &= v\mathbf{b} - a\mathbf{w} - \begin{pmatrix} c \\ 0 \end{pmatrix}, \end{aligned} \tag{Σ}$$

of ordinary differential equations in $\mathbb{R}^5$, where the usual physical notations have been adopted; the longitudinal components of momentum $m := \rho v$ and magnetic field a are known to be constant. The constants $\mu - \nu > 0$, $\nu > 0$, $\eta > 0$ represent the fluid's longitudinal and transversal viscosity, and electrical resistivity, respectively. Furthermore, $p(v) = \hat{p}(m/v)$ is derived from a general barotropic pressure law $\hat{p} = \hat{p}(\rho)$. Only two constants of integration have to be considered, namely $j \in (-\infty, +\infty)$ and $c \in [0, +\infty)$.

A heteroclinic orbit connecting two rest points $\mathbf{u}^{\pm}$ of Σ corresponds to a *viscous profile* for the standing shock wave with the states $\mathbf{u}^{-}$ and $\mathbf{u}^{+}$.

We present results from [3,4] which show that (i) system Σ is gradient-like, (ii) there are up to four isolated rest points, (iii) heteroclinic orbits between the rest points do or do not exist depending on the respective ratios of (μ, ν, η). For the proof of (iii) we use the Conley index as in [1] and geometric singular perturbation theory as in [2].

References

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