Shock waves in quasi-thermal-incompressible materials

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Incompressibility is a useful idealization for materials characterized by an extreme resistance to volume changes. For pure mechanical problems, i.e. where no change in temperature is involved, an incompressible material is easily understood as a material whose density is constant; in this case the solutions of model equations for incompressible fluids are obtainable as the limit case of the corresponding models involving compressible fluids [1].

For thermomechanical problems, i.e. when the processes are not isothermal, the definition itself of incompressibility is not straightforward and several models have been proposed. The first model of incompressibility was characterized by the independence of all the constitutive equations on the pressure [2], which leads to the conclusion – in strike contrast with experimental evidence – that the density must be constant [3]. A second, less restrictive, model requires that the only constitutive function independent of the pressure is the specific volume [4]. Such a definition of incompressibility allows to avoid the problems raised by Müller's definition but it is still not satisfactory as in this model instabilities affect wave propagation: since the chemical potential is not concave, the sound velocity might become imaginary, therefore losing the hyperbolicity of the system of Euler equations.

In order to solve these inconveniences, a new model of incompressibility has recently been proposed by Ruggeri & Gouin [5]. According to this model, a material is called *quasi-thermal-incompressible* (QTI) if the specific volume, V, and the specific internal energy, ϵ, differ to order δ^2 from functions depending only on the temperature T:

$$V\left(p,T\right) = V_0 + \delta W\left(T\right) - \delta^2 U\left(p,T\right), \qquad \epsilon\left(p,T\right) = e\left(T\right) - \delta T W'\left(T\right) + \mathcal{O}\left(\delta^2\right),$$

where δ is a small dimensionless parameter, V_0 is a constant, $W\left(T\right)$, $U\left(p,T\right)$ and $e\left(T\right)$ are constiutive functions chosen in agreement with thermodynamics restrictions. It is remarkable that QTI materials are compressible fluids that approximate incompressible fluids to order δ^2 in the sense of Müller's definition.

The purpose of the present work is to analyze wave propagation, in particular shock waves, is QTI materials. The limit case with $\delta \to 0$ (corresponding to incompressible materials) is going to be investigated as well.

References

[1] P. L. Lions, N. Masmoudi, "Incompressible limit for a viscous compressible fluid", *Journal de Mathématiques Pures et Appliquées* 77, 585–627 (1998)

[2] I. Müller, *Thermodynamics*, Pitman, London (1985)

[3] H. Gouin, A. Muracchini, T. Ruggeri, "On the Müller paradox for thermal-incompressible media", *Continuum Mech. Thermodyn.*, DOI: 10.1007/s00161-011-0201-1 (2012), in press

[4] K. R. Rajagopal, M. Ruzika, A. R. Srinivasa, "On the Oberbeck-Boussinesq approximations", *Mathematical models and methods in applied sciences* 6, 1157–1167 (1996)

[5] H. Gouin, T. Ruggeri, "A consistent thermodynamical model of incompressible media as limit case of quasi-thermal-incompressible materials", *Internat. J. Non-Linear Mech.*, DOI:10.1016/j.ijnonlinmec.2011.11.005 (2012), in press

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