Shallow water equations for horizontal-shear flows: characteristics, analytical and numerical solutions

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The talk focuses on the theoretical analysis of the equations

$$\begin{cases} u_t + uu_x + vu_y + gh_x = 0, \quad h_y = 0, \\ h_t + (uh)_x + (vh)_y = 0, \quad uY_i'(x) - v\big|_{y=Y_i} = 0, \quad (i = 1, 2), \end{cases}$$

describing open channel flows of ideal incompressible fluid with horizontal velocity shear in the long wave approximation [1]. Here (u, v) is the fluid velocity; h is the free-surface height over the flat bottom $z = 0$; $y = Y_1(x)$ and $y = Y_2(x)$ are the lateral channel walls; and g is the constant gravity acceleration. In the semi-Lagrangian frame of reference this model transforms to the integrodifferential equations. Theoretical analysis of the model is based on the proposed by V. M. Teshukov concept of hyperbolicity for systems of equations with operator coefficients. A distinctive feature of integrodifferential models is the presence of both discrete and continuous spectrum of characteristic velocities.

Necessary and sufficient conditions of generalized hyperbolicity for the equations of motion are formulated. An example of verification of the hyperbolicity conditions is given, and an analogy with the well-known stability criterion for shear flows is noted. Exact (in particular, periodical) solutions of the model are constructed and interpreted physically for the class of traveling waves. It is shown that traveling waves are stable in the linear approximation only in the case of an insignificant change in the fluid depth. Differential balance laws approximating the basic integrodifferential equation are proposed. These equations are used to perform numerical calculations of the waves propagation.

The concepts of sub- and supercritical flows are introduced for the model describing the steady-state horizontal-shear shallow flows of an ideal incompressible fluid with a free boundary in a channel of variable cross-section [2]. Internal structure of flow developed in a local channel contraction or expansion is analyzed. Continuous and discontinuous exact solutions describing different flow regimes are constructed and their properties are studied. Analytical solutions for flows with the formation of recirculation zones are obtained.

References

[1] Chesnokov A.A., Liapidevskii V.Yu., Wave motion of an ideal fluid in a narrow open channel, *J. App. Mech. Tech. Phys.*, **50** (2009), pp. 220–228.

[2] Liapidevskii V.Yu., Chesnokov A.A., Sub- and supercritical horizontal-shear flows in an open channel of variable cross-section, *Fluid Dynamics*, **44** (2009), pp. 903–916.

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