A finite volume approximation of a 2 Layer system for growth of sandpile based on schemes for discontinuous flux for hyperbolic conservation laws

Aekta Aggarwal
TIFR Centre for Applicable Mathematics,Bangalore,India
aekta@math.tifrbng.res.in

We propose an explicit finite volume numerical scheme for a system of partial differential equations proposed in [3], a model for growing sandpiles under a vertical source on a flat bounded table, based on schemes for discontinuous flux for hyperbolic conservation laws. In such a system, an eikonal equation for the standing layer of the pile is coupled to an advection equation for the rolling layer. The model in one dimension is given by

$$v_t - (vu_x)_x = -(1 - |u_x|)v + f, \text{on}[0,1] \times (0,T) \tag{1}$$

$$u_t = (1 - |u_x|)v, \text{in}[0,1] \times (0,T) \tag{2}$$

with initial condition

$$u(x,0) = 0 = v(x,0) \forall x \in (0,1)$$

and boundary condition

$$u(0,t) = u(1,t) = 0$$

where, f is a given poisitive source.

The idea here is to include the source term f in the form of an integral with the flux term,i.e.$-(vu_x)_x$ and use the idea of well balanced schemes proposed by Mishra [2]. Since the equation (1) is a first order pde with discontinous coefficient $u_x(x,t)$ in space variable,we approximate (1) using the idea of discontinuous flux for hyperbolic conservation laws proposed by Gowda and Adimurth [1]. Our schemes are monotone and can be extended to higher dimensions. We prove some basic estimates about the physical properties of the model. We compare our scheme and the results of the numerical experiments established in 1 and 2 dimension with the finite difference schemes proposed by Falcone and Vita[4]. Our schemes work for larger CFL.

References

[1] Adimurthi, J. Jaffre and G.D.Veerappa Gowda, Godunov type methods for conservation laws with flux function discontinuous in the space variable, *SIAM J. of Numerical Analysis* **42** (2004), pp. 179-208

[2] Kenneth Hvistendahl Karlsen, Siddhartha Mishra and Nils Henrik Risebro, Well-balanced schemes for conservation laws with source terms based on a local discontinuous flux formulation, *Math. Comp.* **78** (2009), pp. 55-78.

[3] K.P. Hadeler and C. Kuttler, Dynamical models for granular matter, *Granular Matter* **2** (1999), pp. 9-18.

[4] M. Falcone and S. Finzi Vita, A finite-difference approximation of a two-layers system for growing sandpiles, SIAM 2006.

Joint work with: Second Author G D V Gowda (*Professor, TIFR Centre For Applicable Mathematics*)