The Fourtheenth International Conference on

HYPERBOLIC PROBLEMS: Theory Numerics and Applications

June 25-29, 2012



BOOK OF ABSTRACTS

UNIVERSITY OF PADOVA



Sponsors

We would like to gratefully acknowledge partial support from the following sponsors:

- \star ERC Starting Grant 2009 "Hyperbolic Systems of Conservation Laws: Singular Limits, Properties of Solutions and Control Problems"
- \star Research Project MIUR PRIN 2009 "Systems of Conservation Laws and Fluid Dynamics: Methods and Applications"
- \star GDRE CONEDP European Research Group on PDE control
- * FP7-PEOPLE-2010-ITN European Research Network "Sensitivity Analysis for Deterministic Controller Design"
- $\star\,$ Progetto di Eccellenza Fondazione Cariparo 2009-2010 "Nonlinear PDEs: Models, Analysis, and Control-theoretic Problems"
- * ERC Starting Grant 2010 "Traffic Management by Macroscopic Model"
- \star Dipartimento di Matematica, Università di Padova
- \star Dipartimento di Matematica Pura ed Applicata, Università degli Studi dell'Aquila
- ★ Università di Padova
- ★ University of Zürich
- \star University of Basel



Contents

1	Committees	11
2	Welcome from OC and SC Chairs	13
3	Abstracts of plenary lectures	15
	Camillo De Lellis	15
	Siddhartha Mishra	16
	Nader Masmoudi	17
	Igor Rodnianski	19
	Sijue Wu	19
	Yoshio Sone	20
	Song Jiang	20
	Enrique Zuazua	21
	Eduard Feireisl	21
	James Sethian	24
	Giovanni Russo	24
	Alexis Vasseur	27
		~ ~
4	Abstracts of invited lectures	29
	Mikhail Perepelitsa	29
	David Gerard-Varet	29
	Seung-Yeal Ha	30
	Michael Shearer	30
	Gigliola Staffilani	31
	Kun Xu	33
	Corrado Lattanzio	34
	Karine Beauchard	35
	Shih-Hsien Yu	35
		36
	Michael Herty	37
	Shigeru Takata	38
	Steve Shkoller	39
	Jean-Luc Guermond	40
	Paolo Secchi	40
	Moritz Reintjes	42
	Xavier Raynaud	42
	Gianluca Crippa	43
	Joachim Krieger	44
5	Abstracts of contributed lectures — Monday 15,15–16,15	47
-	5.1 Session $1 - Room F - Numerical Methods I$	47
	Arun Koottungal	47
	Uijwal Koley	48
	5.2 Session $2 - Room D - Navier-Stokes and Euler Equations I \dots \dots \dots \dots$	49
	Nikolav Gusev	49
	Matthias Kotschote	50
	5.3 Session 3 — Room H — Numerical Methods II	50
	Raul Borsche	50
	Andreas Hiltebrand	51
	5.4 Session λ — Room I — Numerical Methods III	52
	Manuel I. Castro Díaz	52
	Clement Cances	52
	5.5 Session 5 — Room G — Numerical Methods for Atmospheric and Geophysical Mod-	55
	els I	54
		54

		Ivar Lie
		Ilja Kroeker
	5.6	Session 6 — Room E — Multi Physics Models I $\ldots \ldots $
		Panters Rodriguez-Bermudez
		James Glimm
	5.7	Session 7 — Room A — Theory of Conservation Laws I
		Young-Pil Choi
		Stephane Junca
	5.8	Session 8 — Room B — Reaction-Convection-Diffusion Equations $\ldots \ldots \ldots$
		Mohamed Gazibo
		Andrea Terracina
	5.9	Session 9 — Room C — Control Problems for Hyperbolic Equations I $\ldots \ldots \ldots $ 62
		Alberto Bressan
		Shyam Ghoshal
6	\mathbf{Abs}	tracts of contributed lectures — Monday 17.20–19.20 65
	6.1	Session $10 - Room F - Numerical Methods IV \dots $
	-	Christiane Helzel
		Jun Luo
		Arnaud Duran
		Helen Yee
	6.2	Session $11 - Room D - Navier-Stokes$ and Euler Equations II
		Helena J. Nussenzveig Lopes
		Donatella Donatelli
		Konstantina Trivisa
		Fucai Li
	6.3	Session 12 — Room H — Numerical Methods V $\ldots \ldots $
		Yohan Penel
		Wei Junxia
		Knut Waagan
		Friedemann Kemm
	6.4	Session 13 — Room I — Numerical Methods VI $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots $ 75
		Jochen Schuetz
		Laurent Boudin
		Mathieu Girardin
		Philip Lawrence Roe
	6.5	Session 14 $-$ Room G $-$ Numerical Methods for Atmospheric and Geophysical
		<i>Models II</i>
		Oswald Knoth
		Thomas Mueller
		Maria Lukacova
		Dante Kalise
	6.6	Session 15 — Room E — Multi Physics Models II
		Dan Marchesin
		Julio Silva
		Vitor Matos
	o =	Mirko Kraenkel
	6.7	Session 10 — Koom A — Theory of Conservation Laws II
		Franziska weber
		Evgeniy Panov
	60	Darko Mitrovic
	0.8	Olindo Zanotti
		Mahmoud Abdolrahman
		Wannouu Abuenannan 90 Vuri Trakhinin 01
		1011 11a.xiiiiiiii

		Baver Okutmustur
	6.9	Session $18 - Room C - Control Problems for Hyperbolic Equations II 93$
		Peipei Shang
		Sebastian Pfaff
		Fabio Priuli
		Mohamed Kanso
7	Abs	tracts of contributed lectures — Tuesday 15.15–16.15 97
	7.1	Session 19 — Room E — Numerical Methods VII $\ldots \ldots \ldots \ldots \ldots \ldots \ldots $ 97
		Philip Lawrence Roe
		Rodolphe Turpault
	7.2	Session 20 — Room C — Navier-Stokes and Euler Equations III $\ldots \ldots \ldots $ 99
		Quansen Jiu
		Paola Trebeschi
	7.3	Session 21 — Room G — Numerical Methods VIII $\ldots \ldots \ldots$
		Nikolaos Sfakianakis
		Espen Jakobsen
	7.4	Session 22 — Room H — Numerical Methods IX $\ldots \ldots \ldots$
		Vivien Desveaux
		Marica Pelanti
	7.5	Session 23 — Room F — Complex and Social Models
		Suncica Canic
		Rinaldo Colombo
	7.6	Session 24 — Room I — Multi Physics Models III
		Olivier Rouch
		Khaled Saleh
	7.7	Session 25 — Room A — Theory of Conservation Laws III
		Florent Renac
		Zhi-Qiang Shao
	7.8	Session 26 — Room D — Kinetic Models I $\ldots \ldots $
		Giuseppe Savaré
		Eitan Tadmor
	7.9	Session 27 — Room B — Control Problems for Hyperbolic Equations III 112
		Zhiqiang Wang
		Vincent Perrollaz
_		
8	Abs	tracts of contributed lectures — Tuesday 17.20–19.20 114
	8.1	Session $28 - Room E - Numerical Methods X \dots $
		Smadar Karni
		Dietmar Kroener
		Christoph Gersbacher
		Monika Twarogowska
	8.2	Session 29 — Room C — Navier-Stokes and Euler Equations IV $\ldots \ldots \ldots$
		Sylvie Benzoni-Gavage
		Elisabetta Chiodaroli
		Luigi Berselli
		Stefano Spirito
	8.3	Session $30 - Room \ G - Numerical \ Methods \ XI \ \ldots \ \ldots \ \ldots \ \ldots \ \ldots \ 119$
		Livio Pizzocchero
		Ulrik Fjordholm
		Guoxian Chen
		Matteo Semplice
	8.4	Session 31 — Room H — Numerical Methods XII
		Jonas Sukys
		Maya Briani
		Christoph Zeiler

		Aslan Kasimov	
	8.5	Session 32 — Room F –	- Relaxation Processes and Complex Models
		Marta Strani	
		Emmanuel Audusse	
		Helene Mathis	
		Christophe Chalons	
	8.6	Session 33 — Room I –	- Multi Physics Models IV
		Pablo Castaneda	
		Sudarshan Kumar Kenettink	ara
		Raimund Buerger	
		Riccardo Adami	
	8.7	Session 34 — Room A -	- Theory of Conservation Laws $IV \ldots \ldots \ldots \ldots \ldots 136$
		Edwige Godlewski	
		Marguerite Gisclon	
		Barbara Lee Keyfitz	
		Boris Haspot	
	8.8	Session 35 — Room D -	- Kinetic Models II
		Yi Wang	
		Frederique Charles	
		$\begin{array}{cccc} 1 \text{-Kun Chen} & \ldots & \ldots & \ldots \\ K & C & \vdots & W \end{array}$	
	0.0	Kung-Chien Wu	
	8.9	Session 36 — Room B –	- Control and Geometric Problems for Hyperbolic Equations143
		Matthias Kawski	
		Giuseppe Maria Cociite	
		Paulo Amonim	
		Paulo Amorini	
9	Abs	tracts of contributed lect	ıres — Thursday 9.15–9.45 146
	9.1	Session 37 — Room F –	- Numerical Methods XIII
		Bruno Després	
		Manuel J. Castro Díaz	
	9.2	Session 38 — Room B –	- Navier-Stokes and Euler Equations $V \ldots \ldots \ldots \ldots \ldots 148$
		Nikolai Chemetov	
		Joshua Ballew	
	9.3	Session 39 $-$ Room G $-$	- Numerical Methods XIV
		Grady Lemoine	
		Fausto Cavalli	
	9.4	Session 40 $-$ Room A $-$	- Convective Flows
		Christian Klingenberg	
		Marco Di Francesco	
	9.5	Session 41 — Room E –	- Electromagnetic Flows $I \dots $
		Paolo Corti	
		Andrew McMurry	
	9.6	Session 42 — Room D -	$- PDEs in Mathematical Physics \ldots \ldots \ldots \ldots \ldots 154$
		Felipe Linares	
	~ -	Irina Kmit	
	9.7	Session 43 — Room C –	- Theory of Conservation Laws V
		Hermano Frid \ldots	
	0.0	Benjamin Boutin	Die Fluide Medele I
	9.8	Session 44 — Koom H -	
		Jan Ernest	
	0.0	Francois James	Hudrodenamian and Simulations
	9.9	Bajan Arora	11901-11901-1100 מוום אווישטונטונג
		Rortram Taotz	

10 Ab	stracts of contributed lect	ures — Thursday 11.20–12.50	161
10.1	1 Session 46 - Room F -	$- Numerical Methods XV \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$. 161
	Eitan Tadmor		. 161
	Alina Chertock		. 162
	Christophe Berthon		. 163
10.2	2 Session 47 - Room B -	- Navier-Stokes and Euler Equations VI	. 164
	Masashi Ohnawa	· · · · · · · · · · · · · · · · · · ·	. 164
	Joseph Roberts		. 165
	Corentin Audiard		167
10 5	Session $l8 - Boom G$ -	– Numerical Methods XVI	168
10.0	Friend Brissid Storreston		168
	Vittorio Bispoli		160
	Armanda Casa		170
10		TTT A	170
10.4	Exercise Herbinster	$-$ wave Analysis 1 \ldots \ldots \ldots \ldots \ldots \ldots	. 1/1
	Itsuko Hasnimoto		. 1/1
			. 172
10.	Jan Giesselmann		. 173
10.5	5 Session 50 — Room E –	- Electro-Magnetic Flows & High Frequency Phenomena .	. 174
	Denise Aregba-Driollet		. 174
	Yong Lu		. 176
	Lorenzo Zanelli		. 177
10.6	5~Session~51-Room~D -	- Traffic Flow and Population Dynamics	. 177
	Maria Laura Delle Monache		. 177
	Massimiliano Rosini		. 179
	Stefan Berres		. 181
10.7	7 Session 52 — Room C -	- Theory of Conservation Laws VI	. 181
	Boris Andreianov		. 181
	Laura Spinolo		. 183
	Khai Nguyen		. 183
10.8	3 Session 53 - Room H -	– BioFluids Models II	. 184
	Sulevman Ulusov		. 184
	Jinhuan Wang		. 185
	Shumaila Javeed		186
10 0	Session $5l - Boom I -$	- Models and Simulations in Mechanics	187
10.0	Filipe Carvalho		187
	Maren Hantke		187
	Martin Rybicki		188
	Martin Rybicki		. 100
11 Ab	stracts of contributed lect	ures — Thursday 14.55–15.55	189
11.1	Session 56 — Room A -	– Nonlinear Wayes I	. 189
	Francoise Foucher		. 189
	Johannes Hoewing		189
11 5	Session $57 - Boom G$ -	– Numerical Methods XVII	190
11.2	Karol Mikula		190
	Argiris Delis		101
11 9	$\begin{array}{c} \text{Argnis Dens} \\ \text{Section 50} \\ $	– Machanice and Fluide	102
11.0	Alerei Meilyheev	- mechanics and Flatas	. 192
	Alexander Kha		100
11	Alexander Ale \dots \mathcal{A}	Waya Dattama Anglusia I	102
11.4	E Session OU - KOOM U -	— wave Famerns Analysis I	. 193
	Manir Hadzic		1. 193
4 4 4	$\begin{array}{c} \text{Ramon Plaza} \\ \text{C} \\$. 193
11.8	$ Session \ b1 - Room \ E - $	- Ineory of Conservation Laws VII	. 194
	Laura Caravenna		. 194
	Daniela Tonon	······································	. 195
11.6	Session 62 — Room I –	- BioFluids Models III	. 196
	Patrizia Bagnerini		. 196

8

	11.7	Anne-Celine Boulanger Session 63 — Room	 F —	Mean Curvature Motion and Moving Interfaces	 	 	. 197 . 198
		Yves Achdou Peter Frolkovic	· · · · · · · ·		· · · ·	· · · ·	. 198 . 199
12	Abs	tracts of contributed	lectur	res — Thursday 17.00–19.30			200
	12.1	Session 64 — Room	D -	Numerical Methods XVIII			. 200
		Roger Kaeppeli					. 200
		Michael Dudzinski					. 201
		Sigrun Ortleb					. 201
		Sebastiano Boscarino .					. 203
	12.2	Session $65 - Room$	A -	Nonlinear Waves II			. 203
		Alessia Ascanelli					. 203
		Katrin Grunert					. 204
		Frederic Lagoutiere					. 205
		Wladimir Neves					. 206
		Simonetta Abenda					. 206
	12.3	Session 66 — Room	G -	Numerical Methods XIX			. 207
		Jacques Sainte-Marie .					. 207
		Barry Koren					. 208
		Frederike Kissling					. 209
		Bojan Popov					. 210
		Alexander Kurganov .					. 211
	12.4	Session 67 — Room	B -	Conservation Laws and Applications I			. 212
		Kazuaki Nakane		•••			. 212
		Thomas Auphan					. 214
		Andrea Corli					. 215
		Graziano Guerra					. 216
		Mauro Garavello					. 217
	12.5	Session 68 — Room	H -	Kinetic Models III			. 218
		Zhenving Hong					. 218
		Gabriella Puppo					. 220
		Xiuqing Chen					. 221
		Renjun Duan					. 221
		Bugra Kabil					. 222
	12.6	Session 69 — Room	c -	Wave Patterns Analysis II			. 224
		Denis Serre		· · · · · · · · · · · · · · · · · · ·			. 224
		Heinrich Freistuhler .					. 224
		Benjamin Texier					. 225
		Dmitry Tkachev					. 226
		Allen Tesdall					. 227
	12.7	Session 70 - Room	E -	Conservation Laws and Applications II			. 228
		Franz Achleitner					. 228
		Hantaek Bae					. 229
		Caroline Bauzet					. 229
		Nicolas Seguin					. 230
		Johannes Waechtler .					. 232
	12.8	Session 71 - Room	I - I	Phase Field Models			. 232
		Frederico Furtado					. 232
		Eduardo Abreu					. 233
		Sebastian Noelle					. 234
		Fumioki Asakura					. 235
		Olga Rozanova					. 236
	12.9	Session 72 $-$ Room	F -	Numerical Methods for Hamilton-Jacobi Equation	ns		. 237
		Roberto Ferretti					. 237
		Olivier Bokanowski					. 239

		Kristian Debrabant	40
		Adriano Festa	41
		Martin Nolte	42
19	Abs	tracts of posters	12
10	A DS 13 1	Monday 14 00-16 35 Via Bassi Rooms first floor	±0 43
	10.1	$\begin{array}{c} \text{Abbas Banaissa} \end{array} \begin{array}{c} 2 \end{array}$	40 // 3
		Carlo Bianca	43 43
		Giancarlo Facchi 2	10 44
		Temple Fav	45
		Mauro Gaggero	46
		Michael Jaehn	10 47
		Mehmet Koksal	47
		Thomas Maerz	47
		Aurora Marica	48
		Nadine Naidi	49
		Gilbert Peralta	50
		Ferdinand Thein	50
		Amit Tomar	51
	13.2	Tuesday, 14.00–16.35, Via Bassi Rooms, first floor	52
		Alexander Chesnokov	52
		Igor Fedotov	53
		Ee Han	53
		Andreas Klaiber	54
		Mingjie Li	55
		Tohru Nakamura	55
		Se-Eun Noh	56
		Michael Shatalov	56
		Vladimir Shelkovich	57
		Changhui Tan	59
		Yoshihiro Ueda	60
		Yakov Yakubov	61
	13.3	Thursday, 14.00–16.15, Via Bassi Rooms, first floor	61
		Aekta Aggarwal	61
		Slavko Brdar	62
		Guillaume Clair	63
		Olivier Delestre	64
		Katharma Elsen	64
		Jean-Marc Herard	65
		Stephan Joubert	00
		Lilla Krivodonova	00 67
		narisii Kuillar	01 67
		Andrea Mentrem	07 60
		Robabah Sahandi	09 60
			09

1 Committees

Organizing Committee

Co-Chair: Fabio Ancona, Padova **Co-Chair:** Andrea Marson, Padova

Members:

Debora Amadori, L'Aquila Franco Cardin, Padova Annalisa Cesaroni, Padova Giuseppe M. Coclite, Bari Gianluca Crippa, Basel Camillo De Lellis, Zürich Carlotta Donadello, Besançon Donatella Donatelli, L'Aquila Mauro Garavello, Milano Paola Goatin, Sophia Antipolis Graziano Guerra, Milano

Scientific Committee

Co-Chair: Alberto Bressan, University Park PA **Co-Chair:** Pierangelo Marcati, L'Aquila

Members:

Martino Bardi, Padova Francois Bouchut, Marne-la-Vallée Stefano Bianchini, Trieste Sunčica Čanić, Houston Gui-Qiang Chen, Oxford Constantine Dafermos, Providence Pierre Degond, Toulouse Björn Engquist, Austin Hermano Frid, Rio de Janeiro Helge Holden, Oslo Rolf Jeltsch, Zürich Song Jiang, Beijing Shuichi Kawashima, Fukuoka Axel Klar, Kaiserslautern Dietmar Kröner, Freiburg Ta-Tsien Li, Shanghai Tai-Ping Liu, Taipei Helena Nussenzveig Lopes, Rio de Janeiro Roberto Natalini, Roma Alfio Quarteroni, Milano Laure Saint-Raymond, Paris Denis Serre, Lyon Chi-Wang Shu, Providence Eitan Tadmor, College Park MD Blake Temple, Davis Athanasios Tzavaras, Heraklion Zhou Ping Xin, Hong Kong Shih-Hsien Yu, Singapore

2 Welcome from OC and SC Chairs

Welcome to Padova and to HYP2012 - the fourteenth International Conference on Hyperbolic Problems!

This bi-annual series of international conferences, devoted to theory, numerics and applications of hyperbolic problems, has now become one of the highest quality and most successful conference series in Applied Mathematics. Its main objective is to bring together researchers, practitioners and students with interest in all aspects of hyperbolic differential equations and related models. This year we look forward to hosting a record number of over 350 participants, led by a world renowned list of plenary and invited speakers.

This book collects thirtyone abstracts of plenary and invited lectures, two hundred and sixteen abstracts of talks, to be delivered in nine parallel sessions, and more than thirty abstracts of contributed posters. These cover a broad spectrum of topics, with particular highlights on:

 \star Singular limits (zero-viscosity, relaxation, incompressible limit, semi-classical limits) and dispersive equations in mathematical physics.

 \star Nonlinear wave patterns in several space dimensions.

 \star Particle/molecular dynamics (kinetic methods, passage from microscopic to macroscopic, numerical issues in transitional regimes, magneto-hydrodynamics).

 \star Theory and numerics of multiphases and interfaces (boundary layers, phase boundaries, multiphase wave propagation).

 \star Transport in complex environments (homogenization, semiclassical limits, scattering in random media, porous media, biological applications, network and traffic flows).

 \star Control problems for Hyperbolic PDEs (controllability/stabilizability properties, optimal control) and Hamilton-Jacobi related problems.

 \star General relativity and Geometric PDEs.

We hope that the conference will provide a forum to stimulate and exchange new ideas from different disciplines, and to formulate new models and problems that will have impact in engineering, in physical and biological sciences, as well as industrial applications.

We would like to take this opportunity to thank the University of Padova and Magnifico Rettore Prof. Giuseppe Zaccaria. Moreover, we would like to express a special thank to Prorettore Vicario Prof. Francesco Gnesotto, for all his invaluable support and helps in organizing this event. We thank as well the Director of the Department of Mathematics Prof. Bruno Viscolani and the Department of Mathematics staff, particularly Alessandro Lanza for designing the logo of the conference. We would like also to thank the Major of Padova Dott. Flavio Zanonato and the member of the city council Prof. Gianni Di Masi for their help in the organization of the social events of the conference, particularly the Conference Banquet at Palazzo della Ragione. Finally, we would like to thank the staff of PadovaMeeting for their dedication and professional work in helping to organize every logistic aspect of this conference.

We look forward to your participation in a stimulating and productive environment of the HYP2012 conference.

Fabio Ancona, Alberto Bressan, Pierangelo Marcati, Andrea Marson

History. Hyperbolic Problems: Theory, Numerics and Applications is a bi-annual series of conferences which bring together researchers and students with interests in theoretical and computational aspects of hyperbolic PDEs and of related time-dependent models in the applied sciences. The previous editions were hosted in St-Etienne, **France** (1986), Aachen, **Germany** (1988), Uppsala, **Sweden** (1990), Taormina, **Italy** (1992), Stony Brook, NY **USA** (1994), Hong Kong (1996), Zürich, **Switzerland** (1998), Magdeburg, **Germany** (2000), Pasadena, CA **USA** (2002), Osaka, **Japan** (2004), Lyon, **France** (2006), College Park, MA **USA** (2008) and Beijing, **China** (2010).

Last update of this online version: June 23, 2012. Possible changes will also be included in the program available on the website of the conference at the address: http://www.hyp2012.eu/program and posted during the conference.

3 Abstracts of plenary lectures

Monday, Aula Magna Galilei, Palazzo Bo, 9.30-10.15

Non-standard solutions of isentropic Euler with Riemann data

Camillo De Lellis Institut für Mathematik, Universität Zürich camillo.delellis@math.uzh.ch

We consider the isentropic compressible Euler equations of gas dynamics in two space dimensions and in the Eulerian formulation. The gas is described by the state vector (ρ, v) , where ρ is the density and v the velocity. The balance laws for mass and linear momentum give therefore the following system of 3 scalar equations

$$\begin{aligned} \partial_t \rho + \operatorname{div}_x(\rho v) &= 0 \\ \partial_t(\rho v) + \operatorname{div}_x(\rho v \otimes v) + \nabla[p(\rho)] &= 0. \end{aligned}$$
 (1)

The pressure p is required to be a smooth function with p' > 0. A largely studied class of examples is given by the pressure law $p(\rho) = \kappa \rho^{\gamma}$ where $\gamma > 1$. However, the results presented in this talk are, for the moment, not valid for such laws.

We will focus our attention on the Cauchy problem for (1), i.e. on solutions on $\mathbb{R}^2 \times [0, \infty[$ satisfying the initial conditions

$$(\rho, v)(x, 0) = (\rho_0(x), v_0(x)).$$
(2)

Moreover, we will consider Riemann data having the following very specific form

$$\rho_0(x) = \begin{cases} \rho^+ & \text{if } x_2 > 0\\ \rho^- & \text{if } x_2 < 0 \end{cases}$$
(3)

$$v_0(x) = \begin{cases} v^+ & \text{if } x_2 > 0\\ v^- & \text{if } x_2 < 0 \,. \end{cases}$$
(4)

As it is well known solutions to (1) are in general not unique, unless the system is complemented with suitable admissibility criteria. Perhaps the most popular one is the so-called *entropy condition*, which in the case at hand requires the following inequality for the energy density and the energy flux:

$$\partial_t \left(\rho \varepsilon(\rho) + \rho \frac{|v|^2}{2} \right) + \operatorname{div}_x \left[\left(\rho \varepsilon \rho + \rho \frac{|v|^2}{2} + p(\rho) \right) v \right] \le 0,$$
(5)

where the internal energy density ε is linked to the pressure p by the identity $p(r) = r^2 \varepsilon'(r)$.

Since the pioneering work of Riemann it is known that, if we restrict our attention to the 1-dimensional Riemann problem, i.e. to pairs (ρ, v) which are admissible solutions of (1)-(2)-(3)-(4) and depend only on $\frac{x_2}{t}$, then (with some more assumptions of technical natural) there is a unique solution (see for instance [7, Section 4.7]). Surprisingly the situation is radically different if we drop the requirement that (ρ, v) depends only on $\frac{x_2}{t}$.

Theorem 1. There are a smooth pressure law p with p' > 0 and constants ρ^{\pm} and v^{\pm} for which there exist infinitely many admissible bounded solutions (ρ, v) of (1), (2), (3) - (4) with $\inf \rho > 0$.

The proof builds upon the methods of papers [2]-[3], where László Székelyhidi and the first author had already shown that the admissibility condition (5) does not imply the uniqueness of L^{∞} solutions of the Cauchy problem. However, the examples in the paper [3] had very rough initial data and it was not at all clear whether more regular data could be achieved. We indeed were inspired by the recent work of Székelyhidi, who in [6] recasts the vortex-sheet problem of incompressible fluid dynamics in the framework of [2]-[3]. The situation here is, though, considerably more complicated and hence requires some new ideas.

We note a few important things.

- The pressure law p is constructed ad hoc and hence it is still open whether special choices of p might obstruct our construction.
- The data of Theorem 1 cannot be generated by Lipschitz compression waves and hence the question whether Theorem 1 might hold for regular initial data is still open and currently under investigation.
- In view of the results in [6] and because Theorem 1 shares many similarities with them, it seems likely that the Dafermos' entropy rate admissibility criterion does not select the "classical" solution to the Riemann problem, i.e. there might be a "non-standard" solution which is more dissipative than the classical one.
- Finally, though the solutions of Theorem 1 are very irregular, it is rather unclear where one wishes to set a boundary. On the one hand the space of BV functions does not seem suitable for an existence theory in more than one space dimension (see the papers [5] and [1]; however, explicit examples of blow-up are, to my knowledge, still missing for the system (1)). On the other hand the recent paper [4] shows the existence of *continuous* solutions to the incompressible Euler equations which dissipate the kinetic energy. This may suggest that the framework of [2]-[3] is likely to produce "strange" piecewise continuous solutions to hyperbolic systems of conservation laws.

My talk will discuss all these issues and give an outlook of several natural questions which these considerations naturally rise.

References

- C. De Lellis, Blowup of the BV norm in the multidimensional Keyfitz and Kranzer system. Duke Math. J. 127 (2005), pp. 313–339.
- [2] C. De Lellis and L. Székelyhidi Jr., The Euler equations as a differential inclusion. Ann. of Math. (2) 170 (2009), pp. 1417–1436.
- [3] C. De Lellis and L. Székelyhidi Jr., On admissibility criteria for weak solutions of the Euler equations. Arch. Ration. Mech. Anal. 195 (2010), pp. 225–260.
- [4] C. De Lellis and L. Székelyhidi Jr., Continuous dissipative Euler flows. Preprint (2012).
- [5] J. Rauch, BV estimates fail for most quasilinear systems in dimension greater than one. Comm. Math. Phys. 106 (1986), 481–484.
- [6] L. Székelyhidi Jr., Weak solutions to the incompressible Euler equations with vortex sheet initial data. Preprint (2011), to appear in C. R. Acad. Sci. Paris, Ser. I.
- [7] D. Serre, Systems of conservation laws I. Cambridge University Press, 1999.

Joint work with: Elisabetta Chiodaroli (Universität Zürich), László Székelyhidi Jr. (Universität Leipzig)

Monday, Aula Magna Galilei, Palazzo Bo, 10.45–11.30

* * * -

Efficient numerical methods for quantifying uncertainty in solutions of systems of conservation laws

Siddhartha Mishra

Center of Mathematics for Applications, University of Oslo, Norway and Seminar for Applied Mathematics, ETH Zürich, Switzerland. smishra@sam.math.ethz.ch Inputs to systems of conservation laws such as initial data, boundary conditions, source terms, flux and diffusion coefficients are characterized by uncertainty, due to measurement errors. This input uncertainty results in uncertainty in the solutions of the underlying systems. We model input uncertainty as well as the resulting solutions by random fields. The well-posedness theory of random entropy solutions for scalar conservation laws with random initial data, sources and fluxes is presented and possible extensions to systems indicated. The focus of the lecture will be on reviewing state of the art numerical methods for quantifying uncertainty in random conservation laws. We will consider statistical sampling methods such as the Monte Carlo methods and the recently developed Multi-level Monte Carlo (MLMC) methods, present the underlying convergence and computational complexity theories and describe various numerical experiments that show the robustness and efficiency of these methods. In particular, Euler and MHD equations with random initial data, shallow water equations with random bottom topography, Euler equations with uncertain equations of state and two-phase flow equations with random relative permeabilities will be presented. The experiments will demonstrate that MLMC methods are totally non-intrusive, can handle large number of sources of uncertainty and scale to a very large number of processors in a parallel computing architecture. Some open issues regarding statistical sampling methods will be discussed and these methods will be compared with a novel deterministic class of numerical methods, the so called stochastic finite volume (SFV) methods. The convergence theory of SFV methods will be indicated and possible advantages of these methods, on problems with small number of sources of uncertainty, will be highlighted.

Joint work with: Christoph Schwab, Jonas Šukys and Svetlana Tokareva (Seminar for Applied Mathematics, ETH Zürich) and Nils Henrik Risebro (Center of Mathematics for Applications, University of Oslo)

Monday, Aula Magna Galilei, Palazzo Bo, 11.45–12.30

Homogenization and boundary layers

Nader Masmoudi Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, NY 10012, USA. masmoudi@cims.nyu.edu

We consider the homogenization of an elliptic system with Dirichlet boundary condition, when the coefficients of both the system and the boundary datum are ε -periodic. We show that, as $\varepsilon \to 0$, the solutions converge in L^2 with a power rate in ε , and identify the homogenized limit system. Due to a boundary layer phenomenon, this homogenized system depends in a non trivial way on the boundary. Our analysis answers a longstanding open problem, raised for instance in the book of Bensoussan, Lions and Papanicolaou. It extends substantially previous results obtained for polygonal domains with sides of rational slopes as well as our previous paper [3] where the case of irrational slopes was considered.

We consider the homogenization of elliptic systems in divergence form

$$-\nabla \cdot \left(A\left(\cdot/\varepsilon\right)\nabla u\right)\left(x\right) = 0, \quad x \in \Omega,\tag{1}$$

set in a bounded domain Ω of \mathbb{R}^d , $d \geq 2$, with an oscillating Dirichlet data

$$u(x) = \varphi(x, x/\varepsilon), \quad x \in \partial\Omega.$$
 (2)

As is customary, $\varepsilon > 0$ is a small parameter, and $A = A^{\alpha\beta}(y) \in M_N(\mathbb{R})$ is a family of functions of $y \in \mathbb{R}^d$, indexed by $1 \leq \alpha, \beta \leq d$, with values in the set of $N \times N$ matrices. Also, u = u(x) and $\varphi = \varphi(x, y)$ take their values in \mathbb{R}^N . We recall, using Einstein convention for summation, that for each $1 \leq i \leq N$,

$$(\nabla \cdot A(\cdot/\varepsilon) \nabla u)_i(x) := \partial_{x_{\alpha}} \left[A_{ij}^{\alpha\beta}(\cdot/\varepsilon) \partial_{x_{\beta}} u_j \right](x)$$

In the sequel, greek letters α, β, \dots will range between 1 and d and latin letters i, j, k, \dots will range between 1 and N.

Systems of type (1) are involved in various domains of material physics, notably in linear elasticity and in thermics In many cases they come with a right hand side f. In the context of thermics, d = 2 or 3, N = 1, u is the temperature, and $\sigma = A(\cdot/\varepsilon)\nabla u$ is the heat flux given by Fourier law. The parameter ε models heterogeneity, that is short-length variations of the material conducting properties. The boundary term φ in (2) corresponds to a prescribed temperature at the surface of the body. In the context of linear elasticity, d = 2 or 3, N = d, u is the unkown displacement, f is the external load and A is a fourth order tensor that models Hooke's law.

We make three hypotheses:

18

i) Ellipticity: For some $\lambda > 0$, for all family of vectors $\xi = \xi_i^{\alpha} \in \mathbb{R}^{Nd}$

$$\lambda \sum_{\alpha} \xi^{\alpha} \cdot \xi^{\alpha} \leq \sum_{\alpha, \beta, i, j} A_{ij}^{\alpha, \beta} \xi_{j}^{\beta} \xi_{i}^{\alpha} \leq \lambda^{-1} \sum_{\alpha} \xi^{\alpha} \cdot \xi^{\alpha}.$$

ii) Periodicity: $\forall y \in \mathbb{R}^d$, $\forall h \in \mathbb{Z}^d$, $\forall x \in \partial \Omega$, A(y+h) = A(y), $\varphi(x,y) = \varphi(x,y+h)$.

iii) Smoothness: The functions A and φ , as well as the domain Ω are smooth. It is actually enough to assume that ϕ and Ω are in some H^s for s big enough, but we will not try to compute the optimal regularity.

We are interested in the limit $\varepsilon \to 0$, *i.e.* the homogenization of system (1)-(2).

For the non-oscillating Dirichlet problem, one shows that u^{ε} weakly converges in $H^1(\Omega)$ to the solution u^0 of the homogenized system

$$\begin{cases} -\nabla \cdot \left(A^0 \nabla u^0\right)(x) = 0, & x \in \Omega, \\ u^0(x) = \varphi(x), & x \in \partial\Omega. \end{cases}$$
(3)

The so-called homogenized matrix A^0 comes from the averaging of the microstructure. It involves the periodic solution $\chi = \chi^{\gamma}(y) \in M_N(\mathbb{R}), \ 1 \leq \gamma \leq d$, of the *cell problem*:

$$-\partial_{y_{\alpha}} \left[A^{\alpha\beta}(y) \,\partial_{y_{\beta}} \chi^{\gamma}(y) \right] = \partial_{y_{\alpha}} A^{\alpha\gamma}(y), \quad \int_{[0,1]^d} \chi^{\gamma}(y) \,dy = 0. \tag{4}$$

The homogenized matrix is then given by:

$$A^{0,\alpha\beta} = \int_{[0,1]^d} A^{\alpha\beta} + \int_{[0,1]^d} A^{\alpha\gamma} \partial_{y_\gamma} \chi^\beta$$

One may even go further in the analysis, and obtain a two-scale expansion of u^{ε} . Denoting

$$u^{1}(x,y) := -\chi^{\alpha}(y)\partial_{x_{\alpha}}u^{0}(x), \qquad (5)$$

it is proved for instance in the book Bensoussan-Louis and Papanicolaou that

$$u^{\varepsilon}(x) = u^{0}(x) + \varepsilon u^{1}(x, x/\varepsilon) + O(\sqrt{\varepsilon}), \text{ in } H^{1}(\Omega).$$
(6)

Actually, an open problem in this area is to compute the next term in the expansion in the presence of a boundary. This is actually another motivation for this work.

The main result of this talk is

Theorem (Homogenization in smooth domains)

Let Ω be a smooth bounded domain of \mathbb{R}^d , $d \geq 2$. We assume that it is uniformly convex (all the principal curvatures are bounded from below).

Let u^{ε} be the solution of system (1)-(2), under the ellipticity, periodicity and smoothness conditions i)-iii).

There exists a boundary term φ_* (depending on φ , A and Ω), with $\varphi_* \in L^p(\partial\Omega)$ for all finite p, and a solution u^0 of (3) with boundary data φ_* , such that:

$$\|u^{\varepsilon} - u^{0}\|_{L^{2}(\Omega)} \leq C_{\alpha} \varepsilon^{\alpha}, \quad \text{for all } 0 < \alpha < \frac{d-1}{3d+5}.$$
(7)

References

- Marco Avellaneda and Fang-Hua Lin, Compactness methods in the theory of homogenization. Comm. Pure Appl. Math., 40, (6), (1987)803–847, 1987.
- [2] Alain Bensoussan, Jacques-Louis Lions, and George Papanicolaou, Asymptotic analysis for periodic structures, *Studies in Mathematics and its Applications*, bf 5, North-Holland Publishing Co., Amsterdam, (1978).
- [3] David Gérard-Varet and Nader Masmoudi, Homogenization in polygonal domains. J. Eur. Math. Soc. (JEMS), 13, no. 5, (2011), 1477-1503.
- [4] David Gérard-Varet and Nader Masmoudi, Homogenization and boundary layers. Acta. Math (to appear), (2012).

Joint work with: David Gérard-Varet (DMA/CNRS, Ecole Normale Supérieure, 45 rue d'Ulm, 75005 Paris, France).

TUESDAY, AUDITORIUM CONSERVATORIO POLLINI, 8.30-9.15

Evolution problem in general relativity

Igor Rodnianski MIT, Boston irod@math.mit.edu

The talk will focus on mathematical aspects of the evolution problem in General Relativity and review its progress and main challenges. A prominent interaction of Geometry and PDE methods in the subject will be illustrated on examples ranging from incompleteness theorems, formation of trapped surfaces and the recent proof of the L^2 curvature conjecture.

TUESDAY, AUDITORIUM CONSERVATORIO POLLINI, 9.30-10.15

Some large time behaviors of surface water waves

Sijue Wu University of Michigan, Ann Arbor sijue@umich.edu

The mathematical problem of *n*-dimensional water wave concerns the motion of the interface separating an inviscid, incompressible, irrotational fluid, under the influence of gravity, from a region of zero density (i.e. air) in *n*-dimensional space. It is assumed that the fluid region is below the air region. Assume that the density of the fluid is 1, the gravitational field is $-\mathbf{k}$, where \mathbf{k} is the unit vector pointing in the upward vertical direction, and at time $t \geq 0$, the free interface is $\Sigma(t)$, and the fluid occupies region $\Omega(t)$. When surface tension is zero, the motion of the fluid is described by

$$\mathbf{v}_t + \mathbf{v} \cdot \nabla \mathbf{v} = -\mathbf{k} - \nabla P \qquad \text{on } \Omega(t), \ t \ge 0, \\
 \text{div } \mathbf{v} = 0, \qquad \text{curl } \mathbf{v} = 0, \qquad \text{on } \Omega(t), \ t \ge 0, \\
 P = 0, \qquad \text{on } \Sigma(t) \\
 (1, \mathbf{v}) \text{ is tangent to the free surface } (t, \Sigma(t)),$$
(1)

where \mathbf{v} is the fluid velocity, P is the fluid pressure.

In this talk, we will survey results and ideas concerning the local and global wellposedness of the Cauchy problem of equation (1), and present some recent work concerning singularities of the solutions.

______ * * * _____

Tuesday, Auditorium Conservatorio Pollini, 10.45–11.30

Review of the original derivation of the Boltzmann equation and its extension to an infinite-range intermolecular potential

Yoshio Sone Kyoto University, Japan yoshio.sone.53c@st.kyoto-u.ac.jp

The original derivation of the Boltzmann equation is reviewed by paying attention to the scale parameters and the limiting parameters in its derivation. The Boltzmann equation for an infinite-range potential is discussed on the basis of the review. The contribution of intrinsic gravitational force between molecules is evaluated along this line.

References

- [1] Y. Sone, Molecular gas dynamics, Birkäuser Boston, (2007)
- Y. Sone, Supplement to Molecular gas dynamics (Birkäuser, 2007), Kyoto University Research Information Repository (2011) (http://hdl.handle.net/2433/66098)

Tuesday, Auditorium Conservatorio Pollini, $11.45{-}12.30$

* * * -

Recent progress in existence theory for the 3D steady compressible Navier-Stokes equations

Song Jiang Institute of Applied Physics and Computational Mathematics, Beijing, China jiang@iapcm.ac.cn

In the last decades, significant progress has been made on the mathematical aspect of the steady Navier-Stokes equations for three-dimensional compressible flows. In this talk, we shall briefly review some recent existence results on weak solutions with large data. The ideas and developed techniques used in the existence theory (such as P.L. Lions' framework of the existence proof, new estimates in Morry spaces of both pressure and kinetic energy) will be presented, and some open questions will be discussed.

Wednesday, Aula Magna Galilei, Palazzo Bo, 8.30–9.15

Optimal placement of sensors, actuators and dampers for waves

Enrique Zuazua BCAM Basque Center for Applied Mathematics zuazua@bcamath.org

In this lecture we address the problem of the optimal placement of sensors, actuators and dampers for wave equations. We first discuss the dissipative wave equation where, due to the non-selfadjoint nature of the generator of the dynamics, characterizing the decay rate of solutions as time tends to infinity needs to take into account both spectral properties and the propagation of bicharacteristic rays. We present the state of the art in what concerns the optimal placement of dampers. We then turn our attention to the conservative wave equation and the optimal placement of sensors and actuators, both fundamental problems from a control theoretical point of view, with many potential applications. Using Fourier series representations the problem can be recast as an optimal design one involving all the spectrum of the laplacian. We develop a complete theory allowing to distinguish, depending on the complexity of the data to be observed/controlled, cases in which the solution is a classical set constituted by a finite number of subdomains, from others in which the optimal set is of Cantor type or those when relaxation occurs. These results will be illustrated by numerical simulations. Most of the work presented in this lecture is part of ongoing research in collaboration with Y. Privat (ENS Cachan, Antenne de Bretagne, France) and E. Trélat (Université Pierre et Marie Curie (Paris 6), Laboratoire Jacques-Louis Lions, Paris, France).

References

- Y. Privat, E. Trélat and E. Zuazua, Optimal observation of the one-dimensional wave equation, preprint (2012).
- [2] Y. Privat, E. Trélat and E. Zuazua, Optimal observation of the one-dimensional wave equation, preprint (2012).

Wednesday, Aula Magna Galilei, Palazzo Bo, 9.30–10.15

* * *

Relative entropy methods in the mathematical theory of complete fluid systems

Eduard Feireisl Institute of Mathematis, Academy of Sciences of the Czech Republic, Prague feireisl@math.cas.cz

1. Navier-Stokes-Fourier system

Relative entropy methods are based on estimating the distance, in a suitable metric, of a solution to a system of partial differential equations to a given function, typically another solution of the same system. We use this approach in the study of *weak solutions* to the full Navier-Stokes-Fourier system describing the motion of a viscous, compressible and heat conducting fluid:

$$\partial_t \varrho + \operatorname{div}_x(\varrho \vec{u}) = 0, \tag{1}$$

$$\partial_t(\varrho \vec{u}) + \operatorname{div}_x(\varrho \vec{u} \otimes \vec{u}) + \nabla_x p(\varrho, \vartheta) = \operatorname{div}_x \mathcal{S} + \varrho \vec{f}, \tag{2}$$

$$\partial_t(\varrho s(\varrho, \vartheta)) + \operatorname{div}_x(\varrho s(\varrho, \vartheta)\vec{u}) + \nabla_x\left(\frac{\vec{q}}{\vartheta}\right) = \sigma,$$
(3)

where $\rho = \rho(t, x)$ is the fluid density, $\vec{u} = \vec{u}(t, x)$ the velocity field, and $\vartheta = \vartheta(t, x)$ the absolute temperature. Furthermore, $p(\rho, \vartheta)$ is the pressure, $s = s(\rho, \vartheta)$ the specific entropy, $S = S(\vartheta, \nabla_x \vec{u})$ the viscous stress determined by Newton's law

$$\mathcal{S}(\vartheta, \nabla_x \vec{u}) = \mu(\vartheta) \left(\nabla_x \vec{u} + \nabla_x^t \vec{u} - \frac{2}{3} \mathrm{div}_x \vec{u} \mathcal{I} \right) + \eta(\vartheta) \mathrm{div}_x \vec{u} \mathcal{I}, \tag{4}$$

and $\vec{q} = \vec{q}(\vartheta, \nabla_x \vartheta)$ is the heat flux,

$$\vec{q}(\vartheta, \nabla_x \vartheta) = -\kappa(\vartheta) \nabla_x \vartheta. \tag{5}$$

Finally, the symbol σ stands for the *entropy production*,

$$\sigma = \frac{1}{\vartheta} \Big(\mathcal{S}(\vartheta, \nabla_x \vec{u}) : \nabla_x \vec{u} - \frac{\vec{q}(\vartheta, \nabla_x \vartheta) \cdot \nabla_x \vartheta}{\vartheta} \Big).$$
(6)

We suppose that the fluid occupies a bounded domain $\Omega \subset \mathbb{R}^3$, the boundary of which is energetically insulated, specifically,

$$\vec{u}|_{\partial\Omega} = 0, \ \vec{q}(\vartheta, \nabla_x \vartheta) \cdot \vec{n}|_{\partial\Omega} = 0.$$
(7)

If, moreover, the external force $\vec{f} = \nabla_x F(x)$ is conservative, there are two obvious constants of motion: The total mass

$$\int_{\Omega} \varrho(t, \cdot) \, \mathrm{d}x = M_0$$

and the total energy

$$\int_{\Omega} \left(\frac{1}{2} \varrho |\vec{u}|^2 + \varrho e(\varrho, \vartheta) - \varrho F \right) (t, \cdot) \, \mathrm{d}x = E_0,$$

where $e = e(\rho, \vartheta)$ is the specific internal energy interrelated to the pressure and the entropy by means of Gibbs' relation

$$\vartheta Ds(\varrho,\vartheta) = De(\varrho,\vartheta) + p(\varrho,\vartheta)D\left(\frac{1}{\varrho}\right).$$
(8)

2. Thermodynamic stability, ballistic free energy

The so-called *hypothesis of thermodynamic stability* plays a crucial role in the forthcoming analysis:

$$\frac{\partial p(\varrho,\vartheta)}{\partial \varrho} > 0, \ \frac{\partial e(\varrho,\vartheta)}{\partial \vartheta} > 0.$$
(9)

We introduce *ballistic free energy*

$$H_{\Theta}(\varrho,\vartheta) = \varrho \Big(e(\varrho,\vartheta) - \Theta s(\varrho,\vartheta) \Big), \ \Theta > 0,$$

together with the *relative entropy* functional

$$\mathcal{E}(\varrho,\vartheta|r,\Theta) = H_{\Theta}(\varrho,\vartheta) - \frac{\partial H_{\Theta}(r,\Theta)}{\partial \varrho}(\varrho-r) - H_{\Theta}(r,\Theta).$$
(10)

As a direct consequence of (9), we check that

 $\varrho \mapsto H_{\Theta}(\varrho, \Theta)$ is strictly convex for any fixed Θ ,

$$\vartheta \mapsto H_{\Theta}(\varrho, \vartheta)$$
 is decreasing for $\vartheta < \Theta$ and increasing for $\vartheta > \Theta$.

Consequently,

$$\mathcal{E}(\varrho,\vartheta|r,\Theta) \ge c(K) \Big(|\varrho-r|^2 + |\vartheta-\Theta|^2 \Big) \text{ for } (\varrho,\vartheta) \in K,$$
(11)

$$\mathcal{E}(\varrho,\vartheta|r,\Theta) \ge c(K) \Big(1 + \varrho e(\varrho,\vartheta) + \varrho |s(\varrho,\vartheta)| \Big) \text{ for } (\varrho,\vartheta) \in [0,\infty)^2 \setminus K,$$
(12)

where $K \subset (0, \infty)^2$ is a compact set containing and open neighbourhood of (r, Θ) .

3. Stability of equilibria

Consider the equilibrium solution $\tilde{\rho}, \overline{\vartheta}, \overline{\vartheta}$

$$\nabla_x p(\tilde{\varrho}, \overline{\vartheta}) = \tilde{\varrho} \nabla_x F, \ \tilde{\varrho} = \tilde{\varrho}(x), \ \overline{\vartheta} > 0 \text{ a positive constant},$$

determined by the constraints

$$\int_{\Omega} \tilde{\varrho} \, \mathrm{d}x = M_0, \ \int_{\Omega} \left(\tilde{\varrho} e(\tilde{\varrho}, \overline{\vartheta}) - \tilde{\varrho} F \right) \, \mathrm{d}x = E_0.$$

Solutions of (1 - 3), supplemented with the boundary conditions (7), satisfy the total dissipation balance:

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_{\Omega} \left(\frac{1}{2} \varrho |\vec{u}|^2 + \mathcal{E}(\varrho, \vartheta | \tilde{\varrho}, \overline{\vartheta}) \right) \,\mathrm{d}x + \overline{\vartheta} \int_{\Omega} \sigma \,\mathrm{d}x = 0, \tag{13}$$

where $\tilde{\varrho}, \overline{\vartheta}$ is the equilibrium solution.

Thus the coercivity properties (11), (12) imply that the functional

$$\int_{\Omega} \left(\frac{1}{2} \varrho |\vec{u}|^2 + \mathcal{E}(\varrho, \vartheta | \tilde{\varrho}, \overline{\vartheta}) \right) \, \mathrm{d}x$$

represents a *distance* between the trajectory $t \mapsto \{\varrho(t, \cdot), \vartheta(t, \cdot), \vec{u}(t, \cdot)\}$ to the equilibrium $\{\tilde{\varrho}, \overline{\vartheta}, 0\}$. In particular, relation (13) yields *unconditional* convergence of solutions to equilibria for $t \to \infty$, see [2].

4. Weak solutions and weak-strong uniqueness principle

Weak solutions satisfy equations (1 - 3) in the sense of distributions, where the entropy production rate σ complies with *inequality*

$$\sigma \ge \frac{1}{\vartheta} \Big(\mathcal{S}(\vartheta, \nabla_x \vec{u}) : \nabla_x \vec{u} - \frac{\vec{q}(\vartheta, \nabla_x \vartheta) \cdot \nabla_x \vartheta}{\vartheta} \Big), \tag{14}$$

and the whole system is supplemented by the total energy balance

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_{\Omega} \left(\frac{1}{2} \varrho |\vec{u}|^2 + \varrho e(\varrho, \vartheta) \right) \,\mathrm{d}x = \int_{\Omega} \varrho \vec{f} \cdot \vec{u} \,\mathrm{d}x. \tag{15}$$

Such a definition is

• *compatible* in the sense that regular weak solutions satisfy the system in the classical sense, in particular, they satisfy (14) with equality sign;

• weak solutions exist *globally in time* for any finite energy initial data under suitable structural restrictions imposed on the state equation and the viscosity coefficients.

Finally, it can be shown, by the method of relative entropy, that the weak solutions satisfy the *weak-strong* uniqueness principle. The proof is based on using the relative entropy functional in the form

$$\int_{\Omega} \left(\varrho |\vec{u} - \tilde{\vec{u}}|^2 + \mathcal{E}(\varrho, \vartheta | \tilde{\varrho}, \tilde{\vartheta}) \right) \, \mathrm{d}x,\tag{16}$$

where $\{\tilde{\varrho}, \tilde{\vartheta}, \tilde{\vec{u}}\}$ is a (hypothetical) strong solution emanating from the same initial data. It can be shown that the weak and strong solutions coincide as long as the latter exists, see [1].

References

- [1] E.Feireisl and A.Novotný, Arch. Rational Mech. Anal., 204 (2012), pp. 683-706
- [2] E.Feireisl and D.Pražák, Asymptotic behavior of dynamical systems in fluid mechanics, AIMS Springfield, (2010)

Joint work with: Antonín Novotný (Université du Sud Toulon-Var, France)

Wednesday, Aula Magna Galilei, Palazzo Bo, 10.45–11.30

Tracking Multiphase Physics: Geometry, Foams, and Thin Films

James A. Sethian University of California, Berkeley sethian@math.berkeley.edu

Many scientific and engineering problems involve interconnected moving interfaces separating different regions, including dry foams, crystal grain growth and multi-cellular structures in man-made and biological materials. Producing consistent and well-posed mathematical models that capture the motion of these interfaces, especially at degeneracies, such as triple points and triple lines where multiple interfaces meet, is challenging.

Joint with Robert Saye of UC Berkeley, we introduce an efficient and robust mathematical and computational methodology for computing the solution to two and three-dimensional multi-interface problems involving complex junctions and topological changes in an evolving general multiphase system. We demonstrate the method on a collection of problems, including geometric coarsening flows under curvature and incompressible flow coupled to multi-fluid interface problems.

Finally, we compute the dynamics of unstable foams, such as soap bubbles, evolving under the combined effects of gas-fluid interactions, thin-film lamella drainage, and topological bursting.

FRIDAY, ROOM C, VIA BASSI , 11.00--11.45

- * * * -

Implicit-Explicit methods for hyperbolic systems with hyperbolic and parabolic relaxation

Giovanni Russo Department of Mathematics and Computer Science, University of Catania russo@dmi.unict.it

In this talk we discuss the problem of constructing effective high order methods for the numerical solution of hyperbolic systems of balance laws, in presence of stiff source. Because of the stiffness, the use of implicit integrators is advisable, so that no restrictions on the time step due to small relaxation time will appear. Two different relaxation systems will be considered, namely hyperbolic and parabolic relaxation. Because of the different nature of the problems, the two cases will be considered separately. A common denominator of both treatments is the choice of space discretization. Most schemes for conservation or balance laws are discretized by finite volume (FV), conservative finite difference (FD), or discontinuous Galerkin (DG). Here we choose conservative finite difference it is probably the simplest general approach for the construction of high order shock capturing schemes for such problems.

Hyperbolic relaxation The prototype 2×2 hyperbolic system with hyperbolic relaxation takes the form:

$$\begin{cases} u_t + v_x = 0\\ v_t + p(u)_x = \boxed{-\frac{1}{\varepsilon}(v - q(u))}\\ u(x, 0) = u_0(x), \quad v(x, 0) = v_0(x) \end{cases}$$
(1)

with $p'(u) > 0 \forall u \in \mathbb{R}$. Formally, if $\varepsilon \to 0$, the 2 × 2 system relaxes to the relation v = q(u) and the single scalar equation for u:

$$u_t + q(u)_x = 0 \tag{2}$$

If the subcharacteristic condition $q'(u)^2 \leq p'(u) \ \forall u \in \mathbb{R}$ is satisfied, then the solution of system (1) relaxes to the solution of Eq.(2). If the initial data is "well prepared", i.e. if $v_0(x) = q(u_0(x))$, then the solution will not present any "initial layer".

Numerical solutions of systems of the form (1) can be effectively obtained by using Implicit-Explicit Runge-Kutta methods in time, coupled with conservative finite-difference in space. The hyperbolic part (which may be non linear and is non local because of the space derivative) may be treated explicitly, since the system is non-stiff (if one is interested in resolving all the waves), while the stiff implicit part can be treated implicitly.

The simplest IMEX scheme is obtained by first implicit-explicit Euler scheme, which for system (1) can be written as

$$\begin{cases} u^{n+1} = u^n - \Delta t D v^n \\ v^{n+1} = v^n - \Delta t D p(u^n) - \frac{\Delta t}{\varepsilon} (v^{n+1} - q(u^{n+1})) \end{cases}$$

where D represent a discretization of the space derivative. As $\varepsilon \to 0$, the numerical solution is projected onto the manyfold v = q(u), and the scheme relaxes to the Explicit Euler scheme fo the relaxed equation (2).

IMEX-Runge Kutta schemes with s-stages will guarantee higher order accuracy. They are characterized by two coupled Runge-Kutta schemes, the implicit one identified by the $s \times s$ matrix A and the vectors $b, c \in \mathbb{R}^s$, while the explicit scheme is defined by matrix \tilde{A} and vectors \tilde{b} and \tilde{c} . Usually the implicit scheme is diagonally implicit, i.e. matrix A is a lower triangular matrix, while \tilde{A} is lower triangular with zeroes on the diagonal.

In the design of effective IMEX schemes for problems with hyperbolic relaxation several requirements are considered, namely:

- 1. Accuracy. High order in time is achieved by imposing the so-called order conditions obtained by matching Taylor expansion in time or exact and numerical solution. In addition to the usual order conditions of the two RK schemes, one has to satisfy some additional coupling conditions (see [4]). Such conditions guarantee the so called *classical order*, valid for $\varepsilon \approx 1$.
- 2. Asymptotic preservation. We require that the method applied to system (1) becomes a consistent discretization of the relaxed equation (2) as $\varepsilon \to 0$, possibly maintaining the same order of accuracy in the limit. This property is related to the *L*-stability of the implicit scheme (see, for example, [6]).
- 3. Uniform accuracy. The accuracy of the method depends on ε , and a degradation of the accuracy is observed for intermediate values of ε . It would be desirable to reduce such degradation. The accuracy dependence is analyzed by comparing the asymptotic expansion in ε of the exact and numerical solution [1]. Based on such comparison, additional conditions are derived, and used to construct new schemes with better uniform accuracy in ε .

All such points will be addressed during the talk. Two classes of IMEX-RK will be considered. The first one, called type A, has the property that the matrix A is invertible. For such methods it is easy to prove that the IMEX relaxes to the explicit RK applied to the relaxed equation, thus maintaining the order of accuracy in the variable u. The second class is called CK [4]. For them $a_{11} = 0$. Such methods are more difficult to analyze, but are somehow easier to construct than methods of type A, because some simplifying conditions can be applied to their coefficients. Several numerical tests on various problems will illustrate the relative merits of the IMEX schemes presented.

Parabolic relaxation Parabolic relaxation is obtained when one is interested in the long time behavior of the solution of hyperbolic systems with relaxation. The prototype system takes the form

$$\begin{cases} u_t + v_x = 0\\ \varepsilon^2 v_t + p(u)_x = -(v - q(u))\\ u(x, 0) = u_0(x), \quad v(x, 0) = v_0(x) \end{cases}$$
(3)

As the relaxation parameter vanishes, the variable v obeys the relation $v = q(u) - p(x)_x$, while and the asymptotic behavior of the system is governed by a scalar convection-diffusion equation.

$$u_t + q(u)_x = p(u)_{xx} \tag{4}$$

Notice that the characteristic speeds $\lambda_{\pm} = \pm \sqrt{p'(u)}/\varepsilon$ diverge as $\varepsilon \to 0$, which makes the numerical treatment of the system more delicate.

Two different kinds of IMEX Runge-Kutta schemes will be considered. The first will be denoted as *partitioned* [2]: the stiffness is associated to the *variable*. The equation for the non stiff variable u will be treated explicitly, while the equation for the stiff variable v will be treated implicitly, according to the following scheme (here for simplicity we consider the case q = 0 and p(u) = u)

$$u_t = -v_x$$
 [Explicit]
 $v_t = -(u_x + v)/\varepsilon^2$ [Implicit] (Partitioned)

The second family will be denoted *additive*: the right hand side is given by the sum of two terms, one of which is treated explicitly, and one implicitly, according to the scheme

$$\begin{array}{rcl} u_t &=& \hline -v_x \\ v_t &=& \hline -u_x/\varepsilon^2 \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & &$$

Methods based on the first approach have been more studied in the literature for the diffusion relaxation, because the hyperbolic part becomes stiff when the system relaxes towards the parabolic equation. In fact, the characteristic speeds are $c_{\pm} = \pm 1/\varepsilon$, and classical explicit schemes for the hyperbolic part would suffer by a CFL restriction $\Delta t \leq \varepsilon C \Delta x$, where the maximum CFL number C is of order unity and depends on the particular scheme.

Two main issues will be discussed here. First, we shall show that the use of classical IMEX schemes for hyperbolic systems with stiff relaxation, with L-stable implicit part (see, for example, [3]), applied to system (3) in partitioned form will lead to consistent explicit discretization of the limit convection-diffusion equation (4). Because of this, the limit scheme will suffer of the typical parabolic CFL restriction $\Delta t \propto \Delta x^2$. To overcome such a drawback, in both approaches one can make a wise use of the asymptotic limit, by adding and subtracting the same term, one treated implicitly and one explicitly, so that, in the limit $\varepsilon \to 0$, the scheme converges to an implicit method for the diffusion equation (see [2,3]).

The second part concerns the analysis of the *additive approach* [3]. This approach is attractive, because it is the more commonly used one for hyperbolic systems with relaxation, however it has the serious drawback that the hyperbolic part itself is stiff, and therefore it appears almost hopeless to treat a stiff term with an explicit scheme, and get around with prohibitive stability conditions. We show that the simple Explicit-Implicit Euler scheme applied to system (3) (with q = 0 and p = u) in the additive form converges to explicit Euler schemes applied to the diffusion equation (apart from higher order terms), while other classical IMEX schemes fail. The behavior is explained by an analysis based on an asymptotic expansion in ε of the exact and numerical solution. The analysis introduces additional conditions that need to be satisfied. Using such conditions it is possible to derive new second order IMEX additive schemes that posses the desired AP property.

Several applications to various test cases, including non linear diffusion, and model kinetic equations will be presented.

References

- S. Boscarino, Error Analysis of IMEX Runge-Kutta Methods Derived from Differential Algebraic Systems, SIAM Journal on Numerical Analysis 45 (2007), pp.1600–1621.
- [2] S. Boscarino, L. Pareschi, G. Russo, Implicit-Explicit Runge-Kutta schemes for hyperbolic systems and kinetic equations in the diffusion limit, preprint arXiv:1110.4375v1.
- [3] S. Boscarino, G. Russo, Flux-Explicit ImEx Runge-Rutta Schemes for Hyperbolic to Parabolic Relaxation Problems, SIAM J. Numer. Anal., submitted.
- [4] M. H. Carpenter, C. A. Kennedy, Additive Runge-Kutta schemes for convection-diffusion-reaction equations, Appl. Numer. Math. 44 (2003), pp.139–181.
- [5] G. Naldi, L. Pareschi, Numerical Schemes for Hyperbolic Systems of Conservation Laws with Stiff Diffusive Relaxation SIAM J. on Numer. Anal., 37, (2000), pp.1246-1270
- [6] L. Pareschi, G. Russo, Implicit-Explicit Runge-Kutta schemes and applications to hyperbolic systems with relaxations, *Journal of Scientific Computing* 25, 2005, pp.129-155

Joint work with: Sebastiano Boscarino (University of Catania), Lorenzo Pareschi (University of Ferrara)

FRIDAY, ROOM C, VIA BASSI, 12.00-12.45

Strong stability of shocks in L^2 for conservation laws, and application to asymptotic analysis

Alexis F. Vasseur University of Texas at Austin vasseur@math.utexas.edu

We develop a theory based on relative entropy to show the uniqueness and L^2 stability (up to a translation) of extremal entropic Rankine-Hugoniot discontinuities for systems of conservation laws (typically 1-shocks, n-shocks, 1-contact discontinuities and n-contact discontinuities of large amplitude) among bounded entropic weak solutions having an additional trace property. The existence of a convex entropy is needed. No BV estimate is needed on the weak solutions considered. The theory holds without smallness condition. The assumptions are quite general. For instance, strict hyperbolicity is not needed globally. For fluid mechanics, the theory handles solutions with vacuum.

Note that the relative entropy method is also an important tool in PDEs in the study of asymptotic limits. Applications of the relative entropy method in this context began with the work of Yau and have been studied by many others. However, for the compressible limit to conservation laws, up to now, this method was successful as long as the limit solution stayed Lipschitz. We will present, also, some new result of asymptotic analysis to shocks of conservation laws using the relative method.

Joint work with: Kyudong Choi (University of Texas at Austin), Nicholas Leger (Carnegie Mellon)

4 Abstracts of invited lectures

Monday, Room C, Via Bassi, 14.30–15.10

On variational kinetic Formulations for scalar conservation laws and the equations of gas dynamics

Mikhail Perepelitsa University of Houston misha@math.uh.edu

The kinetic formulation of weak entropy solutions of scalar conservation laws, developed by Lions-Perthame-Tadmor(1994), can be equivalently expressed in a variational form. This property was discovered by Panov(1996) in his theory of kinetic measure-valued solutions and, independently, in a recent paper of Brenier(2009). We discuss a number of interesting properties of such variational kinetic solutions, in particular, the geometric interpretation of solutions as curves in a suitable Hilbert space for which the tangent vector minimizes an interaction functional. In the second part of the talk we will describe a variational kinetic formulation for the Euler equations of gas dynamics that has the geometric structure similar to the structure of the kinetic form of scalar conservation laws.

Monday, Room D, Via Bassi, 14.30–15.10

* * * -

Domain continuity for the Euler and Navier-Stokes equations

David Gérard-Varet Université Paris 7 and Institut de Mathématiques de Jussieu gerard-varet@math.jussieu.fr

The aim of the talk is to understand the effect of rough walls or rough obstacles on fluid flows. It has various physical motivations, including drag reduction in microfluidics. Mathematically, there are two natural ways to model the roughness:

- 1. by considering fluid domains with non-smooth boundaries.
- 2. by considering fluid domains with oscillating boundaries, the oscillation being of small amplitude and wavelength.

The first model often raises numerical and mathematical difficulties (like a lack of Cauchy theory), which requires to consider smooth approximations Ω^{ε} of the irregular domain Ω^{0} . As regards the second model, denoting by ε the small wavelength or amplitude of the oscillating boundary, one is also led to consider a sequence of domains Ω^{ε} parametrized by ε .

This leads naturally to questions of domain continuity for fluid models, broadly: if $\Omega \varepsilon$ converges to Ω^0 , does the associated fluid velocity u^{ε} converge to u^0 ? Are the boundary conditions preserved in the limit?

We shall investigate these questions in the context of the Euler and Navier-Stokes equations.

References

- [1] Anne-Laure Dalibard and David Gerard-Varet, Effective Boundary conditions at a rough surface starting from a slip condition, *Journal of Differential equations*, Volume no. 251 (2011), pp. 3450-3487
- [2] David Gerard-Varet and Christophe Lacave, The 2D Euler equations on singular domains, Preprint 2012.

Joint works with: Christophe Lacave (Univ. Paris 7 and IMJ), Anne-Laure Dalibard (DMA, ENS)

Monday, Room C, Via Bassi, 16.35–17.15

* * *

Complete synchronization of particle and kinetic Kuramoto models on networks

Seung-Yeal Ha

Department of Mathematical Sciences, Seoul National University syha@snu.ac.kr

In this talk, we will discuss the complete synchronization of particle and kinetic Kuramoto models with general couplings. The synchronization of many weakly coupled oscillators often appears in natural systems, e.g., two pendulum clocks suspended from the same bar, the flashing of fireflies, the singing of crickets and hand clapping by audiences in a concert hall, etc. These phenomena can be modeled by the coupled oscillators on some networks. The network associated with the Kuramoto model with all-to-all coupling is simply the Kuramoto oscillators on the complete graph. It is easy to say that the network structure will affect the synchronizability of Kuramoto oscillators scattered on the vertices of networks numerically. Then a natural question is how the network structure affects the complete synchronization of oscillators. For this, we consider two cases "symmetric connected graph and an asymmetric graph with hierarchical leadership" and we will provide some quantitative theorems for the synchronizability of particle and kinetic Kuramoto models.

Monday, Room D, Via Bassi, 16.35–17.15

- * * * -

Two-Phase Flow in Porous Media: Shock Waves and Stability

Michael Shearer North Carolina State University shearer@ncsu.edu

In this talk, I discuss a variety of contexts in which undercompressive shock waves have been discovered recently. The main focus will be on models of two-phase flow in porous media. Plane waves are modeled by the onedimensional Buckley-Leverett equation, a *scalar* conservation law. The Gray-Hassanizadeh model for ratedependent capillary pressure adds dissipation and a BBM-type dispersion, giving rise to undercompressive waves. Two-phase flow in porous media is notoriously subject to fingering instabilities, related to the classic Saffman-Taylor instability. However, a two dimensional linear stability analysis of sharp planar interfaces reveals a criterion predicting that weak Lax shocks may be stable or unstable to long-wave two-dimensional perturbations. This surprising result depends on the hyperbolic-elliptic nature of the *system* of linearized equations. Numerical simulations of the full nonlinear system of equations, including dissipation and dispersion, verify the stability predictions at the hyperbolic level.

Joint work with: Kim Spayd and Zhenzheng Hu (North Carolina State University).

TUESDAY, ROOM C, VIA BASSI, 14.30–15.10

Almost sure existence of global weak solutions for supercritical Navier-Stokes equations

Gigliola Staffilani Massachusetts Institute of Technology gigliola@math.mit.edu

In this talk we show that after suitable data randomization there exists a large set of supercritical periodic initial data for both 2D and 3D Navier-Stokes equations for which global energy bounds are proved. As a consequence we obtain almost sure supercritical global weak solutions. We also show that in 2D these global weak solutions are unique.

To explain the problem more in details let's start by considering the initial value problem for the incompressible Navier-Stokes equations given by

$$\begin{cases} \partial_t \vec{u} = \Delta \vec{u} - \mathbb{P} \nabla \cdot (\vec{u} \otimes \vec{u}); & x \in \mathbb{T}^d \quad \text{or} \quad \mathbb{R}^d, \, t > 0 \\ \nabla \cdot \vec{u} = 0 & \\ \vec{u}(x,0) = \vec{f}(x), \end{cases}$$
(1)

where f is divergence free and \mathbb{P} is the projection into divergence free vector fields given via

$$\mathbb{P}\vec{h} = \vec{h} - \nabla \frac{1}{\Delta} (\nabla \cdot \vec{h}).$$
⁽²⁾

It is well-known that global well-posedness of (1) when the space dimension d = 3 is a long standing open question. This is related to the fact that the equations (1) are so called super-critical when d > 2. Indeed recall that if the velocity vector field $\vec{u}(x,t)$ solves the Navier-Stokes equations (1) in \mathbb{T}^d then $\vec{u}_{\lambda}(x,t)$ with

$$\vec{u}_{\lambda}(x,t) = \lambda \vec{u}(\lambda x, \lambda^2 t),$$

is also a solution to the system (1) for the initial data

$$\vec{u}_{0\lambda} = \lambda \vec{u}_0(\lambda x) . \tag{3}$$

The spaces which are invariant under such a scaling are called critical spaces for the Navier-Stokes equations. Examples of critical spaces for the Navier-Stokes in \mathbb{T}^d are:

$$\dot{H}^{\frac{d}{2}-1} \hookrightarrow L^{d} \hookrightarrow \dot{B}^{-1+\frac{d}{p}}_{p|p<\infty,\infty} \hookrightarrow BMO^{-1}.$$
(4)

In particular, for Sobolev spaces, $\|\vec{u}_{\lambda}(x,0)\|_{\dot{H}^{s_c}} = \|\vec{u}(x,0)\|_{\dot{H}^{s_c}}$, when $s_c = \frac{d}{2} - 1$. We recall that the exponents s are called *critical* if $s = s_c$, sub-critical if $s > s_c$ and super-critical if $s < s_c$.

On the other hand, classical solutions to the (1) satisfy the decay of energy which can be expressed as:

$$\|u(x,t)\|_{L^2}^2 + \int_0^t \|\nabla u(x,\tau)\|_{L^2}^2 d\tau = \|u(x,0)\|_{L^2}^2.$$
(5)

Note that when d = 2 the energy $||u(x,t)||_{L^2}$, which is the globally controlled thanks to (5), is exactly the scaling invariant $\dot{H}^{s_c} = L^2$ -norm. In this case the equations are said to be *critical*. When d = 3, the energy $||u(x,t)||_{L^2}$ is at the super-critical level with respect to the scaling invariant $\dot{H}^{\frac{1}{2}}$ -norm, and hence the Navier-Stokes equations are said to be *super-critical* and the lack of a known bound for the $\dot{H}^{\frac{1}{2}}$ contributes in keeping the global well-posedness question for the initial value problem (1) still open.

In this talk we consider the periodic Navier-Stokes problem in (1) and in particular we address the question of long time existence of weak solutions for supercritical initial data both in d = 2, 3, see also [8]. For d = 2we address uniqueness as well. Our goal is to show that by randomizing in an appropriate way the initial data in $H^{-\alpha}(\mathbb{T}^d)$, d = 2, 3 (for some $\alpha = \alpha(d) > 0$) which is below the critical threshold space $H^{s_c}(\mathbb{T}^d)$, one can construct a global in time weak solution to (1). Such solution is unique when d = 2. We note that similar well-posededness results were obtained for the super-critical wave equations by Burq and Tzvetkov in [1,2,3]. The approach of Burq and Tzvetkov was applied in the context of the Navier-Stokes in order to obtain local in time solutions to the corresponding integral equation for randomized initial data in $L^2(\mathbb{T}^3)$, as well as global in time solutions to the corresponding integral equation for randomized initial data that are small in $L^2(\mathbb{T}^3)$ by Zhang and Fang [9] and by Deng and Cui [4] Also in [5], Deng and Cui obtained local in time solution to the corresponding integral equation for randomized initial data that are small in $L^2(\mathbb{T}^3)$ by Zhang integral equation for randomized initial data in $H^s(\mathbb{T}^d)$, for d = 2, 3 with -1 < s < 0. However our result is the first to offer a construction of a **global in time** weak solution to (1) for randomized initial data (without any smallness assumption) in negative Sobolev spaces $H^{-\alpha}(\mathbb{T}^d)$, d = 2, 3, for some $\alpha = \alpha(d) > 0$.

Roughly speaking the idea of the proof is the following: we start with a divergence free and mean zero initial data $\vec{f} \in (H^{-\alpha}(\mathbb{T}^d))^d$, d = 2, 3 and suitably randomize it to obtain \vec{f}^{ω} which in particular preserves the divergence free condition. Then we seek a solution to the initial value problem (1) in the form $\vec{u} = e^{t\Delta} \vec{f}^{\omega} + \vec{w}$. In this way we single out the linear evolution $e^{t\Delta} \vec{f}^{\omega}$ and identify the difference equation that \vec{w} should satisfy. At this point it becomes convenient to state an equivalence lemma between the initial value problem for the difference equation and the integral formulation of it. This equivalence is similar to Theorem 11.3 in [7], see also [6]. We will be using the integral equation formulation near time zero and the other one away from zero. The key point of this approach is the fact that although the initial data are in $H^{-\alpha}$ for some $\alpha > 0$, the heat flow of the randomized data gives almost surely improved L^p bounds). These bounds in turn yield improved nonlinear estimates arising in the analysis of the difference equation for \vec{w} almost surely, and consequently a construction of a global weak solution to the difference equation is possible.

References

- Burq, N. and Tzvetkov, N. Random data Cauchy theory for supercritical wave equation I: Local theory, Invent. math. 173, No. 3 (2008), 449–475.
- [2] Burq, N. and Tzvetkov, N. Random data Cauchy theory for supercritical wave equation II: A global existence result, *Invent. math.* 173, No. 3 (2008), 477–496.
- [3] Burq, N. and Tzvetkov, N. Probabilistic well-posedness for the cubic wave equation, Preprint arXiv:1103.2222v1
- [4] C. Deng and S. Cui, Random-data Cauchy problem for the Navier-Stokes equations on T³, J. Differential Equations 251, No. 4-5 (2011), 902–917.
- [5] C. Deng and S. Cui, Random-data Cauchy Problem for the Periodic Navier-Stokes Equations with Initial Data in Negative-order Sobolev Spaces, *Preprint arXiv:1103.6170v.1*.
- [6] G. Furioli, P.G. Lemarié-Rieusset and E. Terraneo, Unicité dans L³(ℝ³) et d'autres espaces fonctionnels limites pour Navier-Stokes [Uniqueness in L³(ℝ³) and other functional limit spaces for Navier-Stokes equations], Rev. Mat. Iberoamericana 16, No. 3 (2000), 605–667.
- [7] P.G. Lemarié-Rieusset, Recent developments in the Navier-Stokes problem, Chapman & Hall/CRC Reserach Notes in Mathematics 431, 2002.
- [8] T. Tao, A quantitative formulation of the global regularity problem for the periodic Navier-Stokes equation, Dyn. Partial Differ. Equ. 4 (2007), no. 4, 293–302.
- [9] T. Zhang and D. Fang, Random data Cauchy theory for the incompressible three dimensional Navier-Stokes equations, *Proc. AMS* 139, No. 8, (2011), 2827–2837.

Joint work with: Andrea Nahmod (University of Massachusetts, Amherst) and Natasa Pavlovic (University of Texas, Austin).

TUESDAY, ROOM D, VIA BASSI, 14.30–15.10

High-order gas evolution model for computational fluid dynamics

Kun Xu Hong Kong University of Science and Technology makxu@ust.hk

The foundation for the development of modern computational fluid dynamics (CFD) is based on the Riemann solution of the Euler equations. The high-order schemes are basically related to high-order spatial reconstruction. In order to overcome the low-order wave interaction mechanism due to the Riemann solution, the temporal accuracy of the scheme is improved through the Runge-Kutta time stepping method. The close coupling between the spatial and temporal evolution in the original nonlinear governing equations seems weakened due to its spatial and temporal decoupling. For the viscous flow, the piece-wise discontinuous initial data and the hyperbolic-parabolic nature of the Navier-Stokes equations seem inconsistent mathematically, such as the divergence of the viscous and heat conducting terms due to initial discontinuity. Therefore, in order to alleviate this difficulty, the inviscid and viscous terms in the NS equations are numerically treated differently in most CFD methods.

Based on the Boltzmann equation, we are going to present a high-order gas dynamic model, the so-called time-dependent flux function at a cell interface, from a high-order discontinuous initial reconstruction. The theoretical basis for such an approach is due to the fact that the Boltzmann equation has no specific requirement on the smoothness of the initial data and the kinetic equation has dynamic mechanism to construct a dissipative wave structure starting from an initially discontinuous flow condition on a time scale of particle collision time. More specifically, the gas-kinetic scheme covers a whole spectrum of scales, from the kinetic to the hydrodynamic ones. This talk will present a hierarchy to construct high-order gas-kinetic scheme (GKS).

In comparison with the Riemann solver, the GKS provides a valid physical evolution process from a discontinuity. The GKS first presents particle free transport process, then through the particle collision it generates the dissipative wave structure. With intensive particle collisions within a time step, such as in the hydrodynamic scale, a Navier-Stokes gas distribution function can be obtained from the GKS. The Euler solution is considered as a limiting case when intensive particle collisions take palce. Numerically, the GKS formulation makes a smooth transition from the upwind to the central difference scheme in the process of gas evolution. It is a unification of two different algorithm development methodologies in the traditional CFD methods. This kind of mechanism can be hardly described using any macroscopic governing equation. Theoretically, the gas-kinetic equation provides a mechanism for the transport in all scales from kinetic to hydrodynamic. On the other hand, the macroscopic governing equations, such as the NS equations, describe the flow evolution in the hydrodynamic one only. How to handle the discontinuity becomes a fundamental problem in CFD. Even though the Riemann solution of the Euler equations can mathematically handle the initial discontinuity, the validity of using such a solution in the description of a numerical shock in a discretized space is questionable from a physical modeling point of view.

A numerical shock layer on the scale of a few mesh points needs to be considered as an enlarged physical shock structure. Since a physical shock has the thickness on the order of particle mean free path, an enlarged numerical shock layer means the size to become the length scale of numerical particle mean free path. Therefore, on the scale of cell size, the non-equilibrium flow physics has to be taken into account in the gas evolution process in the shock region. As we know, as a particle moves across a physical shock layer, there is only limited number of particle collisions. The non-equilibrium shock structure is constructed through the competition between particle free transport and collision. This non-equilibrium process provides the appropriate dissipation for the smooth transition from one equilibrium state at upstream to another equilibrium one at downstream. But, inside the numerical shock layer, the exact Riemann solver replaces the non-equilibrium physical reality by an equilibrium one with the assumption of infinite number of particle collisions and the generation of distinguishable waves. As a result, the numerical process of the Riemann solution has no a dynamic dissipative mechanism, especially in multi-dimensional case. Therefore, the use of the Euler equations in the flux modeling must have problem in the non-equilibrium region, such as the triggering of shock instability in Godunov method in high Mach number flow simulation. The GKS follows closely the flow physics. The initial free transport, which provides numerical dissipation, depends closely on the jump of the discontinuity. For the contact discontinuity wave, the exact Euler solution assumes infinite number of particle collisions which prevent the penetration of particles crossing each other. For the GKS, the particle penetration exists all the time in its underlying modeling.

In a discretized space, any physical discontinuity is enlarged to the cell size scale due to the limited resolution. Since the Riemann problem is truthfully solving the Euler equations, the absence of non-equilibrium mechanism indicates that the Euler solution cannot be properly used as dynamic evolution model for the enlarged dissipative region. One may think of using the NS equations with dissipative terms to capture the corresponding nonequilibrium layer. But, this cannot be fully valid, because the NS equations have only the physical dissipation on the hydrodynamic scale, which cannot be used to describe the flow behavior in the kinetic scale, such as the initial particle free transport process from an discontinuity.

The main idea we would like to deliver is that for a shock capturing scheme we need a correct physical mechanism to model the gas evolution from a discontinuity in the discretized space. There is no valid macroscopic governing equations to describe such a physical mechanism yet. The value of gas-kinetic scheme is that it provides a new way for the CFD algorithm development. A valid physical process will become more important in the construction of high-order CFD methods. The numerical examples will demonstrate the importance of high-order gas evolution model.

TUESDAY, ROOM C, VIA BASSI, 16.35–17.15

* *

Relative entropy in diffusive relaxation

Corrado Lattanzio University of L'Aquila corrado@univaq.it

As it is well-known, the presence of convex dissipative entropies in hyperbolic models with relaxation gives a stabilizing effect to the system and leads to global existence results, at least near equilibria. In that cases, it is possible to produce a relative entropy identity, which among other applications, can control the hyperbolicto-hyperbolic relaxation process and give a simple and direct convergence framework, at least in the case of smooth equilibria.

The aim of this talk is to describe this approach in the case of diffusive relaxations, again for smooth solutions to the parabolic equilibria. Thanks to the relative entropy identity, we obtain a stability estimate and convergence for the relaxation limit. The results are obtained in various different cases, and in particular for multidimensional Euler equations.

Joint work with: Athanasios E. Tzavaras (University of Crete)

Invited lectures
Tuesday, Room D, Via Bassi, 16.35--17.15

Controllability results for degenerate parabolic operators.

Karine Beauchard CNRS, CMLS, Ecole Polytechnique Karine.Beauchard@math.polytechnique.fr

Unlike uniformly parabolic equations, parabolic operators that degenerate on subsets of the space domain exhibit very different behaviors from the point of view of controllability. For instance, null controllability in arbitrary time may be true or false according to the degree of degeneracy and there are also examples where a finite time is needed to ensure such a property. This talk will survey most of the theory that has been established so far for operators with boundary degeneracy, and discuss recent results for operators of Grushin type and of Kolmogorov type, which degenerate in the interior.

Joint work with: Piermarco Cannarsa (Universita degli Studi di Roma Tor Vergata), Roberto Guglielmi (Universita degli Studi di Roma Tor Vergata).

Thursday, Room C, Via Bassi, 8.30–9.10

* * * -

Boundary kernels for dissipative systems

Shih-Hsien Yu National University of Singapore matysh@nus.edu.sg

In this talk we will present a study on the kernel functions of the Dirichlet-Neumann maps for dissipative systems in a half space. We start from the consideration of the Greens function for an initial-boundary value problems for linear dissipative systems. With the fundamental solutions of the dissipative systems, one can reduce the initial-boundary value problems into boundary value problems so that the well-posedness of the system gives linear algebraic systems over the polynomials in the Fourier and Laplace variables for the Dirichlet-Neumann datum at the boundary, where Fourier variables are in the directions of boundary, and the Laplace is for the time variable. In order to invert the Dirichlet-Neumann map from the transformation variables to the space-time variables we introduce a path, which contains the spectral information of the systems, in the complex plan for the time Laplace variable. On this path, the Laplace-Fourier variables can be recombined, through the Cauchys complex contour integral, into a form resemble to that for a whole space problem. Thus, the classical results for the whole space problem can be used to obtain the pointwise spac-time structure for long wave components of the kernel function of the Dirichlet-Neumann map for points within a finite Mach region. We also apply direct energy estimates to yield the pointwise structure of the kernel functions in any high Mach number region. Finally, we have obtained exponentially sharp estimates for the kernel function in the space-time variables. For example, the kernel functions for both DA lembert wave equation with dissipation and a linearized compressible Navier-Stokes equation can be expressed explicitly in space-time variables with errors which decay exponentially in both space-time variables. This gives a globally quantitative and qualitative wave propagations at boundary. THURSDAY, ROOM D, VIA BASSI, 8.30–9.10

Viscek flocking dynamics and phase transition

Jian-Guo Liu Duke University jliu@math.duke.edu

Consider the following nonlinear Fokker-Planck equation describing self-propelled dynamics for some large biological system such as flocking of birds and schooling of fishes,

$$f_t + \omega \cdot \nabla_x f = d\Delta_\omega f + \nabla_\omega \cdot (f\nabla_\omega \phi)$$

where $f(x, \omega, t)$ is a distribution function on $\mathbb{R}^n \times \mathbb{S}_{n-1}$. The self-propelled speeds for all biological agents are assumed to be uniform and take value one for simplest. ϕ is the interaction potential describing Viscek self-alignment of the orientation towards its local averaged orientation [2, 5]

$$\phi(x,\omega) = \nu(\rho) \int_{\mathbb{R}^n \times \mathbb{S}_{n-1}} \psi(|x-x'|)\omega \cdot \omega' f(x',\omega',t) dx' d\omega'$$

 $\Delta_{\omega}, \nabla_{\omega}, \nabla_{\omega}$ are the laplican, divergence and gradient operator on \mathbb{S}_{n-1} . $\rho = \int f d\omega$ is density.

The flocking behavior in the above Fokker-Planck equation appears in a similar form as orientational phase transitions as in liquid crystal and ferromagnetism at a critical norse level d_c or equivalently at a critical mass density ρ_c . Below this value, the only equilibrium distribution is isotropic for orientations and is stable. Any initial distribution relaxes exponentially fast to this isotropic equilibrium state. By contrast, when the density is above the threshold, a second class of anisotropic equilibria formed by Von-Mises-Fischer distributions of arbitrary orientation appears. The isotropic equilibria become unstable and any initial distribution relaxes towards one of these anisotropic states with exponential speed of convergence.

In this talk, I will present a joint work with Amic Frouvelle [4] on rigorous analysis of the phase transition for the spatial homogeneous dynamics and a joint work with Pierre Degond and Amic Frouvelle [1] on asymptotic analysis for the spatial inhomogeneous case on the hydrodynamics limit for the anisotropic region and nonlinear diffusion approximation in the isotropic region.

References

- [1] P. Degond, A. Frouvelle and J.-G. Liu, Macroscopic limits and phase transition in a system of self-propelled particles, submitted.
- [2] P. Degond and S. Motsch, Continuum limit of self-driven particles with orientation interaction, Mathematical Models and Methods in Applied Sciences, 18 (2008) pp. 1193–1215.
- [3] P. Degond and J.-G. Liu, Hydrodynamics of self-alignment interactions with precession and derivation of the Landau-Lifschitz-Gilbert equation Math. Models Methods Appl. Sci., 22 (2012), pp. 1114001-18
- [4] A. Frouvelle and J.-G. Liu, Dynamics in a kinetic model of oriented particles with phase transition SIAM J. Math Anal, 44 (2012), pp. 791–826.
- [5] T. Vicsek, A. Czirk, E. Ben-Jacob, I. Cohen, and O. Shochet, Novel type of phase transition in a system of self-driven particles, *Phys. Rev. Lett.* **75**, 1226–1229 (1995).

Joint work with: Pierre Degond, (Institut de Mathématiques de Toulouse, Université Paul Sabatier), Amic Frouvelle (Institut de Mathématiques de Toulouse, Université Paul Sabatier)

THURSDAY, ROOM C, VIA BASSI, 10.35–11.15

Hyperbolic Equations on Networks

Michael Herty RWTH Aachen University herty@mathc.rwth-aachen.de

The theory of transportation methods has been an active area of research since almost three decades. Early work has been inspired by studying the physics of road networks or large scale production networks. Later ideas, methods and results have been extended to electric, biological or social networks. A major motivation for studying systems on the rather particular geometry of a network stems from the huge economic and sociologic impact of results in this area. The common ground are networks wherein the dynamics is governed by partial differential equations, in particular conservation laws and balance equations where we use the various types of solutions that have been considered in the mathematical literature. In particular in the context of mathematical modeling and analysis a variety of literature exists concerning network flows. These publications range from application in data networks (e.g. [1]) and traffic flow (e.g. [2]) over supply chains (e.g. [3]) to flow of water in canals (e.g. [4,5]). Also in the engineering community, gas flow in pipelines is in general modeled by transient models, see e.g. [6]. More detailed models based on partial differential equations have also been introduced be found e.g. in [7]. Therein, the flow of gas inside the pipes is modeled by a system of balance laws. At junctions of two or more pipes, algebraic conditions couple the solution inside the pipeline segments and yield a network solution. A different physical problem, leading to a similar analytical framework, is that of the flow of water in open channels, considered [8]. Starting with the classical work, several other recent papers deal with the control of smooth solutions [9]. In all these cases an assumption on the C^1 -norm of initial and boundary data is necessary. Clearly, the given references are incomplete and we refer to the papers and references therein for more details.

In this talk we want to present recent results on conservation laws on networks starting from questions the modeling of physical processes on these networks and going further to tackle questions of control and stablization of network flows. We will summarize existing and new results and present a common framework of the mathematical discussion of network solutions [10]. Based on the analytical results controllability and optimization issues will be discussed. Controlability issues have also been analysed using Lyapunov functions as well as energy estimates and feedback boundary control laws. Currently, most results are based on single equations and have not been extended to networks. In order to obtain a network formulation, coupling conditions which yield boundary conditions are essential. The formulation of well-defined node conditions that are also reasonable model is still a major issue in the mathematical discussion. Further, we discuss the incoporation of results on feedback laws at boundaries of single controlled partial differential equations within suitable coupling conditions. Concerning optimization problems on networks only a few rigorous results exists so far. We present some approaches and discuss recent results in the direction of the characterization of optimal controls with applications to supply chain networks and traffic flow. Numerical examples will be given.

We want to present a broad view on these problems and also give some directions for open problems and possible future research.

- C. D'Apice, R. Manzo, and B. Piccoli. Packet flow on telecommunication networks. SIAM J. Math. Anal., 38(3):717–740, 2006.
- [2] G. Coclite, M. Garavello, and B. Piccoli. Traffic flow on a road network. SIAM J. Math. Anal., 36(6):1862– 1886, 2005.
- [3] S. Göttlich, M. Herty, and A. Klar. Modelling and optimization of supply chains on complex networks. Communications in Mathematical Sciences, 4(2):315–330, 2006.
- [4] J. de Halleux, C. Prieur, J.-M. Coron, B. d'Andréa Novel, and G. Bastin. Boundary feedback control in networks of open channels. *Automatica J. IFAC*, 39(8):1365–1376, 2003.

- [5] M. Gugat and G. Leugering. Global boundary controllability of the de st. venant equations between steady states. Annales de l'Institut Henri Poincaré, Nonlinear Analysis, 20(1):1–11, 2003.
- [6] A. J. Osiadacz. Simulation and analysis of gas networks. Gulf Publishing Company, Houston, 1989.
- [7] M. K. Banda, M. Herty, and A. Klar. Coupling conditions for gas networks governed by the isothermal Euler equations. *Networks and Heterogeneous Media*, 1(2):275–294, 2006.
- [8] G. Leugering and E. J. P. G. Schmidt. On the modeling and stabilization of flows in networks of open canals. SIAM J. Control Opt., 41(1):164–180, 2002.
- [9] T.-T. Li and B.-P. Rao. Exact boundary controllability for quasi-linear hyperbolic systems. SIAM J. Control Optim., 41(6):1748–1755, 2003.
- [10] R. M. Colombo, M. Herty, and V. Sachers. On 2 × 2 conservation laws at a junction. SIAM J. Math. Anal., 40(2):605–622, 2008.

Thursday, Room D, Via Bassi, 10.35–11.15

Singular behavior of a rarefied gas on a planar boundary

Shigeru Takata Department of Mechanical Engineering, Kyoto University takata.shigeru.4a@kyoto-u.ac.jp

We will discuss some singularities in a rarefied gas that should be observed on a planar boundary.

It has already been shown in 1960s and 1970s by Sone [1] and Sone & Onishi [2] that the slope of a macroscopic quantity diverges logarithmically in a rarefied gas on a planar boundary, by using the Bhatnagar-Gross-Krook-Welander (BGK or BKW) model of the Boltzmann equation. Recently, Lilley & Sader [3] have also numerically pointed out the slope divergence of macroscopic quantities on the boundary for the Boltzmann equation. However, the divergence rate is not clear in their discussion. With I-Kun Chen and Tai-Ping Liu [4], we have recently proved for a highly rarefied gas on the basis of the Boltzmann equation that the slope of flow velocity diverges logarithmically in the thermal transpiration between two parallel plates. In this talk, we will show on the basis of the Boltzmann equation that, irrespective of the Knudsen number,

- (i) the slope of a macroscopic quantity diverges logarithmically on a planar boundary;
- (ii) the logarithmic behavior of (i) induces a microscopic divergence on the boundary, namely the derivative of velocity distribution function with respect to the normal component of the molecular velocity diverges logarithmically for the molecular velocities parallel to the boundary;
- (iii) the singularity (i) is related quantitatively to the discontinuity of the velocity distribution function on the boundary.

- Y. Sone, Kinetic theory analysis of linearized Rayleigh problem, J. Phys. Soc. Jpn 19 (1964), pp. 1463– 1473
- [2] Y. Sone and Y. Onishi, Kinetic theory of evaporation and condensation Hydrodynamic equation and slip boundary condition, J. Phys. Soc. Jpn 44 (1978), pp. 1981–1994

- [3] C. R. Lilley and J. E. Sader, Velocity gradient singularity and structure of the velocity profile in the Knudsen layer according to the Boltzmann equation, *Phys. Rev. E* **76** (2007), 026315
- [4] I-K. Chen, T. -P. Liu, and S. Takata, Boundary singularity for thermal transpiration problem of the linearized Boltzmann equation, preprint (2010), submitted to Arch. Rational Mech. Anal.

Joint work with: Hitoshi Funagane (Department of Mechanical Engineering, Kyoto University).

------ * * * ------

Thursday, Room C, Via Bassi, 14.10–14.50

On the finite-time splash and splat singularities for the 3-D free-surface Euler equations

Steve Shkoller University of California Davis shkoller@math.ucdavis.edu

We prove that the 3-D free-surface incompressible Euler equations with regular initial geometries and velocity fields have solutions which can form a finite-time "splash" (or "splat") singularity first introduced in [1], wherein the evolving 2-D hypersurface, the moving boundary of the fluid domain, self-intersects at a point (or on surface). Such singularities can occur when the crest of a breaking wave falls unto its trough, or in the study of drop impact upon liquid surfaces. Our approach is founded upon the Lagrangian description of the free-boundary problem that we used in [2], combined with a novel approximation scheme of a finite collection of local coordinate charts; as such we are able to analyze a rather general set of geometries for the evolving 2-D free-surface of the fluid. We do not assume the fluid is irrotational, and as such, our method can be used for a number of other fluid interface problems, including compressible flows, plasmas, as well as the inclusion of surface tension effects.

References

- A. Castro, D. Córdoba, C. Fefferman, F. Gancedo, and M. Gómez-Serrano, Splash singularity for water waves, arxiv:1106.2120v2.
- [2] D. Coutand and S. Shkoller, Well-posedness of the free-surface incompressible Euler equations with or without surface tension, J. Amer. Math. Soc., Volume no. 20 (2007), pp. 829–930.
- [3] D. Coutand and S. Shkoller, On the finite-time splash and splat singularities for the 3-D free-surface Euler equations, arXiv:1201.4919v2.

Joint work with: Daniel Coutand (Heriot-Watt University)

* *

THURSDAY, ROOM D, VIA BASSI, 14.10–14.50

Entropy viscosity for hyperbolic systems and questions regarding parabolic regularization

Jean-Luc Guermond

Department of Mathematics, Texas A&M University 3368 TAMU, College Station, TX 77843, USA guermond@math.tamu.edu

A numerical method for approximating nonlinear conservation laws is described [1]. The technique consists of augmenting the numerical discretization at hand with a viscous regularization where the nonlinear viscosity is based on the local size of a discrete entropy production. This method is simple to program and does not use any flux or slope limiters. The method can reasonably be justified for scalar conservation equations. Stability results are established for scalar conservation equations using some explicit Runge Kutta techniques.

The implementation is not so clear when dealing with systems, since the question of parabolic regularization is mainly open in this case. The particular question of the Euler system and the Navier-Stokes regularization is addressed. A nonstandard (non-diagonal) regularization is proposed that, in addition to stabilizing the velocity, acts on the density and the internal energy and is such that the entropy sets $\{s(e, \rho) \ge r\}$ are positively invariant.

The technique is illustrated on various benchmark problems using continuous finite elements, discontinuous finite elements, spectral elements, and Fourier series.

References

 Jean-Luc Guermond and Richard Pasquetti and Bojan Popov, Entropy viscosity method for nonlinear conservation laws, *Journal of Computational Physics*, 230 (2011), pp. 4248–4267

Joint work with: Bojan Popov (Department of Mathematics, Texas A&M University 3368 TAMU, College Station, TX 77843, USA).

THURSDAY, ROOM C, VIA BASSI, 16.15–16.55

Stability of the free plasma-vacuum interface

Paolo Secchi Department of Mathematics Brescia University, Italy paolo.secchi@ing.unibs.it

We consider the free boundary problem for the plasma-vacuum interface in ideal compressible magnetohydrodynamics (MHD).

Plasma-vacuum interface problems appear in the mathematical modeling of plasma confinement by magnetic fields (see, e.g., [2]). In this model the plasma is confined inside a perfectly conducting rigid wall and isolated from it by a vacuum region, due to the effect of strong magnetic fields. In astrophysics, the plasma-vacuum interface problem can be used for modeling the motion of a star or the solar corona.

Let us assume that the interface between plasma and vacuum is given by a hypersurface

 $\Gamma(t) := \{ (x', x_3) \in \mathbb{R}^3, \, x_3 = f(t, x') \} \,,$

where $t \in [0, T]$ and $x' = (x_1, x_2)$ and let $\Omega^+(t)$ and $\Omega^-(t)$ be space-time domains occupied by the plasma and the vacuum respectively. Then we have $\Omega^{\pm}(t) = \{x_3 \ge f(t, x')\}$.

In the plasma region $\Omega^+(t)$ the flow is governed by the usual compressible MHD equations:

$$\begin{cases} \partial_t \rho + \operatorname{div} \left(\rho v\right) = 0, \\ \partial_t (\rho v) + \operatorname{div} \left(\rho v \otimes v - H \otimes H\right) + \nabla q = 0, \\ \partial_t H - \nabla \times \left(v \times H\right) = 0, \\ \partial_t \left(\rho e + \frac{1}{2}|H|^2\right) + \operatorname{div} \left(\left(\rho e + p\right)v + H \times \left(v \times H\right)\right) = 0, \end{cases}$$
(1)

where ρ denotes density, $v \in \mathbb{R}^3$ plasma velocity, $H \in \mathbb{R}^3$ magnetic field, $p = p(\rho, S)$ pressure, $q = p + \frac{1}{2}|H|^2$ total pressure, S entropy, $e = E + \frac{1}{2}|v|^2$ total energy, and $E = E(\rho, S)$ internal energy. With a state equation of gas, $\rho = \rho(p, S)$, and the first principle of thermodynamics, (1) is a closed system.

System (1) is supplemented by the divergence constraint

$$\operatorname{div} H = 0 \tag{2}$$

on the initial data.

In the vacuum domain $\Omega^{-}(t)$, as in [1, 2], we consider the so-called *pre-Maxwell dynamics*

$$\nabla \times \mathcal{H} = 0, \qquad \operatorname{div} \mathcal{H} = 0, \tag{3}$$

describing the vacuum magnetic field $\mathcal{H} \in \mathbb{R}^3$. That is, as usual in nonrelativistic MHD, in the Maxwell equations we neglect the displacement current $(1/c) \partial_t E$, where c is the speed of light and $E \in \mathbb{R}^3$ the electric field.

The plasma variable U = U(t, x) = (q, v, H, S) is connected with the vacuum magnetic field \mathcal{H} through the relations [1, 2]

$$\partial_t \varphi = v \cdot N, \quad [q] = 0, \quad H \cdot N = 0, \quad \mathcal{H} \cdot N = 0 \quad \text{on } \Gamma(t),$$
(4)

where $N = (-\partial_1 f, -\partial_2 f, 1)$ and $[q] = q|_{\Gamma} - \frac{1}{2}|\mathcal{H}|_{\Gamma}^2$ denotes the jump of the total pressure across the interface. Therefore the interface $\Gamma(t)$ moves with the plasma, the total pressure is continuous across $\Gamma(t)$, the magnetic field on both sides is tangent to $\Gamma(t)$. Because of (4), the free interface $\Gamma(t)$ is a characteristic boundary for (1). This fact gives a loss of control of derivatives in the normal direction to the boundary.

We assume that the plasma density does not go to zero continuously at the interface (clearly in the vacuum region $\Omega^{-}(t)$ the density is identically zero), but has a jump, meaning that it is bounded away from zero in the plasma region and it is identically zero in the vacuum region. This assumption is compatible with the continuity of the total pressure in (4).

It is well-known that for general data the linearization of (1) - (4) may be either violently unstable or weakly (neutrally) stable because of the failure of the uniform Kreiss-Lopatinski condition. For instance, posing $H = \mathcal{H} = 0$ gives the Euler compressible equations in vacuum and the Rayleigh-Taylor instability may occur. The introduction of the magnetic field may have a stabilizing effect and it is of interest to find under which conditions the problem becomes stable.

In our talk we discuss the well-posedness of the problem in suitable anisotropic Sobolev spaces under the stability condition

$$|H \times \mathcal{H}| > 0 \quad \text{on } \Gamma(t), \tag{5}$$

i.e. provided the magnetic fields on the two sides of the free-boundary are not colinear.

- I. B. Bernstein, E. A. Frieman, M. D. Kruskal, and R. M. Kulsrud, An energy principle for hydromagnetic stability problems, *Proc. Roy. Soc. London. Ser. A.*, 244 (1958), pp. 17-40
- [2] J.P. Goedbloed and S. Poedts, Principles of magnetohydrodynamics with applications to laboratory and astrophysical plasmas, Cambridge University Press, (2004)
- [3] P. Secchi and Y. Trakhinin, Well-posedness of the linearized plasma-vacuum interface problem, preprint (2011), submitted

[4] P. Secchi and Y. Trakhinin, Well-posedness of the plasma-vacuum interface problem, in preparation

Joint work with: Yuri Trakhinin (Sobolev Institute of Mathematics, Novosibirsk, Russia)

THURSDAY, ROOM D, VIA BASSI, 16.15–16.55

* * *

Points of General Relativistic Shock Wave Interaction are "Regularity Singularities" where Spacetime is Not Locally Flat

Moritz Andreas Reintjes University Regensburg moritzreintjes@googlemail.com

In this talk I am going to present the results of a recent paper [2], in which we show that the regularity of the gravitational metric tensor cannot be lifted from $C^{0,1}$ to $C^{1,1}$ by any $C^{1,1}$ coordinate transformation in a neighborhood of a point of shock wave interaction in General Relativity, without forcing the determinant of the metric tensor to vanish at the point of interaction. This is in contrast to Israel's celebrated 1966 Theorem, which states that such coordinate transformations always exist in a neighborhood of a point on a smooth *single* shock surface [1]. The results imply that points of shock wave interaction represent a new kind of singularity in spacetime, singularities that make perfectly good sense physically, that can form from the evolution of smooth initial data, but at which spacetime is not *locally Minkowskian* under any coordinate transformation. In particular, at such singularities, delta function sources in the second derivatives of the gravitational metric tensor exist in all coordinate systems, but due to cancelation, the Riemann curvature tensor remains uniformly bounded.

References

- W. Israel, Singular hypersurfaces and thin shells in general relativity, *Il Nuovo Cimento*, Volume XLIV B no. 1 (1966), pp. 1-14.
- [2] M. Reintjes and B. Temple, Points of General Relativistic Shock Wave Interaction are "Regularity Singularities" where Spacetime is Not Locally Flat, *Proc. R. Soc. A* (accepted), arXiv:1105.0798.

Joint work with: (John), Blake Temple (University of California - Davis).

* * * ------

FRIDAY, ROOM C, VIA BASSI, 8.30–9.10

Lipschitz stability for the Hunter-Saxton and Camassa-Holm equation

Xavier Raynaud Center of Mathematics for Applications, Oslo xavierra@cma.uio.no In this talk, we will present the construction of a metric for two related nonlinear partial differential equations, the Hunter-Saxton equation and the Camassa-Holm equation. This metric makes the semigroup of solutions Lipschitz continuous. The solutions typically break down in finite time. After breakdown, they are no longer unique and can be prolongated in several consistent ways. By using a change of variable - from Eulerian to Lagrangian coordinates - and introducing an extra energy density variable, we obtain an equivalent system which is well-posed as a system of ordinary differential equations in a Banach space. Going back to the original Eulerian variable, we can construct a semigroup of conservative solutions (solutions which for almost every time conserve the total energy). However, we observe that this semigroup of solutions is not stable with respect to any standard norm. To construct a metric which yields stability, we transport the topology of the equivalent system in Lagrangian variables to the Eulerian setting. The difficulty is that the mapping between Lagrangian and Eulerian variables is not a bijection. However, by precisely identifying the discripency between the two sets as the action of a group (group of diffeomorphism or relabelling group), we can eliminate the redundancy which is introduced by the Lagrangian variables.

References

- Bressan, Alberto and Holden, Helge and Raynaud, Xavier, Lipschitz metric for the Hunter-Saxton equation J. Math. Pures Appl., 1 (2010).
- [2] Grunert Katrin and Holden, Helge and Raynaud, Xavier, Lipschitz metric for the periodic Camassa-Holm equation, J. Differential Equations, **250** (2011).
- [3] Grunert, Katrin and Holden, Helge and Raynaud, Xavier, Lipschitz metric for the Camassa–Holm equation on the line *arXiv-1010.0561v1* (2010).

Joint work with: Alberto Bressan (Penn State University, USA), Helge Holden (NTNU and CMA, Norway) Katrin Grunert (NTNU, Norway)

FRIDAY, ROOM C, VIA BASSI, 9.15–9.55

Two uniqueness results for the two-dimensional continuity equation with velocity having L^1 or measure curl

Gianluca Crippa Departement Mathematik und Informatik – Universität Basel gianluca.crippa@unibas.ch http://www.math.unibas.ch/crippa

In this seminar I will present two results regarding the uniqueness (and further properties) for the two-dimensional $continuity \ equation$

$$\begin{cases} u_t + \operatorname{div} (bu) = 0\\ u(0, x) = \bar{u}(x) \end{cases}$$

and the ordinary differential equation

$$\begin{cases} \frac{\partial \Phi}{\partial t}(t,x) = b(t,\Phi(t,x)) \\ \Phi(0,x) = x \end{cases}$$

in the case when the vector field

$$b(t,x): [0,T] \times \mathbb{R}^2 \to \mathbb{R}^2$$

is bounded, divergence free and satisfies additional conditions on its distributional curl:

$$\operatorname{curl} b = -\partial_{x_2} b^1 + \partial_{x_1} b^2$$

Such settings appear in a very natural way in various situations, for instance when considering two-dimensional incompressible fluids.

(1) The case when b is time-independent and its curl is a (locally finite) measure (without any sign condition). Uniqueness of bounded distributional solutions to the continuity equation follows from a series of papers by Alberti, Bianchini and myself. In such papers, we provide a characterization of two-dimensional bounded divergence free vector fields enjoying uniqueness, in terms of the so-called *weak Sard property*. The weak Sard property is a suitable measure theoretical version of the usual Sard property satisfied by "sufficiently differentiable" maps between Euclidean space. It is possible to prove that the weak Sard property is enjoyed by vector fields with measure curl.

(2) The case when b is time-dependent and its curl belongs to $L^1(\mathbb{R}^2)$. This case is covered by a joint result with Bouchut, extending previous works with De Lellis. This time the idea is to work at the level of the ordinary differential equation, deriving effective estimates of stability and compactness under suitable bounds on the velocity. The result with De Lellis was addressing the case of $W^{1,p}$ velocities, with p > 1, and the upgrade with Bouchut sets the case in which the derivative of the velocity is a singular integral of a summable function, including in particular the case of L^1 curl. This proves uniqueness, stability and compactness of Lagrangian solutions to the continuity equation.

In the seminar I will present the main steps in the proofs in the two cases described above. I will also sketch some possible improvements and extensions and describe some applications.

References

- [1] G. Alberti, S. Bianchini and G. Crippa, A uniqueness result for the continuity equation in two dimensions, preprint (2011), to appear on *Journal of the European Mathematical Society (JEMS)*.
- [2] G. Alberti, S. Bianchini and G. Crippa, Structure of level sets and Sard-type properties of Lipschitz maps, preprint (2011), to appear on Annali della Scuola Normale Superiore, Classe di Scienze.
- [3] G. Alberti, S. Bianchini and G. Crippa, On the L^p-differentiability of certain classes of functions, preprint (2012).
- [4] F. Bouchut and G. Crippa, Lagrangian flows for vector fields with gradient given by a singular integral, in preparation.
- [5] G. Crippa and C. De Lellis, Estimates and regularity results for the DiPerna-Lions flow, J. Reine Angew. Math., 616 (2008), pp. 15-46.

Based on joint works with: Giovanni Alberti (Università di Pisa), Stefano Bianchini (SISSA Trieste), François Bouchut (CNRS & Université Paris-Est-Marne-la-Vallée) and Camillo De Lellis (Universität Zürich).

FRIDAY, ROOM C, VIA BASSI, 10.00–10.40

* * *

Threshold phenomena for critical wave equations

Joachim Krieger EPFL, Lausanne, Switzerland joachim.krieger@epfl.ch

Invited lectures

I'll discuss recent results on stability/instability of soliton type solutions for certain critical wave equations, such as the critical wave maps problem or the focussing critical NLW in three dimensions. In particular, I'll focus on exotic blow up dynamics, and threshold dynamics associated with center stable manifolds. These results aim at the goal of revealing all possible dynamics resulting from data in a small neighborhood (in a suitable topology) of the soliton type solution.

The author was partially supported by the SNF-grant 200021-13752.

5 Abstracts of contributed lectures — Monday 15.15–16.15

5.1 Session 1 — Room F — Numerical Methods I

S1 – Numerical Methods I – Room F, 15.15–15.45

Finite volume evolution Galerkin schemes for wave propagation in heterogeneous media

Koottungal Revi Arun

Institut für Geometrie und Praktische Mathematik, RWTH Aachen, Templergraben 55, D-52056 Aachen, Germany.

arun@igpm.rwth-aachen.de

The propagation of hyperbolic waves in heterogeneous media arises in the modelling of several physical phenomena, e.g. the traffic flow with varying conditions, acoustic or elastic waves in heterogeneous materials, to name a few. In many fields of applications, such as the exploration seismology, one studies the propagation of small amplitude man made waves in earth and their reflection off geological structures. Here, the hope is to determine the underlying geological configuration from the available measurements at the surface. In such cases new phenomena can appear since reflections of waves at interfaces can lead to discontinuities in the wave speeds, even for linear equations. For related works dealing with the numerical modelling of wave propagation in heterogeneous media we refer the reader, e.g. to [1] and the references therein.

In the present work we use the finite volume evolution Galerkin (FVEG) method to model the propagation of acoustic waves in a heterogeneous material with spatially varying wave speeds and impedance. The FVEG method, originally developed by Lukáčová and coworkers, cf. e.g. [2,3,4,5], is a predictor-corrector method combining a finite volume corrector step with an evolutionary predictor step. In order to evolve fluxes along the cell interfaces a multidimensional approximate evolution operator which takes into account of the infinitely many directions of wave propagation is used. The latter is constructed using the theory of bicharacteristics of multidimensional hyperbolic systems. In the previous works of Lukáčová and others, cf. [3,4], the evolution operator was derived only for locally linearised systems where the bicharacteristics reduce to straight lines.

The goal of this work is to derive the FVEG scheme for linear hyperbolic systems with spatially varying Jacobians without any local linearisation as also done in [6]. Due to the space dependence of the material parameters and any nonzero velocity field in the ambient medium, the bicharacteristics no longer remain straight lines. This introduces new difficulties in the derivation of the exact integral representation as well as in the numerical approximation. We overcome these difficulties and present a systematic derivation of the FVEG scheme; particularly for an acoustic wave equation system with space dependent wave speeds and impedance. However, the results presented here are general and hence they can also be employed for any quasi-linear hyperbolic system of first order.

Using the general theory of bicharacteristics we derive exact solution evolution operators for the wave equation system and approximate them using quadratures. We show that in order to obtain stable nonoscillatory results it is appropriate to approximate the heterogeneous medium by a staggered grid that is assigned to the vertices of a finite volume grid. However, the conservative variables are collocated on a nonstaggered grid. We present several numerical experiments for wave propagation with continuous as well as discontinuous wave speeds that confirm the accuracy, robustness and reliability of the new FVEG scheme.

- T. R. Fogarty, R. J. LeVeque, High-resolution finite volume methods for acoustics in periodic or random media, J. Acoust. Soc. Amer., 106 (1999), pp. 17-28.
- [2] M. Lukáčová-Medvidová, K. W. Morton, G. Warnecke, Evolution Galerkin methods for hyperbolic systems in two space dimensions, *Math. Comp.*, 69 (2000), pp. 1355–1384.

- [3] M. Lukáčová-Medvidová, J. Saibertová, G. Warnecke, Finite volume evolution Galerkin methods for nonlinear hyperbolic systems, J. Comp. Phys., 183 (2002), pp. 533-562.
- [4] M. Lukáčová-Medvidová, K. W. Morton, G. Warnecke, Finite volume evolution Galerkin (FVEG) methods for hyperbolic problems, SIAM J. Sci. Comput., 26 (2004), pp. 1-30.
- [5] M. Lukáčová-Medvidová, S. Noelle, M. Kraft, Well-balanced finite volume evolution Galerkin methods for the shallow water equations, J. Comp. Phys., 221 (2007), pp. 122-147.
- [6] K. R. Arun, M. Kraft, M. Lukáčová-Medvidová, Phoolan Prasad, Finite volume evolution Galerkin method for hyperbolic conservation laws with spatially varying flux functions, J. Comp. Phys., 228 (2009), pp. 565-590.

Joint work with: Sebastian Noelle (Institut für Geometrie und Praktische Mathematik, RWTH Aachen, Templergraben 55, D-52056 Aachen, Germany.)

S1 – Numerical Methods I – Room F, 15.45–16.15

Convergence of finite difference scheme for symmetric Keyfitz-Kranzer system

Ujjwal Koley University of Würzburg toujjwal@gmail.com

In this talk, we consider the Cauchy problem for the $n \times n$ symmetric system of Keyfitz-Kranzer type

$$\begin{cases} u_t + (u\phi(|u|))_x = 0, & x \in \Omega = \mathbb{R} \times (0,T), \\ u(x,0) = u_0(x), & x \in \mathbb{R}, \end{cases}$$
(1)

where T > 0 is fixed, $u = (u^{(1)}, \ldots, u^{(n)}) : \mathbb{R} \times [0, T) \to \mathbb{R}^n$ is the unknown vector map with $|u| = \sqrt{u^{(1)^2} + \cdots + u^{(n)^2}}$, $u_0 = (u_0^{(1)}, \ldots, u_0^{(n)})$ the initial data, and $\phi : \mathbb{R} \to \mathbb{R}$ is given (sufficiently smooth) scalar function. This type of system is a model system for some phenomena in magnetohydrodynamics, elasticity theory and enhanced oil-recovery. We propose an upwind semi discrete finite difference scheme and prove the convergence of the approximate solution to the weak solution of (1). We also test our numerical scheme and provide some numerical results.

Joint work with: Nils Henrik Risebro (University of Oslo).

5.2 Session 2 - Room D - Navier-Stokes and Euler Equations I

S2 – NAVIER-STOKES AND EULER EQUATIONS I – ROOM D, 15.15–15.45

Incompressible Limit of the Linearized Navier–Stokes Equations

Nikolay Anatolievich Gusev Moscow Institute of Physics and Technology n.a.gusev@gmail.com

Let $D \subset \mathbb{R}^d$ be a bounded domain with a piecewise-smooth boundary ∂D , $d \in \mathbb{N}$. Let T > 0 and denote $D_T = \overline{D} \times [0, T]$.

Consider a fluid with *linear* equation of state $\rho = \rho_0 + \alpha p$ where p and ρ denote the pressure and the density respectively, $\alpha > 0$, $\rho_0 > 0$. (Such form of equation of state was suggested in [1] for low compressible fluids.) The linearisation of the Navier–Stokes equations for such fluid in the neighbourhood of an incompressible state $(\rho_0, \mathbf{b}, \overline{p})$ with constant density ρ_0 , divergence-free velocity $\mathbf{b}: D_T \to \mathbb{R}^d$ and pressure \overline{p} can be written as

$$\rho_t + \operatorname{div}(\mathbf{b}\rho) + \operatorname{div}\mathbf{u} = 0, \qquad \rho = \alpha p,$$

$$\mathbf{u}_t + (\mathbf{b}, \nabla)\mathbf{u} + (\mathbf{u}, \nabla)\mathbf{b} + \nabla p = \nu \Delta \mathbf{u} + \kappa \nabla \operatorname{div}\mathbf{u} - \rho(\mathbf{b}_t + (\mathbf{b}, \nabla)\mathbf{b}),$$
(1)

where $\nu > 0$, $\kappa \ge 0$. The unknowns in the equations (1) are the fields $\mathbf{u} : D_T \to \mathbb{R}^d$, $\rho : D_T \to \mathbb{R}$ and $p: D_T \to \mathbb{R}$, which are proportional to the variations of the velocity, density and pressure respectively.

Consider the following initial and boundary conditions for (1):

$$\mathbf{u}|_{t=0} = \mathbf{u}^{\circ}, \qquad \mathbf{u}|_{\partial D} = 0, \qquad p|_{t=0} = p^{\circ}.$$

$$\tag{2}$$

The problem (1)-(2) is usually studied when $\mathbf{b} \equiv 0$ (see, e.g., [2,3]). I will briefly present the results (obtained in [4]) on existence and uniqueness of weak solutions to (1)-(2) in general case ($\mathbf{b} \neq 0$) and then I will focus on passage to the limit in (1)-(2) as $\alpha \to 0$. Sufficient conditions for *weak* and *strong* convergence of the "velocity" \mathbf{u} and the "pressure" p (in $L^2(0,T;H_0^1)$ and $L^{\infty}(0,T;L^2)$ respectively) will be presented and compared against the well-known results for the Navier–Stokes equations, obtained in [5,6] and other papers. The necessity of these sufficient conditions will be demonstrated using explicit solutions to (1)-(2), which are available for some special simplified data.

- Shifrin E.G., Unsteady Flows of Viscous Slightly Compressible Fluids: the Condition of Continuous Dependence on Compressibility, Doklady Physics, 44, No. 3 (1999), pp. 189–192
- [2] Ikehata R., Koboyashi T. and Matsuyama T., Remark on the L₂ Estimates of the Density for the Compressible Navier–Stokes Flow in R³, Nonlinear Analysis, 47 (2001), pp. 2519–2526
- [3] Mucha P.B. and Zajaczkowski W.M., On a L_p -estimate for the linearized compressible Navier–Stokes equations with the Dirichlet boundary conditions, J. Differential Equations, **186** (2002), pp. 377–393
- [4] Gusev N.A., Asymptotic Properties of Linearized Equations of Low Compressible Fluid Motion, J. Math. Fluid Mech. (2011), DOI: 10.1007/s00021-011-0084-8
- [5] Lions P.-L. and Masmoudi N., Incompressible limit for a viscous compressible fluid, J. Math. Pures Appl., 77(6) (1998), pp. 585–627
- [6] Feireisl E. and Novotný A., The Low Mach Number Limit for the Full Navier–Stokes–Fourier System, Arch. Rational Mech. Anal., 186 (2007), pp. 77–107

50

* * *

$\mathrm{S2}$ – Navier-Stokes and Euler Equations I – Room D, 15.45--16.15

Dynamical stability of non-constant equilibria for the compressible Navier-Stokes equations in Eulerian coordinates

Matthias Kotschote University of Constance matthias.kotschote@uni-konstanz.de

In this talk we show global existence and uniqueness of strong solutions to the isothermal compressible Navier-Stokes equations. The initial data have to be near equilibria which may be non-constant due to considering large external forces. We are able to prove exponential stability of equilibria in the phase space and, above all, to study the problem in Eulerian coordinates. The latter seems to be a novelty, since in earlier works strong L_p -solutions are studied only in Lagrangian coordinates; Eulerian coordinates have even been declared as impossible to treat, cf. on p. 418 in [1]. The proof is based on a careful study of the associated linear problem.

References

 P.B. Mucha and W.M. Zajączkowski, Global existence of solutions of the Dirichlet problem for the compressible Navier-Stokes equations, ZAMM, 84, no. 6 (2004), pp. 417-424.

5.3 Session 3 — Room H — Numerical Methods II

S3 – Numerical Methods II – Room H, 15.15-15.45

ADER-schemes for networks of 1D hyperbolic equations

Raul Borsche Technische Universität Kaiserslautern borsche@mathematik.uni-kl.de

In the last years numerical methods of high order, such as ADER-schemes [1], have been developed. In the following we present an extension of such methods to networks of 1D hyperbolic equations. Networks of this type are, e.g. gas-pipelines, sewer-systems, roads or supply chains. A single junction of such networks can be described by

$$\begin{cases} u_t + f(u)_x = 0, & x > 0 \\ \Phi(t, u(t, 0)) = 0, & \end{cases}$$

where u contains the states of all connected arcs and Φ represents the coupling conditions [3]. Solving this system numerically, a regular ADER-scheme can be applied in cells apart from the junction. In the vicinity of x = 0 we extend the spatial information over the boundary by an approach similar to [2]. For the overlapping stencils of WENO reconstruction [4] we determine the missing data by first applying the Cauchy-Kowalewski procedure, computing the temporal derivatives of the coupling conditions and then re-transforming these by an inverse Cauchy-Kowalewski procedure. The assumptions needed for these steps match with the requirements for the well-posedness of the coupling conditions [2]. The accuracy of the resulting method is studied in different numerical examples.

- [1] Toro, Eleuterio F., Riemann solvers and numerical methods for fluid dynamics, Springer-Verlag, (2009)
- [2] Sirui Tan and Chi-Wang Shu, Inverse Lax-Wendroff procedure for numerical boundary conditions of conservation laws, *Journal of Computational Physics*, 229(21) (2010), pp. 8144-8166
- [3] R. Borsche, R. M. Colombo and M. Garavello, Mixed systems: ODEs Balance laws, Journal of Differential Equations, 252(3) (2012), pp. 2311-2338
- [4] Jiang, Guang-Shan and Shu, Chi-Wang, Efficient implementation of weighted ENO schemes, Journal of Computational Physics, 126 (1996), pp. 202-228

Joint work with: Jochen Kall (Technische Universität Kaiserslautern), Axel Klar (Technische Universität Kaiserslautern)

* * * ------

S3 – Numerical Methods II – Room H, 15.45–16.15

Entropy-stable discontinuous Galerkin finite element method with streamline diffusion and shock-capturing

Andreas Eduard Hiltebrand Seminar for Applied Mathematics, ETH Zurich, Zurich, Switzerland andreas.hiltebrand@sam.math.ethz.ch

We consider conservation laws in multiple spatial dimensions, e.g. in two dimensions:

$$u_t + f(u)_x + g(u)_y = 0$$

in a spatial domain D and with t in [0, T] together with suitable boundary conditions. u is the unknown vector of conserved quantities and f resp. g is the flux function in x resp. y-direction.

The spatial elements are denoted T_j with $j \in J$, while $I^n = [t^n, t^{n+1}]$ is the temporal grid with $n \in \{0, \ldots, N-1\}, t^0 = 0$ and $t^N = T$.

We work in entropy variables (entropy symmetrisation): Choose an entropy function S(u), then the entropy variables v are given by $v = S_u$; so u is a function of v. The semilinear form for the space-time discontinuous Galerkin finite elements method is

$$B^{DG}(v,w) = -\sum_{n=0}^{N-1} \sum_{j \in J} \int_{I^n} \int_{T_j} (u \cdot w_t + f(u) \cdot w_x + g(u) \cdot w_y) dx dt$$

+
$$\sum_{n=0}^{N-1} \sum_{j \in J} \int_{T_j} (u(t_-^{n+1}) \cdot w(t_-^{n+1}) - u(t_-^n) \cdot w(t_+^n)) dx$$

+
$$\sum_{n=0}^{N-1} \sum_{j \in J} \sum_{i \in N_j} \int_{I^n} \int_{\partial T_{ij}} H(u^-, u^+; n_{ij}) \cdot w^- dS dt$$

where N_j are the indices of neighbouring cells of cell j, ∂T_{ij} is the common boundary of cell i and j and n_{ij} is the outward normal of cell j. The numerical flux H is chosen to be entropy-stable, i.e. it is an entropy-conservative flux [4] together with a numerical diffusion (Rusanov diffusion mostly). Using this form in the weak formulation already ensures entropy stability and a (formally) arbitrarily high order. But as we are interested in solutions with shocks we have to deal with spurious oscillations at discontinuities.

Therefore, we include a streamline-diffusion and a shock-capturing term [2,3], where the streamline-diffusion term gives some control on the residual while the shock-capturing leads to additional diffusion at shocks. The streamline diffusion term is

$$B^{SD}(v,w) = \sum_{n=0}^{N-1} \sum_{j \in J} \int_{I^n} \int_{T_j} (u_v w_t + f(u)_v w_x + g(u)_v w_y) \cdot Dr dx dt$$

where D = hI, $r = u_t + f(u)_x + g(u)_y$ is the residual and h is the mesh width. The shock-capturing term is

$$B^{SC}(v,w) = \sum_{n=0}^{N-1} \sum_{j \in J} \int_{I^n} \int_{T_j} \epsilon_j^n (u_t \cdot w_t + u_x \cdot w_x + u_y \cdot w_y) dx dt$$

It adds diffusion proportional to ϵ_j^n which is an integral quantity of the norm of the residual r normalized by the norm of the gradient of u.

Choosing the space of test and trial functions V (piecewise polynomials) this leads to the weak formulation: Find $v \in V$ such that

$$\forall w \in V : B^{DG}(v, w) + B^{SD}(v, w) + B^{SC}(v, w) = 0$$

Note that because the streamline diffusion and the shock-capturing terms are non-negative for w = v entropy stability carries over to this formulation.

We investigate the convergence properties of the method theoretically and experimentally for a range of problems. In particular we have solved the linear advection equation, Burgers' equation, the wave equation and the Euler equations in one or two spatial dimensions (cf. [1]).

References

- A. Hiltebrand and S. Mishra, Entropy stable shock capturing space-time Discontinuous Galerkin schemes for systems of conservation laws, preprint (2012)
- [2] C. Johnson and A. Szepessy, On the convergence of a finite element method for a nonlinear hyperbolic conservation law, *Mathematics of Computation*, 49 (1987), pp. 427-444
- [3] C. Johnson, A. Szepessy, and P. Hansbo, On the convergence of shock-capturing streamline diffusion finite element methods for hyperbolic conservation laws, *Mathematics of Computation*, **54** (1990), pp. 107-129
- [4] E. Tadmor, The numerical viscosity of entropy stable schemes for systems of conservation laws. I, Mathematics of Computation, 49 (1987), pp. 91-103

Joint work with: Siddhartha Mishra (Seminar for Applied Mathematics, ETH Zurich, Zurich, Switzerland)

5.4 Session 4 — Room I — Numerical Methods III

S4 – Numerical Methods III – Room I, 15.15–15.45

Entropy-Stable Path-Conservative Numerical Schemes.

Manuel J. Castro Díaz Universidad de Málaga. Dpto. Análisis Matemático castro@anamat.cie.uma.es In [4] Tadmor introduced a sufficient condition for the numerical flux of a conservative method to be entropypreserving. The goal of this work is to generalize this theory to strictly hyperbolic nonconservative systems of the form

$$u_t + A(u)u_x = 0, \quad x \in \mathbb{R}, \ t > 0, \tag{1}$$

equipped with an entropy pair, i.e. a pair of functions $(\eta, q), \eta$ being convex, such that

$$\nabla q(u) = \nabla \eta(u) \cdot A(u), \quad \forall \ u \in \mathbb{R}^n.$$

More precisely, the goal is to design semi-discrete path-conservative numerical schemes

$$\frac{d}{dt}u_i + \frac{1}{\Delta x}(D_{i-1/2}^+ + D_{i+1/2}^-) = 0$$

(see [3]) that are entropy-preserving in the following sense: there exists a consistent numerical entropy flux $Q_{i+1/2}$ such that the numerical solutions also satisfy the equation:

$$\frac{d}{dt}\eta(u_i) + \frac{1}{\Delta x}(Q_{i+1/2} - Q_{i-1/2}) = 0.$$
(2)

An entropy-preserving scheme is not expected to be stable in presence of shocks and thus some numerical viscosity has to be added. What we propose here is to stabilize entropy-preserving path-conservative numerical schemes by using the physical viscosity of the problem. The resulting methods are expected to overcome, at least partially, the difficulty of convergence of the numerical solutions to the physical one discussed in [2], [1].

References

- Abgrall R. and Karni S. A comment on the computation of non-conservative products. J. Comput. Phys., 45 (2010), pp. 382–403.
- [2] Castro M.J., LeFloch P.G., Muñoz M.L., and Parés C.. Why many theories of shock waves are necessary: convergence error in formally path-consistent schemes. J. Comput. Phys. 227 (2008), pp. 8107–8129.
- [3] Parés C., Numerical methods for nonconservative hyperbolic systems: a theoretical framework. SIAM J. Num. Anal. 44 (2006), pp. 300–321.
- [4] Tadmor E. The numerical viscosity of entropy stable schemes for systems of conservation laws. I. Math. Comp. 103 (1987), 49–91.

Joint work with: C. Pares (Universitad de Málaga, Spain), U.S. Fjordholm and S. Mishra, (ETH Zürich, Switzerland).

* * * _____

S4 – Numerical Methods III – Room I, 15.45–16.15

Numerical coupling between systems of balance laws and their late-time asymptotic behavior

Clément Cancès LJLL - UPMC Paris 06 clement.cances@upmc.fr Some hyperbolic systems of balance laws with highly dissipative source terms

$$\epsilon \partial_t U^\epsilon + \partial_x F(U^\epsilon) = \frac{1}{\epsilon} R(U^\epsilon) + S(U^\epsilon), \tag{1}$$

may satisfy in the late-time limit $\epsilon \to 0$ a parabolic equation of the form

$$\partial_t u + \partial_x \left(f(u) - \partial_x \phi(u) \right) = 0. \tag{2}$$

We refer to [1] and [2] for example of models leading to such an asymptotic.

Firstly, we discuss the design of asymptotic preserving Finite Volume schemes based on the HLL formalism [3] following the method proposed in [1,4,5], with a particular emphasis on the simple case of the barotropic gas dynamics.

Then it can be relevant, as for example in the case of model adaptation [5], to couple spatially the "finer" model (1) with the coarser one (2). A method for deriving relevant coupling conditions between the scheme is then proposed.

References

- C. Chalons, F. Coquel, E. Godlewski, P.-A. Raviart and N. Seguin, Godunov-type schemes for hyperbolic systems with parameter-dependent source. The case of Euler with friction, *Math. Models Methods Appl. Sci.*, **20** (2010), pp. 2109-2166.
- [2] C. Berthon and R. Turpault, Asymptotic preserving HLL schemes, Numer. Meth. P.D.E., 27 (2010), pp. 1396-1422.
- [3] A. Harten, P. D. Lax and B. van Leer, On upstream differencing and Godunov-type schemes for hyperbolic conservation laws, SIAM rev., 25 (1983), pp. 35-61.
- [4] L. Gosse and G. Toscani, An asymptotic preserving well balanced scheme for the hyperbolic heat equations, C. R. Acad. Sci. Paris Sér. I Math., 323 (2002), pp. 534-546.
- [5] A.-C. Boulanger, C. Cancès, H. Mathis, K. Saleh and N. Seguin, OSAMOAL: Optimized simulations by adapted models using asymptotic limits, preprint (2012), to appear on *ESAIM: Proceedings*.

Joint work with: Anne-Céline Boulanger (INRIA Paris Rocquencourt), Hélène Mathis (Université de Nantes), Khaled Saleh (UPMC Paris 06 – EDF), Nicolas Seguin (UPMC Paris 06).

5.5 Session 5 — Room G — Numerical Methods for Atmospheric and Geophysical Models I

S5 – Numerical Methods for Atmospheric and Geophysical Models I – Room G, 15.15–15.45

Governing equations and discretization in conservative form for atmospheric models

Ivar Lie StormGeo, Bergen, Norway Ivar.Lie@stormgeo.com

We consider the governing equations for atmospheric models, which in essence consists of the Euler equations and transport equations for the thermodynamic variables and the moist variables. The usual formulations of these equations are derived from conservation principles and a general transport theorem. These basic principles are formulated in integral form, and the PDEs are derived using integral theorem like the divergence theorem.

In this paper we use the governing equations in integral form, hence using the basic principles as they are. There are several advantages of doing this:

- Explicit expression for conservation
- Integral operators are much "nicer" than differential operators.
- It is easy (and natural) to use finite volume methods so one has discrete conservation
- Conservative time discretization can be constructed easily.

Another important topic is the imposition of boundary conditions. It is well known that the so-called primitive equations, which in essence are the differential form if the equations consider, are not well-posed with any classical boundary conditions. However, These results are are for direct imposition of the boundary conditions. The integral formulation of the equations is using a weak imposition of the boundary conditions, and one can show that an imposition of transparent lateral boundary conditions in weak form gives a well-posed problem.

The paper will discuss the integral formulation of the governing equations for weather models in general, see e.g. [1], and use the extensively used WRF model equations [2] as a concrete example. We will show how a combination of finite volume and mixed finite element discretization can be used in a natural way, and present some conservative time discretization schemes. The formulation of transparent boundary conditions will be presented, with an indication of how higher order boundary conditions can be combined with higher order finite volume methods and mixed finite element methods.

References

- P.R.Bannon, Theoretical Foundations for Models of Moist Convection, J.Atmos.Sci. 59(2002), pp.1967-1982.
- [2] http://www.wrfmodel.org

S5 – Numerical Methods for Atmospheric and Geophysical Models I – Room G, 15.45–16.15

Multi-Resolution Methods for Quantifying Uncertainties in Geophysical Applications

Ilja Kröker University of Stuttgart Ilja.Kroeker@mathematik.uni-stuttgart.de

Let us consider for T > 0 the following model problem in the open set $D \subset \mathbb{R}^2$ and probability space (Ω, P)

$$\begin{cases} S_t + \operatorname{div}(\mathbf{v}_s f(S)) = 0 & \text{ in } D \times (0, T) \times \Omega, \\ S(x, 0) = S_0(x) & \text{ in } D. \end{cases}$$
(1)

Here $S: D \times [0,T) \times \Omega \rightarrow [0,1]$ is the unknown saturation of the wetting liquid. The non-linear fractional flow function f and the initial saturation S_0 of the wetting fluid. A random perturbation is given by the velocity field

$$\mathbf{v}_s = (v^1 + \xi(\omega), v^2)^t$$

with a uniformly distributed random variable ξ . The given velocity field $\mathbf{v}_s = \mathbf{v}_s(x)$ satisfies

$$\operatorname{div}(\mathbf{v}_s) = 0 \quad \text{in} \quad D \times \Omega. \tag{2}$$

We understand (1) as a model problem as it ocurs in e.g. porous media flow or sedimentation. The well known Monte-Carlo method can be applied to (1) but requires especially in two or more space dimensions,

high computational effort to quantify the uncertainty. The Polynomial Chaos (PC) stochastic discretisation see e.g. [1], [3] is one way to quantify uncertainty but leads to a high dimensional hyperbolic system. A further application of PC method on an hyperbolic problem is given in [4]. The numerical solution of this system is less expensive then the Monte-Carlo method but it still challenging. We suggest a multi-resolution stochastic discretization a further improvement of the PC-discretization. This approach is similar to the method in [2] applied to problems with randomly perturbed initial values.

In this work we consider the application of a second order finite-volume numerical scheme with the multiresolution stochastic discretization on (weakly) hyperbolic problems with a randomly perturbed nonlinear flux in one and two space dimensions. Furthermore we discuss advantages of the multi-resolution approach from the point of view of parallel computing.

We present numerical examples of the application of the method on the random perturbed quarter five-spot problem (1), (2) and sedimentation problem.

References

- I. Kröker R. Bürger and C. Rohde, Uncertainty quantification for a clarifier-thickener model with random feed, in *Finite volumes for complex applications VI*, J. Fort, J. Fürst, J. Halama, R. Herbin, F. Hubert (eds.),(2011), pp. 195-203
- [2] J. Tryoen, and O. Le Maître, and M. Ndjinga, and A. Ern, Intrusive Galerkin methods with upwinding for uncertain nonlinear hyperbolic systems, J. Comput. Phys., 229, (2010), no. 18, 6485–6511.
- [3] Ghanem, Roger G., and Spanos, Pol D., Stochastic finite elements: a spectral approach, Springer-Verlag, x+214, (1991)
- [4] Gaël Poëtte and Bruno Després and Didier Lucor, Uncertainty quantification for systems of conservation laws, J. Comput. Phys., 228(7). (2009), pp. 2443-2467.

Joint work with: Raimund Bürger (Universidad de Concepción), Wolfgang Nowak (University of Stuttgart), Christian Rohde (University of Stuttgart).

5.6 Session 6 — Room E — Multi Physics Models I

S6 – Multi Physics Models I – Room E, 15.15–15.45

Loss of strict hyperbolicity and Riemann solutions for vertical three-phase flow in porous media

Panters Rodriguez-Bermudez Instituto Nacional de Matemática Pura e Aplicada (IMPA) panters@impa.br

Systems of conservation laws modeling three-phase flow in porous media typically fail to be strictly hyperbolic. This is the case for horizontal one-dimensional convective flow, where this failure occurs at four umbilic points: the three vertices of the saturation triangle and an interior point [1], [2]. When gravity is considered, the situation can be even more complex.

We study the hyperbolicity for systems of two conservation laws, which model vertical three-phase flow, where both gravity and convection are considered. Besides the above mentioned umbilic points, we found new types of points where characteristic values coincide, located at the boundary of the saturation triangle. We have characterized these points using Schaeffer-Shearer cones [4].

For the particular case where convection is negligible, we present the structures of Riemann solutions in terms of fluid density differences. It turns out that these structures are organized around the special cases where two of the fluids have equal densities [3]. In such cases there appears a whole edge of the saturation triangle along which the Jacobian is a multiple of the identity. For certain Riemann data such edges represent contact waves in the solution.

References

- Isaacson, E. L., Marchesin, D., Plohr, B. J. and Temple, J. B., Multiphase flow models with singular Riemann problems. *Mat. Aplic. Comp.* (1992) 11, no. 2, pp. 147-166.
- [2] Medeiros, H. B., Stable hyperbolic singularities for three-phase flow models in oil reservoir simulation. Acta Applicandae Mathematicae (1992) 28, pp. 135-159.
- [3] Rodriguez-Bermudez, P., Buoyancy Driven Three-Phase Flow in Porous Media, PhD Thesis, IMPA, Rio de Janeiro, (2010), www.preprint.impa.br.
- [4] Schaeffer, D. G., Shearer, M., The classification of 2 × 2 systems of non-strictly hyperbolic conservation laws, with application to oil recovery. *Comunications on Pure and Applied Mathematics* (1987) XL, pp. 141-178.

Joint work with: Dan Marchesin (Instituto Nacional de Matemática Pura e Aplicada, IMPA)

* * * ------

S6 – Multi Physics Models I – Room E, 15.45–16.15

Multiple Species Mixing at High Reynolds Number

James Glimm Stony Brook University glimm@ams.sunysb.edu

We study the multiple species Navier-Stokes equation in the limit of high Reynolds number. Acceleration driven flows of this nature define the classical Rayleigh-Taylor and Richtmyer-Meshkov instabilities. We have achieved systematic agreement across many experiments for the overall growth of the mixing region, based on the following two algorithmic features:

- 1. tracking, or some lagrangian dynamics at regions of large solution gradients, to avoid Eulerian advection mass diffusion,
- 2. dynamic (parameter free) subgrid models to account for turbulence of unresolved scales.

Built on this capability, we have reached a number of conclusions:

- uncertainty regarding initial conditions is a minor issue for most but not all experiments;
- the growth rate of the mixing layer is not universal;
- mixing flow properties show only a mild Reynolds number dependence in the range from the experimental value 35,000 to infinite Re.

The molecular properties of the mixture are described by probability density functions (PDFs). We show norm convergence of the indefinite integral of these, the cumulative distribution functions (CDFs). A formulation of w^{*} convergence as Young measures was applied to LES turbulent solutions to express these facts. A theoretical description of this high Re limit was analyzed [1] on the basis of assumed Kolmogorov 1941 statistics. K41 is equivalent to a Sobolev inequality of fractional order, and allows convergence of the incompressible Navier-Stokes equations to a limiting solution of the Euler equations. Likewise, these equations coupled to passive scalars converge w^{*} to a limit which is a Young measure. Taking a renormalization group (RNG) point of view, we expect the compressible Euler equations with n species to have n + 1 RNG fixed points, nonunique solutions of the multispecies Euler equations, with the distinct solutions labeled by turbulent Schmidt and Prandtl numbers and a bulk to shear viscosity ratio.

Contributions of collaborators are gratefully acknowledged.

References

 G.-Q. Chen and J. Glimm, Kolmogorov's Theory of Turbulence and the Invisid Limit of the Navier-Stokes Equations in R³, To appear in CMP,

5.7 Session 7 — Room A — Theory of Conservation Laws I

S7 - Theory of Conservation Laws I - Room A, 15.15-15.45

Time-asymptotic interaction of flocking particles and an incompressible viscous fluid

Young-Pil Choi Department of Mathematical Sciences, Seoul National University freelyer@snu.ac.kr

In this talk, we present a new coupled kinetic-fluid model for the interactions between Cucker-Smale(C-S) flocking particles and incompressible fluid on the periodic spatial domain \mathbb{T}^d . Let $f = f(x, \xi, t)$ be the one-particle distribution function at a periodic spatial domain $x \in \mathbb{T}^d, \xi \in \mathbb{R}^d$ at time t, and u = u(x, t) be the bulk velocity of fluid. In this situation, our model for C-S particles-fluid reads as follows:

$$\partial_t f + \xi \cdot \nabla_x f + \nabla_\xi \cdot \left[(F_a(f) + F_d) f \right] = 0, \quad (x,\xi) \in \mathbb{T}^d \times \mathbb{R}^d, \quad t > 0,$$

$$\partial_t u + (u \cdot \nabla_x) u + \nabla_x p - \mu \Delta_x u = -d \int_{\mathbb{R}^d} F_d f d\xi, \quad \nabla_x \cdot u = 0,$$
(1)

subject to initial data:

$$(f(x,\xi,0),u(x,0)) = (f_{in}(x,\xi),u_{in}(x)), \quad (x,\xi) \in \mathbb{T}^d \times \mathbb{R}^d,$$
(2)

where μ is the kinematic viscosity of the fluid, and F_a, F_d denote the alignment(flocking) force and drag force per unit mass, respectively:

$$F_a(f)(x,\xi,t) := \int_{\mathbb{T}^d \times \mathbb{R}^d} \psi(|x-y|)(\xi_* - \xi) f d\xi_* dy, \quad F_d(x,\xi,t) := u(x,t) - \xi.$$

Here, the communication weight $\psi : \mathbb{R} \to \mathbb{R}_+$ satisfies

$$\psi > 0, \quad (\psi(s_1) - \psi(s_2))(s_1 - s_2) \le 0, \quad \|\psi\|_{\mathcal{C}^1} < \infty.$$
 (3)

Our coupled system consists of the kinetic Cucker-Smale equation and the incompressible Navier-Stokes equations, and these two systems are coupled through the drag force. For the proposed model, we provide a global existence of weak solutions and a priori time-asymptotic exponential flocking estimates for any smooth flow, when the kinematic viscosity of the fluid is sufficiently large. The velocity of an individual C-S particles and fluid velocity tend to the averaged time-dependent particle velocities exponentially fast.

Joint work with: Hyeong-Ohk Bae (Department of financial engineering, Ajou university), Seung-Yeal Ha (Department of mathematical sciences, Seoul national university), Moon-jin Kang (Department of mathematical sciences, Seoul national university).

S7 - Theory of Conservation Laws I - Room A, 15.45-16.15

* * * -

High frequency waves and the maximal smoothing effect for nonlinear scalar conservation laws

Stéphane Junca Laboratoire J.A.D., Université de Nice Spohia-Antipolis, UMR CNRS 6621 junca@unice.fr

First, the propagation of well prepared supercritical high frequency waves for entropy solutions of multidimensional nonlinear scalar conservation laws is studied and simplified from [2].

Second, such oscillating solutions are used to highlight a conjecture of Lions, Perthame, Tadmor, (1994), [7], about the maximal regularizing effect for nonlinear conservation laws. For this purpose, a new definition of nonlinear flux is stated and compared to classical definitions, see [1,2,3,5,7]. Then it is proved that the smoothness expected by [7] in Sobolev spaces cannot be exceeded.

- F. Berthelin, S. Junca, Averaging lemmas with a force term in the transport equation, J.Math.Pures Appl.,(9), 93, No 2,(2010), pp. 113-131.
- [2] G.-Q. Chen, S. Junca, M. Rascle, Validity of Nonlinear Geometric Optics for Entropy Solutions of Multidimensional Scalar Conservation Laws, J. Differential. Equations, 222, (2006), pp. 439-475.
- [3] G. Crippa, F. Otto, M. Westdickenberg, Regularizing effect of nonlinearity in multidimensional scalar conservation laws. *Transport equations and multi-D hyperbolic conservation laws*, Lect. Notes Unione Mat. Ital., 5, Springer, Berlin, (2008), pp. 77-128,.
- [4] C. De Lellis, F. Otto, M. Westdickenberg, Structure of entropy solutions for multidimensional scalar conservation laws, Arch. Ration. Mech. Anal., 170, no2, (2003), pp. 137-184.
- [5] B. Engquist, W. E, Large time behavior and homogenization of solutions of two-dimensional conservation laws, Comm. Pure Appl. Math., 46, (1993), pp. 1-26.
- [6] P.-E. Jabin, Some regularizing methods for transport equations and the regularity of solutions to scalar conservation laws. 2008-2009, Exp. No. XVI, Sémin. Équ. Dériv. Partielles, École Polytech., (2010).
- [7] P.-L. Lions, B. Perthame, E. Tadmor, A kinetic formulation of multidimensional scalar conservation laws and related equations, J. Amer. Math. Soc., 7, (1994), pp. 169-192.
- [8] E. Tadmor, and T. Tao, Velocity averaging, kinetic formulations, and regularizing effects in quasi-linear PDEs. Comm. Pure Appl. Math., 60, no. 10, (2007), pp. 1488-1521.

5.8 Session 8 — Room B — Reaction-Convection-Diffusion Equations

S8 - REACTION-CONVECTION-DIFFUSION EQUATIONS - ROOM B, 15.15-15.45

On the well-posedness of Entropy Solutions to the Degenerate Parabolic Equation with a zero-flux boundary condition

Mohamed Karimou Gazibo University of Franche-Comte mgazibok@univ-fcomte.fr

We consider the general degenerate hyperbolic-parabolic equation:

$$(P) \begin{cases} u_t + \nabla f(u) - \Delta \phi(u) = 0 & \text{in } Q =]0, T[\times \Omega, \\ u(0, x) = u_0(x) & \text{in } \Omega, \\ (f(u) - \nabla \phi(u)) \eta = 0 & \text{on } \Sigma =]0, T[\times \partial \Omega., \end{cases}$$

where Ω be a bounded set of \mathbb{R}^N with a lipschitz boundary $\partial\Omega$ and η the unit normal to $\partial\Omega$ outward to Ω . The initial datum $u_0(x)$ is a measurable function taking values in $[0, u_{\max}]$. Here ϕ is a continuous function non decreasing. Following [2] we assume that f is compactly supported and we define an appropriate notion of entropy solution. We use a vanishing viscosity approximation and get the a priori estimates useful for passing to the limit in the approximate problem. The main point for passing to the limit is based on a local compacity argument (see [3]). In [3], Panov under a local compacity argument on the truncature of approximates solutions obtains a strong precompactness result of entropy solution. We adapt this result in our case and prove that the limit of entropy solution of approximate problem is an entropy solution of (P). We focus on the question of uniqueness of entropy solution for (P). Then it is easy to prove uniqueness of solutions such that boundary condition is satisfied in the sense of strong boundary trace. Unfortunately, we are able to establish this additional solution regularity only for the stationary problem (S) associate to (P) and in the case of one space dimension. Therefore we adapt the hint from the paper [1] and compare a general solution to (P) with a regular solution to (S). We conclude by a standard application of the notion of integral solution coming from the nonlinear semigroup theory . Eventually, we prove the uniqueness result in one space dimension.

References

- B. Andreianov and F. Bouhsiss, Uniqueness for an elliptic-parabolic problem with Neumann boundary condition, J. Evol. Equ 4 (2004) 273-295.
- [2] R. Burger, H. Frid and K. H. Karlsen, On the well-posedness of entropy solution to conservation laws with a zero-flux boundary condition, J. Math. Anal. Appl., 326. (2007), 108-120
- [3] E.Yu. Panov, On the strong pre-compactness property for entropy solutions of a degenerate elliptic equation with discontinuous flux, J. Differential Equations 247 (2009) 28212870.

* * * ·

Joint work with: Boris Andreianov (University of Franche-Comte).

S8 - REACTION-CONVECTION-DIFFUSION EQUATIONS - ROOM B, 15.45-16.15

Entropy formulation for forward–backward parabolic equation

Andrea, Terracina Sapienza Università di Roma terracin@mat.uniroma1.it

In this talk we consider the following forward-backward parabolic equation

$$u_t = \phi(u)_{xx}$$

where the function $\phi(u)$ change monotonicity, this equation has several applications, for instance, in models of phase separation, image processing and population dynamic.

Obviously the Cauchy problem associated to this equation is ill-posed. In analogy with conservation laws, it is possible to consider a viscous regularization of the original problem, that agrees with a pseudo-parabolic approximation of it (see [1]), obtaining an entropy formulation that allows to select a well posed concept of solution for some class of initial data.

In [2] we consider a piecewise linear diffusion function ϕ and initial data that takes values only in the stable regions (where ϕ is increasing). In this case it is easier to understand the meaning of the entropy condition that correspond to admissibility condition for the evolution of the interface that separates different stable regions (again analogy with conservation laws). In this contest we prove local existence and uniqueness. In [3] we study extension in time of the solution and analyze qualitative properties of the interface between two different regions.

In this talk we present some recent results obtained in collaboration with F. Smarrazzo in which we study the singular limit of the pseudo parabolic equation in the case in which ϕ is nonlinear. When initial data takes values only in the stable regions, it is possible to prove more accurate estimate for the third order approximation problem, in particular we obtain a maximum principle for that equation. Using this results, we prove that, for some classes of initial data, the pseudo parabolic problem with Neumann boundary condition converges strongly to the solution of the forward-backward Neumann boundary value problem.

This is a first result of existence in the nonlinear case and is also a results of strong convergence of the approximation problem to the original one.

References

- Plotnikov P. I., Passing to the limit with respect to viscosity in an equation with variable parabolicity direction, *Diff. Equ.*, **30** (1994), pp. 614-622
- [2] Mascia C., Terracina A. & Tesei A., Two-phase entropy solutions of a forward-backward parabolic equation, Arch. Ration. Mech., 194 (2009), pp. 887-925
- [3] Terracina A., Qualitative Behavior of the Two-Phase Entropy Solution of a Forward-Backward Parabolic Problem, SIAM J. Math. Anal., 43 (2011), pp. 228-252

Joint work with: Flavia Smarrazzo (Sapienza Università di Roma).

5.9 Session 9 — Room C — Control Problems for Hyperbolic Equations I

S9 – Control Problems for Hyperbolic Equations I – Room C, 15.15–15.45

Nash equilibria for traffic flow on a network

Alberto Bressan Penn State University bressan@math.psu.edu

In connection with the Lighthill-Whitham conservation law model of traffic flow, a cost functional can be introduced, depending on the departure time and on the arrival time of each driver. Under natural assumptions, there exists a unique globally optimal solution, minimizing the sum of the costs to all drivers.

In a realistic situation, however, the actual traffic is better described by a Nash equilibrium solution, where no driver can lower his individual cost by changing his own departure time. In the case of a single group of drivers traveling on the same road, a characterization of the Nash solution can be provided, establishing its existence and uniqueness.

The talk will also deal with the case of several groups of drivers with different costs, who can choose among different routes on a network of roads in order to reach destinations. For this case, the existence of at least one Nash equilibrium can be still be proved.

The issue of stability of Nash equilibria will also be considered, discussing the results of some numerical simulations.

S9 - Control Problems for Hyperbolic Equations I - Room C, 15.45-16.15

Exact controllability of scalar conservation laws with strict convex flux

Shyam Sundar Ghoshal *Tifr-Cam,India* ssghoshal@math.tifrbng.res.in

We consider the following scalar conservation law in one space dimension. Let $f : \mathbb{R} \to \mathbb{R}$ be a strictly convex C^1 function satisfying the super linear growth,

$$\lim_{|u| \to \infty} \frac{f(u)}{|u|} = \infty.$$
(1)

Let $T > 0, 0 \le \delta < T, A < B, I = (A, B), \Omega = I \times (\delta, T), u_0 \in L^{\infty}(I), b_0, b_1 \in L^{\infty}((0, T))$ and consider the problem

$$u_t + f(u)_x = 0 \qquad (x,t) \in \Omega, \tag{2}$$

$$u(x,\delta) = u_0(x) \quad x \in I, \tag{3}$$

$$u(A,t) = b_0(t) \quad t \in (\delta,T), \tag{4}$$

$$u(B,t) = b_1(t) \quad t \in (\delta,T).$$
(5)

Existence and uniqueness are heavyly understood for the above problem. In spite of being well studied, the problem for exact controllability (for optimal controllability, see [5]) of initial and initial-boundary value problem was open for quite a long time. Normally for the non linear evolution equations, technique of linearization is adopted to study controllability problems. Unfortunately this method does not work (see Horsin [3]) and very few results are available (see Ancona et al. [1],[2]) on this subject . In this talk we will discuss the following three problems of controllability. Let $u_0 \in L^{\infty}(\mathbb{R})$ and

(I) Controllability for pure initial value problem: Assume that $I = \mathbb{R}, \Omega = \mathbb{R} \times (0, T)$. Let $J_1 = (C_1, C_2), J_2 = (B_1, B_2), g \in L^{\infty}(J_1)$, a target be given. The question is, does there exists a $\bar{u}_0 \in L^{\infty}(J_2)$ and u in $L^{\infty}(\Omega)$ such that u is a solution of (2) satisfying

$$u(x,T) = g(x) \quad x \in J_1, \tag{6}$$

$$u(x,0) = \begin{cases} u_0(x) & \text{if } x \notin J_2, \\ \bar{u}_0(x) & \text{if } x \in J_2. \end{cases}$$

$$\tag{7}$$

(II) Controllability for one sided initial boundary value problem: Assume that $I = (0, \infty)$, $\Omega = \mathbb{R} \times (0, T)$, J = (0, C) and a target function $g \in L^{\infty}(J)$ be given. The question is, does there exists a $u \in L^{\infty}(\Omega)$ and $b \in L^{\infty}((0, T))$ such that u is a solution of (2) satisfying

$$u(x,T) = g(x) \quad \text{if } x \in J, \tag{8}$$

$$u(x,0) = u_0(x) \text{ if } x \in (0,\infty),$$
 (9)

$$u(0,t) = b(t) \quad \text{if } t \in (0,T).$$
 (10)

(III) Controllability from two sided initial boundary value problem:

(a). Let $\Omega = \mathbb{R} \times (0,T)$, $I_1 = (B_1, B_2)$, $B_1 \leq C \leq B_2$. Given the target functions $g_1 \in L^{\infty}(B_1, C)$, $g_2 \in L^{\infty}(C, B_2)$, does there exists a $\bar{u}_0 \in L^{\infty}(\mathbb{R} \setminus I_1)$ and $u \in L^{\infty}(\Omega)$ such that u is a solution of (2) satisfying

$$u(x,T) = \begin{cases} g_1(x) & \text{if } B_1 < x < C, \\ g_2(x) & \text{if } C < x < B_2. \end{cases}$$
(11)

and

$$u(x,0) = \begin{cases} u_0(x) & \text{if } B_1 < x < B_2, \\ \bar{u}_0(x) & \text{if } x < B_1 \text{ or } x > B_2. \end{cases}$$
(12)

(b). Here we consider controllability in a strip. Let $I = (B_1, B_2)$, $\Omega = I \times (0, T)$, $B_1 < C < B_2$. Let $g_1 \in L^{\infty}((B_1, C))$, $g_2 \in L^{\infty}((C, B_2))$ be given. Then the question is, does there exist $b_0, b_1 \in L^{\infty}((0, T))$ and a $u \in L^{\infty}(\Omega)$ such that u is a solution of (2) and satisfying

$$u(x,0) = u_0(x), \tag{13}$$

$$u(x,T) = \begin{cases} g_1(x) & \text{if } B_1 < x < C, \\ g_2(x) & \text{if } C < x < B_2. \end{cases}$$
(14)

$$u(B_1, t) = b_0(t), (15)$$

$$u(B_2, t) = b_1(t). (16)$$

Now the question is whether the problems (I), (II) and (III) admit a solution? In fact, it is true and we have settled all the three problems in the paper [4].

In the case of problem (II), Ancona et al. [1],[2] studied the problem from the point of view of Hamilton-Jacobi equations and studies the compactness properties of $\{u(\cdot, T)\}$ when u(x, 0) = 0 and $u(\cdot, 0) \in \mathcal{U}$, here \mathcal{U} is a set of controls satisfying some properties.

In our results on controllability, superlinearity of f plays an important role in removing the condition on T and by creating free regions. Next using convexity and backward construction, we explicitly construct solutions in these free regions for particular data which allow to obtain solutions for control problems.

- Ancona,F; A,Marson, On the attainability set for scalar non linear conservation laws with boundary control, SIAM J.Control optim., vol 36, No.1, 1998, 290-312.
- [2] Ancona, F; A.Marson, Scalar non linear conservation laws with integrable boundary data, Nonlinear Analysis. 35, 1999, 687-710.

- [3] T. Horsin, On the controllability of the Burger equation, ESIAM, Control optimization and Calculus of variations, 3, 1998, 83-95.
- [4] Adimurthi, Shyam Sundar Ghoshal, G.D.Veerappa Gowda, Exact controllability of scalar conservation law with strict convex flux - preprint, 2011.
- [5] Adimurthi, Shyam Sundar Ghoshal, G.D.Veerappa Gowda, Optimal controllability for scalar conservation laws with convex flux- preprint 2011.

Joint work with: Adimurthi (Tifr-Cam), G D V Gowda (Tifr-Cam)

6 Abstracts of contributed lectures — Monday 17.20–19.20

6.1 Session 10 — Room F — Numerical Methods IV

S10 – Numerical Methods IV – Room F, 17.20–17.50

Cartesian grid embedded boundary methods for hyperbolic problems

Christiane Helzel Ruhr-University Bochum christiane.helzel@rub.de

We discuss finite volume methods for hyperbolic pdes on Cartesian grids with embedded boundaries. Embedded boundary methods are very attractive for several reasons: The grid generation is simple even in the presence of complicated geometries. Furthermore, such an approach allows the use of regular Cartesian grid methods away from the embedded boundary, which are much simpler to construct and more accurate than unstructured grid methods. In embedded boundary grids with cut cells adjacent to the boundary, the cut cell volumes can be orders of magnitude smaller than a regular Cartesian grid cell volume. The use of standard difference procedures would lead to an unacceptably small integration timestep. Both *accuracy* and *stability* are issues that need to be addressed at these highly irregular cut cells adjacent to solid bodies. The goal is to construct a method which is stable for time steps that are appropriate for the regular part of the mesh and at the same time do not lead to a loss of accuracy. Several different ideas to overcome the small cell problem in an embedded boundary approach have been proposed in the literature and will briefly be discussed in the talk.

Our approach is based on constructing fluxes at cut cells in such a way that a certain cancellation property is satisfied. It means that flux differences must be of the order of the size of the small grid cell. This can be obtained by introducing *h*-boxes (i.e., boxes of the length of a regular grid cell) at cut cell interfaces (see [1]). More recently (see [2]) we have constructed embedded boundary methods in the context of the method of lines which led to several simplifications compared to the previous approach. This simplified *h*-box method will be presented in the talk. Our current goal is to extend the method to higher than second order. Preliminary results in this direction will also be presented.

References

- C. Helzel, M.J. Berger and R.J. LeVeque, A high-resolution rotated grid method for conservation laws with embedded geometries, SIAM J. Sci. Comput., 26 (2005), pp. 785–809.
- [2] M.J. Berger and C. Helzel, A simplified h-box method for embedded boundary grids, to appear in SIAM J Sci. Comput..

* * * -

Joint work with: Marsha J. Berger (Courant Institute of Mathematical Sciences, New York University)

S10 - Numerical Methods IV - Room F, 17.50-18.20

Comparison of WENO scheme and high-order WENO-gas-kinetic scheme for inviscid and viscous flow simulation

Jun Luo

Mathematics Department, Hong Kong University of Science and Technology Clear Water Bay, Kowloon, Hong Kong maluojun@ust.hk

Computational Fluid Dynamics has made great progress in 1970s and 1980s due to the development of the concept of nonlinear limiter and the characteristic wave decomposition of the Euler equations. Due to its accuracy, robustness, and efficiency, the 2nd-order schemes become the working horses in almost all practical engineering applications at the current stage. On the other hand, as the increasing of computer power and the requirement for accurate solutions for more challenging problems, such as compressible turbulent flow and aero-acoustics, high-order methods become good choices. Over the past two decades, significant progress has been made in the designing of high-order numerical methods in the computational fluid dynamics. To go beyond second-order accuracy, a high degree of sophistication is required to determine the additional degree of freedom in the high-order methods. There are at present several approaches that fulfil some of the basic requirements. Examples include the weighted essentially non-oscillatory (WENO) method and the discontinuous Galerkin (DG) Finite Element methods. Among high order schemes, the WENO may be the most reliable one for the fluid motion with both continuous and discontinuous wave structures. The WENO scheme introduces an accurate and robust reconstruction methods for the discrete numerical data.

In this paper, based on the same WENO reconstruction, for the first time we are going to compare the effect of the flux functions on the numerical accuracy for both the Euler and Navier-Stokes solutions. The compared schemes are the standard fifth-order WENO method and the WENO-Gas-kinetic scheme. The fifth-order finite difference WENO scheme uses the Steger-Warming flux splitting for inviscid parts, the sixth-order accurate central difference for viscous terms, and three stage Runge-Kutta time stepping for the time evolution. The finite volume WENO-GKS uses the same WENO reconstruction for the conservative variables, evaluates its spatial and time evolution of a distribution function based on the solution of the gas-kinetic equation, and obtains the flow transport across a cell interface by one step integration along the cell interface in both space and time. The WENO-Steger-Warming scheme(WENO-SW) and WENO-GKS are tested in the following three cases with different mesh sizes: vortex propagation, Mach 3 step problem, and cavity flow at Reynolds number 3200.

Our results show that both WENO-SW and WENO-GKS yield quantitatively similar results and agree well with each other, provided a sufficient grid resolution is presented. With reduced mesh points, the WENO-GKS appears to have less numerical dissipation and gives more accurate solutions than that from the WENO-SW. For the NS solution, the WENO-GKS couples invscid and viscous fluxes in a single flux evaluation from a WENO initial reconstruction. However, the WENO-SW separates the treatment of inviscid and viscous terms. In the cavity flow simulation, the WENO-SW is more sensitive to the boundary treatment. The operatorsplitting discretization of inviscid and viscous terms prevents the explicit WENO-SW from getting steady state solution, especially in the coarse mesh case. The solution of the WENO-SW is more sensitive to the initial data reconstruction. The general conclusion from the numerical comparison is the following. Besides high-order initial reconstruction, an accurate gas evolution model or flux function in a high-order scheme is very important as well in the capturing of physical solutions. The different numerical treatment of inviscid and viscous terms in the NS equations may introduce errors in high-order schemes, especially in the cases where there are a few mesh points in the dissipative physical structure, such as the boundary layer, and with wave interactions. It will be a challenge to develop a high-order scheme by using operator splitting method to discretize all physical terms separately. Theoretically, the viscous dissipation, heat conduction and transport are indistinguishable in a gas evolution process.

References

[1] C.W. Shu, Essentially non-oscillatory and weighted essentially non-oscillatory schemes for hyperbolic conservation laws, *Lecture Notes in Mathematics*, Springer, (1998).

- [2] G.S. Jiang, C.W. Shu, Efficient implementation of Weighted ENO schemes, J. Comput. Phys., 126 (1996) pp. 202-228.
- [3] J. Casper, Finite-volume implementation of high-order essentially non-oscillatory schemes in two dimensions, AIAA Journal, 30 (1992), pp. 2829-2835.
- [4] K. Xu, A gas-kinetic BGK scheme for the Navier-Stokes equations and its connection with artificial dissipation and Godunov method, J. Comput. Phys., 171 (2001) pp. 289-335.
- [5] Q.B. Li, K. Xu, and S. Fu, A high-order gas-kinetic Navier-Stokes solver, J. Comput. Phys., 229 (2010), pp. 6715-6731.
- [6] J. Luo, K. Xu, A high-order WENO-gas-kinetic scheme for hydrodynamic equations, submitted to J. Comput. Phys.

Joint work with: Li Jun Xuan, (Mathematics Department, Hong Kong University of Science and Technology Clear Water Bay, Kowloon, Hong Kong), Kun Xu (Mathematics Department, Hong Kong University of Science and Technology Clear Water Bay, Kowloon, Hong Kong)

S10 – Numerical Methods IV – Room F, 18.20–18.50

- * * * -

An efficient discretization of the shallow water equations with source terms on unstructured meshes

Arnaud Duran Université Montpellier II, I3M, Montpellier, France Arnaud.Duran@math.univ-montp2.fr

The proposed work concerns the numerical approximation of weak solutions for the shallow water equations with varying topography and friction source terms, on unstructured meshes. The discretization of the topography source term is based on a useful reformulation of the classical shallow water equations, namely the "pre-balanced" equations:

$$U_t + \nabla \cdot H(U, z) = \mathcal{S}(U, z) - \sigma(U)R(U), \qquad (1)$$

with

With

$$U = {}^{t}(\eta, q_{x}, q_{y}) \qquad H(U, Z) = \begin{pmatrix} q_{x} & q_{y} \\ uq_{x} + \frac{1}{2}g(\eta^{2} - 2\eta z) & vq_{x} \\ uq_{y} & vq_{y} + \frac{1}{2}g(\eta^{2} - 2\eta z) \end{pmatrix} \text{ and}$$

$$\mathcal{S}(U, Z) = {}^{t}(0, -g\eta z_{x}, -g\eta z_{y}),$$

where η denotes the water free surface, and q_x , q_y , u, v are the respective discharges and velocities in the xand y direction, z a parameterization of the topography variations and $\sigma(U)R(U)$ an additional friction source term, which can be for instance the usual Manning-Chezy source term.

The discretization of the topography source term is performed using a two-dimensional generalization of the well-balanced scheme introduced in [3]. Indeed, the pre-balanced equations (1) allow a simple and robust discretization of the topography source term. Several extensions of the work introduced in [3] are highlighted. For the discretization of the friction term $\sigma(U)R(U)$, we introduce a new method, in the spirit of [2], relying on a modified Godunov-type scheme that directly account for the friction source term, preserves the robustness and does not change the CFL condition. A formally high-order extension is also investigated, that does not require the introduction of additional source terms to preserve consistency, as done in [1] for instance.

The resulting model provides good properties, including the preservation of motionless steady states, high accuracy and robustness. These properties are assessed through extensive numerical validations and we show the capability of our model in dealing with particularly delicate contexts, including wetting and drying with leading frictional effects. After some academical validations and accuracy/convergence studies, some comparisons with experimental data, like the Malpasset dam break data set are performed.

References

- Audusse E., Bristeau M.O., A well-balanced positivity preserving "second order" scheme for shallow water flows on unstructured meshes, *Journal of Computational Physics* 206 (2005), pp. 311-333.
- [2] Berthon C., Marche F., Turpault R., An efficient scheme on wet dry transitions for Shallow Water Equations with friction, *Computers and Fluids* 48 (2011), pp. 192-201.
- [3] Liang Q., Marche F., Numerical resolution of well-balanced shallow water equations with complex source terms, Advances in Water Resources 32 (2009), pp. 873 - 884.

Joint work with: C.Berthon and R.Turpault (Université de Nantes, France), F.Marche (Université de Montpellier, France), Q.Liang (University of Newcastle, UK).

______ * * * _____

S10 – Numerical Methods IV – Room F, 18.50-19.20

Numerical Dissipation and Wrong Propagation Speed of Discontinuities For Stiff Source Terms

Helen C. Yee NASA-Ames Research Center, Moffett Field, CA, 94035, USA Helen.M.YeeQnasa.gov

The appearance of stiff source terms in modeling unsteady flows containing turbulence with strong shocks and finite-rate chemistry/combustion poses difficulties beyond that for solving non-reacting turbulent flows. The amount and procedure in controlling numerical dissipation in the design of numerical methods can affect the degree of accuracy in obtaining the correct propagation speed of discontinuities if the source term is stiff. The dual requirement to achieve both numerical stability and accuracy with zero or minimal use of numerical dissipation for turbulence with strong shocks and combustion is most often conflicting for existing schemes that were designed for non-reacting flows. The goal of this paper is to relate numerical dissipations that are inherited in a selected set of high order shock-capturing schemes with the onset of wrong propagation speed of discontinuities for two representative stiff detonation wave problems.

Joint work with: Dmitry Kotov (Postdoctoral Fellow, Center for Turbulence Research, Stanford University, CA, 94305 USA), Björn Sjögreen (Lawrence Livermore National Laboratory, Livermore, CA, 94551, USA)

6.2 Session 11 — Room D — Navier-Stokes and Euler Equations II

S11 – Navier-Stokes and Euler Equations II – Room D, 17.20–17.50

Bounded vorticity, bounded velocity (Serfati) solutions to the incompressible 2D Euler equations

Helena J. Nussenzveig Lopes Federal University of Rio de Janeiro hlopes@im.ufrj.br

In 1963 V. I. Yudovich proved the existence and uniqueness of weak solutions of the incompressible 2D Euler equations in a bounded domain assuming that the vorticity, which is the curl of velocity, is bounded. This result was later extended by A. Majda to vorticities which are bounded and integrable in the full plane. A few further extensions of this result have been obtained, most notably by Yudovich himself and, also, by M. Vishik, always assuming some decay of vorticity at infinity. In a short note in 1995, Philippe Serfati gave an incomplete, yet brilliant, proof of existence and uniqueness of solutions to the 2D Euler equations in the whole plane when the initial vorticity and initial velocity are bounded, without the need for decay at infinity. In this talk I will report on work in progress aimed at extending Serfati's result to flows in a domain exterior to an obstacle.

Joint work with: David Ambrose (*Drexel University*), James P. Kelliher (*University of California, Riverside*) and Milton C. Lopes Filho (*Federal University of Rio de Janeiro*).

S11 – NAVIER-STOKES AND EULER EQUATIONS II – ROOM F, 17.50–18.20

Analysis of Oscillations and Defect Measures in Plasma Physics

Donatella Donatelli Dipartimento di Matematica Pura ed Applicata Università degli Studi dell'Aquila, 67100 L'Aquila, Italy donatell@univaq.it

A simplified model to describe the dynamics of a plasma is given by the coupling of the compressible Navier Stokes equations with a Poisson equation, where in dimensionless units the coupling constant can be expressed in terms of a parameter λ which represents the scaled Debye length, namely

$$\begin{cases} \partial_s \rho^{\lambda} + div(\rho^{\lambda}u^{\lambda}) = 0\\ \partial_s(\rho^{\lambda}u^{\lambda}) + div(\rho^{\lambda}u^{\lambda} \otimes u^{\lambda}) + \frac{1}{\gamma}\nabla(\rho^{\lambda})^{\gamma} = \overline{\mu}\Delta u^{\lambda} + (\overline{\nu} + \overline{\mu})\nabla divu^{\lambda} + \rho^{\lambda}\nabla V^{\lambda}\\ \lambda^2 \Delta V^{\lambda} = \rho^{\lambda} - 1. \end{cases}$$

In many cases the Debye length is very small compared to the macroscopic length and so it makes sense to consider the quasineutral limit $\lambda \to 0$ of the system. The velocity of the fluid then evolves according to the incompressible Navier Stokes flow. This type of limit has been studied by many authors in the framework of smooth solution or well-prepared initial data. However there is no analysis for the quasineutral limit for the Navier Stokes Poisson system in the contest of weak solutions and in the framework of general ill prepared initial data. The common feature of this kind of limits in the ill prepared data framework is the high plasma oscillations, namely the presence of high frequency time oscillations of the acoustic waves. Another issues which

makes the limiting behaviour analysis very hard is the presence of very stiff terms due to the electric field $E = \nabla V$. We show in different domains (whole space, periodic domains) that, as $\lambda \to 0$, the velocity field strongly converges towards an incompressible velocity vector field and the density fluctuation weakly converges to zero. In general the limit velocity field cannot be expected to satisfy the incompressible Navier Stokes equation, indeed the presence of high frequency oscillations strongly affects the quadratic nonlinearities and we have to take care of self interacting wave packets. We shall provide a detailed mathematical description of the convergence process by using microlocal defect measures and by developing an explicit correctors analysis.

References

 D. Donatelli and P. Marcati, Analysis of Oscillations and Defect Measures for the Quasineutral Limit in Plasma Physics, Submitted preprint (2011), arXiv:1112.2556v1 [math.AP].

Joint work with: Pierangelo Marcati (Dipartimento di Matematica Pura ed Applicata Università degli Studi dell'Aquila, 67100 L'Aquila, Italy)

S11 - NAVIER-STOKES AND EULER EQUATIONS II - ROOM F, 18.20-18.50

A Low Mach Number Limit of a Dispersive Navier-Stokes System

Konstantina Trivisa University of Maryland trivisa@amsc.umd.edu

This work establishes a low Mach number limit for classical solutions over the whole space of a compressible fluid dynamic system that includes dispersive corrections to the Navier-Stokes equations. The limiting system is a *ghost effect system* [[5]]. This type of systems cannot be typically derived from the Navier-Stokes system of gas dynamics, instead they can be formulated using concepts from kinetic theory. This work is part of a program that aims to identify fluid dynamic regimes and to construct a unified model that captures them. Such a model can also be useful in transition regimes where classical fluid equations are inadequate to describe the dynamics of fluids while computations using kinetic models are expensive. The analysis builds upon the framework developed by Métivier and Schochet [[4]] and Alazard [[1]] in the context of non-dispersive systems. The strategy involves establishing a priori estimates for the slow motion as well as a priori estimates for the fast motion. The desired convergence is obtained by establishing the local decay of the energy of the fast motion.

This is joint work with D. Levermore and W. Sun.

- T. Alazard, Low Mach number limit of the full Navier-Stokes equations, Arch. Rational Mech. Anal. 180, 1-73 (2006).
- [2] C. Bardos, C.D. Levermore, S. Ukai, T. Yang, Kinetic equations: fluid dynamical limits and viscous heating, Bull. Inst. Math. Acad. Sin. (N.S.) 3, No. 1, 1-49 (2008).
- [3] C.D. Levermore, W. Sun and K. Trivisa, A Low Mach Number Limit of a Dispersive Navier-Stokes System. To appear in *SIAM J. Math. Anal.* (2012).
- [4] G. Métivier, S. Schochet, The incompressible limit of the non-isentropic Euler equations, Arch. Rational Mech. Anal. 158, 61-90 (2001).
[5] Y. Sone, *Kinetic theory and fluid dynamics*, Modeling and Simulation in Science, Engineering and Technology. Birkhäuser, Boston, 2002.

S11 – NAVIER-STOKES AND EULER EQUATIONS II – ROOM F, 18.50–19.20

* * *

Low Mach number limit for the compressible viscous magnetohydrodynamic equations

Fucai Li Department of Mathematics, Nanjing University, Nanjing 210093, P.R. China fli@nju.edu.cn

Magnetohydrodynamics (MHD) studies the dynamics of compressible quasineutrally ionized fluids under the influence of electromagnetic fields. The applications of magnetohydrodynamics cover a very wide range of physical objects, from liquid metals to cosmic plasmas. In this talk we report our recent results on on the low Mach number limit for the compressible viscous magnetohydrodynamic (MHD) equations including the isentropic and non-isentropic cases. For the isentropic MHD equations, we consider the low Mach number limit in the whole space and periodic domain with well/ill-prepared data. For the non-isentropic MHD equations, we discuss the low Mach number limit in two cases: (i) small variations on density and temperature with well-prepared initial data; (ii) large variations on density and temperature with ill-prepared initial data. The key points in the proofs of rigorous results shall be mentioned the talk.

References

- S. Jiang, Q. C. Ju and F. C. Li, Incompressible limit of the compressible Magnetohydrodynamic equations with periodic boundary conditions, Comm. Math. Phys. 297 (2010), 371-400.
- [2] S. Jiang, Q. C. Ju and F. C. Li, Incompressible limit of the compressible magnetohydrodynamic equations with vanishing viscosity coefficients, SIAM J. Math. Anal. 42 (2010), 2539-2553.
- [3] S. Jiang, Q. C. Ju and F. C. Li, Low Mach number limit for the multi-dimensional full magnetohydrodynamic equations, submitted.
- [4] S. Jiang, Q.C. Ju, F. C. Li and Z.P. Xin, Low Mach number limit for the full compressible magnetohydrodynamic equations with general initial data, submitted.
- [5] S. Jiang, Q.C. Ju and F. C. Li Incompressible limit of the compressible non-isentropic magnetohydrodynamic equations with zero magnetic diffusivity, submitted.

Joint work with: This talk is based on the papers jointed with Song Jiang (Institute of Applied Physics and Computational Mathematics, Beijing), Qiangchang Ju (Institute of Applied Physics and Computational Mathematics, Beijing), and Zhouping Xin (The Chinese University of Hong Kong).

6.3 Session 12 — Room H — Numerical Methods V

S12 - Numerical Methods V - Room H, 17.20-17.50

Practical CFL conditions for MUSCL schemes solving Euler equations

Yohan Penel University Paris 6 (LJLL-LRC MANON) penel@ann.jussieu.fr

We present a practical method to adapt classical MUSCL schemes on unstructured grids for solving Euler equations in order to ensure positivity of density and pressure. This issue has been investigated in [1,4,5,7]. Qualitative results have been proven in the aforementioned papers. The aim of the present work is to provide an explicit and optimized CFL condition. Preserving positivity indeed requires a more restrictive stability condition. Another adaptation is the reconstruction step where modified limiters are used [2,3] as well as an additional damping coefficient. To illustrate this approach, we focus on a severe test detailed in [6] where density and pressure become dramatically small.

References

- C. Berthon, Robustness of MUSCL schemes for 2D unstructured meshes, J. Comput. Phys., 218(2), 495–509 (2006).
- [2] C. Calgaro, E. Chane-Kane, E. Creusé & T. Goudon, L[∞]-stability of vertex-based MUSCL finite volume schemes on unstructured grids; simulation of incompressible flows with high density ratios, J. Comput. Phys., 229(17), 6027–6046 (2010).
- [3] S. Clain & V. Clauzon, L^{∞} -stability of the MUSCL methods, Numer. Math., 116(1), 31–64 (2010).
- [4] T. Linde & P.L. Roe, *Robust Euler codes*, **13th AIAA CFD Conf.**, AIAA-97-2098, (1997).
- [5] B. Perthame & C.-W. Shu, On positivity preserving finite volume schemes for Euler equations, Numer. Math., 73, 119–130 (1996).
- [6] E. Toro, Riemann solvers and numerical methods for fluid dynamics: a practical introduction, Springer Verlag (2009).
- [7] X. Zhang, Y. Xia & C.-W. Shu, Maximum-principle-satisfying and positivity-preserving high order discontinuous Galerkin schemes for conservation laws on triangular meshes, J. Sci. Comput., DOI:10.1007/s10915-011-9472-8 (2011).

Joint work with: Caterina Calgaro (INRIA Lille & Univ. Lille 1), Emmanuel Creusé (INRIA Lille & Univ. Lille 1) and Thierry Goudon (INRIA Sophia-Antipolis)

__ * * * -

S12 – Numerical Methods V – Room H, 17.50-18.20

A discontinuous Galerkin method for neutron transport equations on arbitrary grids

Wei Junxia

Institute of Applied Physics and Computational Mathematics, Department 6, NO.2. Fenghaodong Road, Haidian District, Beijing, 100094, China wei_junxia@iapcm.ac.cn

Time-dependent neutron transport equation is a kind of important hyperbolic partial differential equation in nuclear science and engineering applications. High dimension neutron transport calculation include computing of space grid, angle direction, energy group and time step, is very complex and huge scale scientific calculation problem. Discontinuous finite element discrete ordinates (DFE-Sn) method is very efficient for solution of such equations especially while concerned with complicated physics including multimedia, larger grid distortion, complex initial and boundary conditions. We have developed a serial solver with this method for neutron and photon coupled transport equation under 2-D cylindrical geometry on unstructured triangle or quadrangle grids. Undoubtedly, the traditional discontinuous Galerkin methods have many distinguished features, however, they have a number of their own weaknesses.

A discontinuous Galerkin method based on a Taylor basis is presented for the solution of neutron transport equations on arbitrary grids. Unlike the traditional discontinuous Galerkin methods, where either standard Lagrange finite element or hierarchical node-based basis functions are used to represent numerical polynomial solutions in each element, this DG method represents the numerical polynomial solutions using a Taylor series espansion at the centroid of the cell. As a result, this new formulation has a number of distinct, desirable, and attractive features and advantages in developing a DG method from a practical perspective, which can be effectively used to address some of shortcomings of the DG methods. The developed method is used to solve time-dependent neutron transport equations under 2-D cylindrical geometry on arbitrary grids. The numerical results obtained by this discontinuous Galerkin method are extremely promising and encouraging in terms of both accuracy and robustness, indicating its ability and potential to become not just a competitive but simply a superior approach than the current available numerical methods.

References

- W.H. Reed and T.R. Hill, Triangular mesh methods for the neutron transport equation, *Tech. Report LA-UR-73-479*, Los Alamos Scientific Laboratory, (1973)
- [2] F.Lianxiang and Y.Shulin, Researches on 2-D neutron transport solver NTXY2D, *Technical report*, Institute of Applied Physics and Computational Mathematics in Beijing, 10. (1999)
- [3] E.E. Lewis, W.F. Miller, Computational Methods of Neutron Transport, New York: John Wiley, Sons Publisher, (1984)
- [4] R.S.Baker, K.R.Koch, An Sn algorithm for the massively parallel CM-200 computer, Nuclear Science and Engineering, 128 (1998), pp. 312-320
- [5] Shawn D Pautz, An Algorithm for Parallel Sn Sweeps on Unstructured Meshes, Nuclear Science and Engineering, 140. (2002), pp.111-136
- [6] Mo Z, Fu L. Parallel flux sweeping algorithm for neutron transport on unstructured grid. Journal of Supercomputing, 30. (2004), pp.5-17
- [7] Yang Shulin, Mo Zeyao, Shen Longjun, The Domain Decomposition Parallel Iterative Algorithm for the 3-D Transport Issue, *Chinese J Comput Phys*, 21. (2004), pp. 1-9
- [8] T.A. Wareing, J.M. McGhee, J.E. Morel and S.D. Pautz, Discontinuous Finite Methods Sn Methods on 3-D unstructured Grids, in Proceeding of International Conference on Mathematics and Computation, Reactor Physics and Environment Analysis in Nuclear Applications, Madrid, Spain, (1999)

- [9] H. Luo, J.D. Baum, R.Lohner. A discontinuous Galerkin method using Taylor basis for compressible flows on arbitrary grids. *Journal of Computational Physics*, **227.** (2008), pp. 8875-8893
- [10] Hong Luo, Luqing Luo, and Kun Xu, A discontinuous Galerkin method based on BGK scheme for the Navier-Stokes Equations on arbitrary grids, Adv. Appl. Math. Mech., 1. (2009), pp. 301-318
- [11] H. Luo, Luqing Luo, Robert Nourgaliev, Vincent A. Mousseau, Nam Dinn, A discontinuous Galerkin method for the Navier-Stokes Equations on arbitrary grids. *Journal of Computational Physics*, 229. (2010), pp. 6961-6978
- [12] B. CoCkburn, Gg. Karniadakis, C.W. Shu, Discontinuous Galerkin Methods, Theory, Computation, and Applications. Lecture Notes in Computational Science and Engineering, Springer-Verlag, New York, 11. (2000), pp. 5-50

Joint work with: Shuanghu, Wang(Institute of Applied Physics and Computational Mathematics, NO.2. Fenghaodong Road, Haidian District, Beijing, 100094, China), Shulin, Yang(Institute of Applied Physics and Computational Mathematics, NO.2. Fenghaodong Road, Haidian District, Beijing, 100094, China), Yibing, Chen(Institute of Applied Physics and Computational Mathematics, NO.2. Fenghaodong Road, Haidian District, Beijing, 100094, China)

S12 - Numerical Methods V - Room H, 18.20-18.50

* *

Exporting numerical schemes from compressible gas dynamics to elasticity

Knut Waagan University of Washington kwaagan@uw.edu

Elastic materials are modeled by nonlinear hyperbolic conservation laws with complicated constitutive equations. Efficient numerical techniques have been developed for shocks in gases. We consider the application of such shock-capturing schemes to nonlinear elasticity. These schemes add dissipative mechanisms that lack proper invariance under rigid motions, which is shown to lead to large errors. We present a method for imposing invariant dissipative mechanisms. The result is a rich class of invariant regularizations which may yield new insight on shock waves in solids, both numerically and theoretically.

S12 – Numerical Methods V – Room H, 18.50–19.20

* * *

An enhancement to the AUFS Flux Splitting scheme by Sun and Takayama

Friedemann Kemm Brandenburg University of Technology Institute for Applied Mathematics and Scientific Computing Platz der Deutschen Einheit 1 03046 Cottbus, Germany kemm@math.tu-cottbus.de The AUFS-scheme [2] by Sun and Takayama is a flux vector splitting scheme without breakdown of discrete shock profiles, usually called carbuncle, but still with a fine resolution of entropy waves. It is based on a splitting of the flux vector of the Euler equations already discussed by Steger and Warming [1, p. 271]:

$$\mathbf{f}(\mathbf{q}) = u\,\mathbf{q} + \mathbf{P}(\mathbf{q}) = u\begin{pmatrix}\rho\\\rho u\\E\end{pmatrix} + \begin{pmatrix}0\\p\\pu\end{pmatrix}$$

with the wave speeds for the advective flux $u \mathbf{q}$ assumed to be u, u, u and for the central part, the pressure part, $\mathbf{P}(\mathbf{q})$ assumed to be $0, \pm c$ with the speed of sound c. Sun and Takayama start from the 2d-version of the resulting scheme and modify the numerical viscosity to enhance the resolution of entropy waves. Unfortunately, this does not improve the resolution of shear waves – it is even poorer – but leads to failure of the scheme due to negative pressure.

We provide possible fixes for both deficiencies by going back to the original viscosity or a Local-Lax-Wendroff type viscosity and applying a correction for the shear viscosity, based on a symmetric HLLC-type solver for the pressure flux. We show how this correction term has to be tuned to get a good resolution of shear waves but no carbuncle.

References

- Joseph L. Steger and R.F. Warming, Flux vector splitting of the inviscid gasdynamic equations with application to finite-difference methods, J. Comput. Phys., 40 (1981), pp. 263–293
- M. Sun and K. Takayama, An artificially upstream flux vector splitting scheme for the Euler equations, J. Comput. Phys., 189(1) (2003), pp. 305–329

6.4 Session 13 — Room I — Numerical Methods VI

S13 – Numerical Methods VI – Room I, 17.20–17.50

A Combined Hybridized Discontinuous Galerkin / Hybrid Mixed Method for Viscous Conservation Laws

Jochen Schütz

Institut für Geometrie und Praktische Mathematik, RWTH Aachen University schuetz@igpm.rwth-aachen.de

Keywords: Hyperbolic / Parabolic PDE, Hybridized Discontinuous Galerkin Method, Hybrid Mixed Method, Adjoint Error Control

We present a novel discretization method for steady-state and time-dependent viscous conservation laws. These equations are given as

$$w_t + \nabla \cdot (f(w) - B(w)\nabla w) = 0 \quad \forall x \in \Omega,$$
(1)

subject to appropriate initial and boundary values. One important example and the motivation for the current work are the compressible Navier-Stokes equations, but there are many other equations that fit into this rather general framework.

Considering the convective term alone, i.e., $B(w) \equiv 0$, it is, at least in the one-dimensional or the scalar case, well-known that solutions have to be sought in a discontinuous function space such as $BV(\Omega)$. This has motivated the use of solution procedures using discontinuous ansatz spaces, such as the Finite Volume [4] or the Discontinuous Galerkin methods [2].

Given that the convective term is zero, i.e., $f(w) \equiv 0$, a model problem for (1) is the heat equation, where it is known that the solution to be expected is very regular, thus motivating solution procedures that rely on underlying spaces having some continuity requirement, for example the well-known BDM-spaces [1]. The use of these spaces is to compute the viscous flux $\sigma := B(w)\nabla w$ in a space possessing a (global) divergence, albeit the approximate gradient itself is allowed to be discontinuous.

In our proposed methodology, we combine a Discontinuous Galerkin method for the convective part and a BDM-Mixed method for the diffusive part. In a first step toward the combination, both methods are hybridized. Hybridization means that the approximate solution is expressed in terms of the traces of the solution on the skeleton of the mesh. This procedure has the advantage of reducing the globally coupled degrees of freedom in an implicit method. The discretizations are now made compatible, and can in a second step be easily combined by summing them up.

In the current publications [5,7], we have extensively validated the method for the steady-state case and showed the fidelity of this approach. For the approximate variable w_h , we obtain optimal convergence rates for both a scalar equation and the compressible Navier-Stokes equations. Further testcases include drag prediction on a cylinder in comparison to results from the literature, and standard NACA0012-airfoil computations.

If σ is such that there is a one-to-one relation with ∇w , e.g., $\sigma = \varepsilon \nabla w$, our method offers the possibility of postprocessing [8]. Postprocessing enhances the order of L^2 -convergence of the approximate solution w_h toward w by 1 or even 2 via a cheap, because local, reconstruction procedure. We have elaborated on convergence orders of both σ_h and w_h in [7].

In [6], we have demonstrated that our proposed method is adjoint consistent [3]. Adjoint consistency is a key property when using adjoint error control, and is as such desirable.

In this talk, we present both steady-state and time-dependent results, and discuss various mathematical properties of the proposed Hybrid Mixed method, such as the adjoint consistency property. The extension of already existing (steady-state) code to time-dependent problems can, given the fact that we solve the steady system using an implicit Euler method, be done via dual timestepping. We will, however, also consider other possible time discretizations.

References

- F. Brezzi, J. Douglas and L.D. Marini, Two families of mixed Finite Elements for second order elliptic problems, *Numerische Mathematik*, 47 (1985), pp. 217-235
- B. Cockburn and C.-W. Shu, The Runge-Kutta Local Projection P¹-Discontinuous Galerkin Finite Element Method for Scalar Conservation Laws, RAIRO Modélisation mathématique et analyse numérique, 25 (1991), pp. 337-361
- [3] R. Hartmann, Adjoint consistency analysis of Discontinuous Galerkin Discretizations, SIAM Journal on Numerical Analysis, 45 (2007), pp. 2671-2696
- [4] R. LeVeque, Finite-Voume Methods for Hyperbolic Problems, Cambridge University Press, (2002)
- [5] J. Schütz and G. May, A Hybrid Mixed Method for the Compressible Navier-Stokes Equations, AICES Technical Report 2011/12-01 (2011), submitted to the *Journal of Computational Physics*
- [6] J. Schütz and G. May, An Adjoint Consistency Analysis for a Class of Hybrid Mixed Methods, IGPM preprint Nr. 336 (2012), submitted to the SIAM Journal on Numerical Analysis
- [7] J. Schütz, M. Woopen and G. May, A Hybridized DG/Mixed Scheme for Nonlinear Advection-Diffusion Systems, Including the Compressible Navier-Stokes Equations, AIAA Paper 2012-0729, (2012)
- [8] R. Stenberg, Postprocessing schemes for some mixed finite elements, Mathematical Modelling and Numerical Analysis, 25 (1991), pp. 151-168

Joint work with: Georg May (Aachen Institute for Advanced Studies in Computational Engineering Science, RWTH Aachen University)

S13 – Numerical Methods VI – Room I, 17.50–18.20

A numerical scheme for the pressureless gases system

Laurent Boudin UPMC - Lab. J.-L. Lions & Inria Paris-Rocquencourt - Reo project team laurent.boudin@upmc.fr

The one-dimensional pressureless gases system has been widely investigated since [1]. For instance, it can be seen as a simplified model of galaxies dynamics in astrophysics or appears in the study of cold plasmas. For T > 0, the gas density $\rho(t, x) \ge 0$ and the momentum $q(t, x) \in \mathbb{R}$ solve the following equations, in $]0, T[\times \mathbb{R},$

$$\partial_t \rho + \partial_x (\rho u) = 0, \partial_t q + \partial_x (q u) = 0,$$

which happen to respectively be conservation laws on mass and momentum. The velocity $u(t,x) \in \mathbb{R}$ which appears in the fluxes must be somehow defined as a quotient of q by ρ . However, this is not always possible, since ρ can be zero. A key contribution about the pressureless gases is the work of Bouchut and James [2]. They pointed out the importance of the OSL (one-sided Lipschitz) condition: $\partial_x u \leq 1/t$.

We here tackle the question of discretizing the pressureless gases system, as more precisely described in [4]. We first prove that the upwind scheme, standardly used for conservation laws, cannot provide the solution to the system, since u, built as q/ρ , cannot satisfy the OSL condition. We then provide a numerical scheme based on the viscous pressureless gases system [3] and preserving, among other properties, the discrete OSL condition, see [5].

References

- F. Bouchut, On zero pressure gas dynamics, in Advances in kinetic theory and computing, Ser. Adv. Math. Appl. Sci., 22 (1994), pp. 171–190
- [2] F. Bouchut and F. James, Duality solutions for pressureless gases, monotone scalar conservation laws, and uniqueness, Comm. Partial Differential Equations, 24 (1999), pp. 2173–2189
- [3] L. Boudin, A solution with bounded expansion rate to the model of viscous pressureless gases, SIAM J. Math. Anal., 32 (2000), pp. 172–193
- [4] L. Boudin and J. Mathiaud, A numerical scheme for the one-dimensional pressureless gases system, to appear in Numer. Methods Partial Differential Equations (2012), 18 pages
- [5] Y. Brenier and S. Osher, The discrete one-sided Lipschitz condition for convex scalar conservation laws, SIAM J. Numer. Anal., 25 (1988), pp. 8–23

Joint work with: Julien Mathiaud (CEA & ENS Cachan)

S13 – Numerical Methods VI – Room I, 18.20–18.50

Large time step and asymptotic preserving numerical schemes for the gas dynamics equations with source terms

Mathieu Girardin

DEN/DANS/DM2S/STMF/LMEC - CEA Saclay, bât. 454 PC 47, 91191 Gif sur Yvette Cedex, France. mathieu.girardin@cea.fr

We are interested in the simulation of subsonic compressible flows in a specific regime where the (main) driving phenomena are stiff source terms and material transport. More precisely, we consider the system of gas dynamics with external body forces and friction

$$\begin{cases} \partial_t \rho + \partial_x (\rho u) = 0, \\ \partial_t (\rho u) + \partial_x (\rho u^2 + p) = \rho (g - \alpha u), \\ \partial_t (\rho E) + \partial_x ((\rho E + p)u) = \rho u (g - \alpha u), \end{cases}$$
(1)

where ρ , u and E denote the density, the velocity and the total energy of the fluid, g the gravitational acceleration and α the friction parameter. The pressure law $p = p(\rho, e)$ is assumed to be a given function of the density ρ and the internal energy e defined by $e = E - \frac{u^2}{2}$. Such flow configuration may be encountered in several industrial processes like the flows involved within the core of a nuclear power plant. We propose a *large time step* and *asymptotic preserving* scheme for the gas dynamics equations with external forces and friction terms.

By asymptotic preserving, we mean that the numerical scheme is able to reproduce at the discrete level the parabolic-type asymptotic behaviour satisfied by the continuous equations [2]. Indeed, when one considers the asymptotic regime obtained for both long time and large friction coefficients, the solution of the system is formally expected to behave like the solution of a typical parabolic system

$$\begin{cases} \partial_{t'}\rho + \partial_x(\rho u^1) = 0, \\ \partial_x p = \rho(g - \alpha u^1), \\ \partial_{t'}(\rho e) + \partial_x((\rho e + p)u^1) = \rho u^1(g - \alpha u^1), \end{cases}$$
(2)

where $u = \epsilon u^1 + O(\epsilon^2)$. We aim at deriving a scheme that preserves this property for the discrete approximate of the solution.

By large time-step, we mean that the scheme is stable under a CFL stability condition driven by the (slow) material waves, and not by the (fast) acoustic waves as it is customary in Godunov-type schemes. We propose a mixed implicit-explicit strategy: the terms responsible for the acoustic waves receive a time implicit treatment while the ones responsible for the transport waves are treated by an explicit update. This task is achieved by means of a Lagrange-Projection algorithm as in [3]. An approximation based on a relaxation strategy [4,1,5] provides a simple mean to circumvent the nonlinearities involved with the equation of state of the fluid.

Numerical evidences are proposed and show a gain of several orders of magnitude in both accuracy and efficiency.

References

- C. Chalons and J.F. Coulombel, Relaxation approximation of the Euler equations, *Journal of Mathematical Analysis and Applications*, 348(2) (2008), pp. 872-893
- [2] C. Chalons, F. Coquel, E. Godlewski, P-A Raviart, N. Seguin, Godunov-type schemes for hyperbolic systems with parameter dependent source. The case of Euler system with friction, *Math. Models Methods Appl. Sci.*, **20(11)** (2010), pp. 2109:2166
- [3] F. Coquel, Q.L. Nguyen, M. Postel, and Q.H. Tran, Entropy-satisfying relaxation method with large time-steps for Euler IBVPs, *Math. Comp.*, **79** (2010), pp. 1493-1533
- [4] S. Jin and Z. P. Xin, The relaxation schemes for systems of conservation laws in arbitrary space dimension, Comm. Pure Appl. Math., 48 (1995), pp. 235-276

78

[5] I. Suliciu, On the thermodynamics of fluids with relaxation and phase transitions. Fluids with relaxation, Int. J. Engag. Sci., 36 (1998), pp. 921-947

Joint work with: Christophe Chalons (Université Paris Diderot-Paris 7 & Laboratoire J.-L. Lions, U.M.R. 7598 UMPC, Boîte courrier 187, 75252 Paris Cedex 05, France.)(LRC MANON, Laboratoire de Recherche Conventionné CEA/DEN/DANS/DM2S and UPMC-CNRS/LJLL.), Samuel Kokh (DEN/DANS/DM2S/STMF/LMEC – CEA Saclay, bât. 454 PC 47, 91191 Gif sur Yvette Cedex, France)

S13 – Numerical Methods VI – Room I, 18.50–19.20

Geometrical Treatment of Geometrical Shock Dynamics

Philip Lawrence Roe Aerospace Engineering, University of Michigan philroe@umich.edu

Geometrical Shock Dynamics (GSD) was introduced by Whitham in 1957 [1] to predict the behavior of a shockwave propagating into a stationary medium. It is a semi-empirical theory that enjoys a surprising degree of practical success. The derivation employs largely geometrical arguments, but can also be formulated as a 2×2 set of hyperbolic conservation laws. The output is the history of the shock front in the form $\mathbf{x}(\xi, t)$ where t is time and ξ measures distance along the shock front. A few simple solutions can be found very directly but more generally a numerical procedure is required. Even then, the simplicity of the equations and the reduction of dimensionality make the method much less costly than, for example, solving the unsteady Euler equations, particularly for propagation over large distences.

Extensions of the method to (a) propagation into moving media [3], or (b) propagation in three dimensions [2], have abandoned these simple formulations in favor of finding a function $\alpha(\mathbf{x})$ where α is the time of arrival. This reformulation leads to less accurate solutions, loses the reduction in dimensionality, lacks a clean interpretation, and fails for the case of a shock that passes the same location more than once. We will show that this reformulation was not necessary, and that very efficient algorithms are got by applying modern methods for hyperbolic conservation laws. The construction remains geometric, supplemented by solutions to Riemann problems.

For the two-dimensional case, the interaction between a shockwave and a cylindrical vortex will be considerd, and compared with Euler solutions. The three-dimensional extension is particularly interesting, as it is closely isomorphic with Lagrangian fluid dynamics, and promotes a reexamination of Godunov-type methods in that context.

References

- Whitham, G. B. A new approach to to the problem of shock dynamics, Part 1: Two-dimensional problems, J. Fluid Mech., 2, 146-171. (1957).
- [2] Whitham, G. B. A new approach to to the problem of shock dynamics, Part 2: Three-dimensional problems, J. Fluid Mech., 4, 369-386. (1959).
- [3] Whitham, G. B. A note on shock dynamics relative to a moving frame J. Fluid Mech., **31**, 449-453. (1968).

Joint work with: Prasanna Amur Varadarajan (Schlumberger International, Cambridge CB3 0EL, UK)

6.5 Session 14 — Room G — Numerical Methods for Atmospheric and Geophysical Models II

S14 – Numerical Methods for Atmospheric and Geophysical Models II – Room G, 17.20–17.50

Split-explicit time integration methods in numerical weather prediction

Oswald Knoth Institute for Tropospheric Research Permoserstrasse 15 D-04318 Leipzig, Germany knoth@tropos.de

Split-explicit integration methods are common time integrators in Eulerian based weather forecast models. In the talk we will give an overview about different types of integrators which are based on Runge-Kutta, multistep and peer integration methods. The methods are compared with respect to their order, stability regions and efficiency. Especially we propose a generalized ansatz of split-explicit integration methods which combine an explicit Runge-Kutta method for the slow wave part and an integrator of your choice for the fast part. The classical RK3 method of Wicker and Skamarock belongs to this class of methods. Order conditions up to order three are derived. For deriving special schemes an optimization problem is formulated which includes as constraints the order conditions and stability restrictions with respect to the linear shallow water equation. The optimization goal is a minimal number of fast wave integration steps. A three stage second order method is obtained, whose underlying Runge-Kutta method has the classical order three. Furthermore a four stage method has been constructed which is of order three for the full Euler equation. Numerical examples are presented for implementations in the atmospheric models ASAM and WRF.

Joint work with: Joerg Wensch (Institute of Scientific Computing, Technical University of Dresden, Zellescher Weg 12-14, D-01069 Dresden)

S14 – Numerical Methods for Atmospheric and Geophysical Models II – Room G, 17.50–18.20

* * * -

Well-balanced simulation of geophysical flows via the Shallow Water Equations with bottom topography: Consistency and Numerical Computation

Thomas Müller Department of Applied Mathematics, University of Freiburg, Germany mueller@mathematik.uni-freiburg.de

The shallow water equations are a simplified mathematical model for incompressible free-surface flows that can be used for the simulation of geophysical problems (tsunamis, avalanches etc.). Especially in large domains, where the consideration of the full model would be computationally too demanding, it is convenient to make use of such simplified models. The system of conservation/balance laws is expressed in terms of the water height hand the depth-averaged horizontal momentum $h\mathbf{u}$ and reads

$$\partial_t h + \nabla \cdot (h\mathbf{u}) = 0 \quad \text{in } \Omega \times [0, T]$$

$$\partial_t (h\mathbf{u}) + \nabla \cdot \left(h\mathbf{u} \otimes \frac{h\mathbf{u}}{h} + \frac{g}{2}h^2 \text{Id}\right) = -gh\nabla b \quad \text{in } \Omega \times [0, T],$$
(1)

where s = s(x, t) denotes the position of the free surface, b = b(x) the bottom profile, h = h(x, t) = s(x, t) - b(x)the water height, $\mathbf{u} = \mathbf{u}(x, t)$ the velocity and g the gravitational constant. Usually, the computational domain Ω is a subset of the horizontal plane, but we also consider the shallow water equations on the sphere \mathbb{S}^2 as a model for global atmospheric flows (e.g. weather forecast, propagation of tsunamis).

For the numerical computation we consider finite volume schemes of higher order. Since classical schemes do not preserve the stable lake at rest state (s = const and $\mathbf{u} \equiv 0$) for a non-flat bottom topography, the well-balancing strategy described in [2] is applied to the finite volume scheme. This strategy is based on a hydrostatic reconstruction, i.e. a modification of the arguments of the numerical flux function. One main focus of this talk is to show that under certain assumptions the order of consistency of the first order scheme (in 1D) is not affected by the modification.

In our simulations with real bottom topography, previously wet regions may drain with time or, conversely, dry regions may be filled with water. Since this can lead to instabilities/unphysical water heights, we employ, additionally to the well-balancing, a wetting and drying strategy proposed by [3]. The numerical simulations were performed in parallel.

For the shallow water equations on the rotating sphere, one has to modify the system (1) in order to take into account Coriolis force due to the rotation of the Earth and the fact, that momentum should be tangential to the sphere (see e.g. [5]). We propose a finite volume scheme that is based on a discrete parallel transport of the momentum before being put into the numerical flux function. For our simulations we use the real bottom topography of the earth [1] and a surface displacement caused by an asteroid impact [4] as initial values. The numerical code for surfaces also enables us to replace the shallow water equations with another hyperbolic equation (e.g. Euler equations of gas dynamics) and to replace the computational domain (e.g. sphere with torus).

References

- Amante, C. and Eakins, B. W., ETOPO1 1 Arc-Minute Global Relief Model: Procedures, Data Sources and Analysis, NOAA Technical Memorandum NESDIS NGDC-24 (2009).
- [2] Audusse, E., Bouchut, F., Bristeau, M.-O., Klein, R. and Benot P., A fast and stable well-balanced scheme with hydrostatic reconstruction for shallow water flows, SIAM J. Sci. Comput., 25(6) (2004), pp. 2050–2065.
- [3] Bollermann, A., Noelle, S. and Lukacova-Medvidova, M., Finite volume evolution galerkin methods for the shallow water equations with dry beds, *Commun. Comput. Phys.* 10(2) (2010), pp. 371–404.
- [4] Ward, S. N. and Asphaug, E., Asteroid Impact Tsunami: A Probalistic Hazard Assessment, *Icarus* 145(1) (2010), pp. 64–78.
- [5] Williamson, D. L., J. B. Drake, J. J. Hack, R. Jakob and Swarztrauber P. N., A standard test set for numerical approximations to the shallow water equations in spherical geometry, J. Comput. Phys., 102(1) (1992) pp.211–224.

Joint work with: Axel Pfeiffer (Department of Applied Mathematics, University of Freiburg, Germany)

S14 – Numerical Methods for Atmospheric and Geophysical Models II – Room G, 18.20–18.50

Adaptive large time step FV and DG methods for some geophysical flows

Mária Lukáčová-Medvid'ová Johannes Gutenberg-University of Mainz lukacova@uni-mainz.de A characteristic feature of many geophysical flows is their *multiscale behaviour* with wave speeds differing by orders of magnitude. If explicit time discretization is used for numerical approximation to a governing system that supports multiscale waves, the maximum stable time step will be limited by wave speed of the most rapidly propagating waves. In order to obtain a reasonably efficient numerical model for simulation of geophysical flows (e.g. atmospheric circulation), it is necessary to circumvent the stability constraint associated with acoustic waves and put the stability limit into closer agreement with the time step limitations arising from accuracy considerations. In [2] we have derived two types of the large time step finite volume evolution Galerkin (FVEG) scheme; the explicit as well as fully implicit scheme. We use here the theory of bicharacteristics yielding a multidimensional evolution operator, which can be interpreted as a multidimensional approximate numerical flux. Numerical simulations confirm the efficiency of the explicit large time step scheme, while the fully implicit approach yielding in some cases more robust scheme is quite costly.

Recently we are working in collaboration with the colleagues from meteorology, [3], on the development of new linear semi-implicit large time step FVEG schemes and Discontinuous Galerkin EG (DGEG) schemes. The fully nonlinear flux is splitted into a linear part governing the fast waves and the rest nonlinear part governing the convective waves. In order to omit the strict stability condition dictated by fast waves, the linear operator is approximated in time in the implicit way, while the nonlinear one is approximated in an explicit way. This yields a desired CFL stability condition depending just on the slow waves. Since most geophysical phenomena show a very localized behaviour, i.e. we have small regions with strong interactions in a larger surrounding area with almost steady solutions, grid adaptation is an inevitable tool. In [1] Bollermann, Lukáčová-Medvid'ová and Noelle extended the multidimensional FVEG scheme to non-uniform, adaptive grids, allowing fine resolutions in the area of interest and a minimal cell number in steady regions. The adaptive grid techniques are used in new semi-implicit DGEG methods as well.

The results that will be presented are based on the collaboration with S. Noelle, K.R. Arun (Aachen) and A. Hundertmark, L. Yelash, A. Müller (Mainz). This research has been supported by the German Research Foundation DFG under the grant LU 1470/2-2.

References

- A. Bollermann, S. Noelle, M. Lukáčová-Medvid'ová: Finite volume evolution Galerkin methods for the shallow water equations with dry beds, *Comm. Comput. Phys.* 10(2) (2011), pp. 371–404.
- [2] A. Hundertmark-Zaušková, M. Lukáčová-Medvid'ová, F. Prill: Large time step finite volume evolution Galerkin methods, J. Sci. Comp. 48 (2011), pp. 227-240.
- [3] A. Müller, J. Behrens, F.X. Giraldo, V. Wirth: Testing refinement criteria in adaptive Discontinous Galerkin simulations of dry atmospheric convection, submitted 2011

S14 – Numerical Methods for Atmospheric and Geophysical Models II – Room G, 18.50–19.20

* * *

A WENO-TVD finite volume scheme for the numerical approximation of atmospheric phenomena

Dante Kalise Dipartimento di Matematica, Sapienza Università di Roma kalise@mat.uniroma1.it

We present a 2D WENO-TVD scheme for the approximation of atmospheric phenomena; our work aims to extend the ideas originally introduced in [1], where a 1D version was developed. The scheme considers a spatial discretization via a second-order TVD flux based upon a flux-centered limiter approach, which makes use of high-order accurate extrapolated values arising from a WENO reconstruction procedure [2]. Time discretization

83

is performed with a third order RK-TVD scheme, and splitting is used for the inclusion of source terms. We present a comprehensive study of the method in atmospheric applications involving advective and convective motion. We develop a set of tests for space-dependent linear advection, where we assess convergence and robustness with respect to the parameters of the scheme. We apply the method to approximate the 2D Euler equations in a series of tests for atmospheric convection; finally, we analyze the performance of the scheme in a layered formulation of the primitive equations [3].

References

- V.A. Titarev and E.F. Toro, WENO schemes based on upwind and centred TVD fluxes, Computers & Fluids, 34 (2005), pp. 705-720
- [2] D.S. Balsara, T. Rumpf, M. Dumbser, and C.D. Munz, Efficient, High Accuracy ADER-WENO Schemes for Hydrodynamics and Divergence-Free Magnetohydrodynamics, *Journal of Computational Physics*, 228 (2008), pp. 2480-2516
- [3] D. Kalise and I. Lie, Modelling and numerical approximation of a 2.5D set of equations for mesoscale atmospheric processes, preprint (2011-024), available at http://www.math.ntnu.no/conservation/2011/024.html

Joint work with: Ivar Lie (StormGeo AS).

6.6 Session 15 — Room E — Multi Physics Models II

S15 – Multi Physics Models II – Room E, 17.20–17.50

Can one obtain numerically a non-existent solution for a viscous system of conservation laws?

Dan Marchesin IMPA, Rio de Janeiro, Brazil marchesi@impa.br

We study the Riemann problem for a mixed elliptic-hyperbolic system of two conservation laws, with quadratic polynomial flux functions. This system is viscous, *i.e.*, it contains small non degenerate spatial second derivative terms.

Numerical simulations using the non-linear Crank-Nicolson scheme with ultra-fine resolution (10^5 grid points) find a stable and persistent solution for the PDEs that involves a transitional (or undercompressive) shock [1]. The traveling wave profile of the transitional shock is a saddle-node saddle connection. However, an analysis based on Bogdanov-Takens bifurcation in the plane proves that the above mentioned viscous system of conservation laws does not have a traveling wave profile between these two states. How do we get out of this conundrum? Increasing resolution does not help.

We believe the numerical solution approximates an orbit for a system of 4 ordinary differential equations. These are the traveling wave equations for a system of two viscous conservation laws containing small third order derivative terms. Such a dispersive system approximates well the finite difference Crank-Nicolson scheme employed in the simulation. Analogous misleading numerical phenomena have already been reported elsewhere; for relevant theory see *e.g.* P. LeFloch [2].

References

 E. Isaacson and D. Marchesin and B. Plohr, Transitional Waves for Conservation Laws, SIAM J. Math. Anal., 21 (1990), pp. 837–866. [2] P. G. LeFloch, Hyperbolic Systems of Conservation Laws. The theory of classical and nonclassical shock waves, ETH Zürich, Birkhäuser, (2002)

Joint work with: Vítor Matos (Faculdade de Economia, Centro de Matemática, Universidade do Porto, Portugal), Julio Daniel Machado Silva (IMPA, Rio de Janeiro, Brazil).

S15 – Multi Physics Models II – Room E, 17.50–18.20

- * * * -

Riemann solutions without intermediate constant states for a system in thermal multiphase flow in porous media

Julio Daniel Silva Instituto Nacional de Matemática Pura e Aplicada, Rio de Janeiro, Brasil jd@impa.br

We consider a nonlinear system of conservation laws arising in petroleum engineering that models the injection of a mixture of gas and oil, in any proportion, into a porous medium filled with a similar mixture. The two mixtures may have different temperatures. An example of such class can be found in [1].

We will focus on a particularly unusual feature found in this model: the Riemann solution for the system is given by a single wave group for a full open set of Riemann data, *i.e.*, there is no constant intermediate state. The key aspect supporting the construction of these Riemann solutions is the existence of structurally stable doubly sonic shock waves, which robustly connect slow rarefaction waves to fast rarefaction waves. Albeit superficially similar to the doubly sonic transitional shock waves predicted in [2], in our case, the doubly sonic shock waves vary when we allow the Riemann data to change.

The solutions are constructed around a coincidence curve, intrinsically associated to most bifurcations in the Riemann solutions for this class of models.

References

- Bruining, Dan Marchesin, Maximal oil recovery by simultaneous condensation of alkane and steam, *Phys. Rev. E*, Volume 75 (2007), pp. 036312
- [2] Stephen Schecter, Dan Marchesin and Bradley Plohr, Structurally stable Riemann solutions, J. Differential Equations, Volume 126 (1996), pp. 303-354

* * * -

Joint work with: Dan Marchesin (Instituto Nacional de Matemática Pura e Aplicada, Rio de Janeiro, Brasil).

S15 – Multi Physics Models II – Room E, 18.20–18.50

Classification of the umbilic point for general immiscible three-phase flow in porous media

Vítor Matos Faculdade de Economia, Centro de Matemática, Universidade do Porto, Portugal vmatos@fep.up.pt

We consider the flow in a porous medium of three fluids that do not mix nor interchange mass. Under simplifying assumptions this is the case for oil, water and gas in a petroleum reservoir. For simple geometry, the horizontal displacement of a pre-existent uniform mixture by another injected mixture gives rise to a Riemann problem for a system of two conservation laws. Such a system depends on laboratory-measured relative permeability functions for each of the three fluids. For Corey models each permeability depends solely on the saturation of the respective fluid [1], giving rise to systems containing an umbilic point in the interior of the physical domain, *i.e.*, the saturation triangle. It has been conjectured that the structure of the Riemann solution in the triangle is strongly influenced by the nature of the umbilic point, which is determined by the quadratic expansion of the flux function nearby. In [2] Schaeffer, Shearer, Marchesin and Paes-Leme proved, for very general Corey permeabilities, that umbilic points lie in Cases I or II of Schaeffer&Shearer's classification.

In the current work we find precisely the boundaries where the transition occurs in permeability parameter space, which was not done in [2]. The novel tool is a constructive method for determining the boundary between Case I and Case II for systems of two conservation laws. In the talk, we present this novel method and we apply it to a family of Corey models.

References

- A. Corey, C. Rathjens, J. Henderson, and M. Wyllie, Three-phase relative permeability, *Trans. AIME*, 207 (1956), pp. 349-351.
- [2] D. Schaeffer and M. Shearer, with appendix of D. Marchesin and P. Paes-Leme, The classification of 2×2 systems of non-strictly hyperbolic conservation laws, with application to oil recovery, *Comm. Pure Appl. Math.*, 40 (1987), pp. 141–178

Joint work with: Pablo Castañeda (IMPA - Instituto Nacional de Matemática Pura e Aplicada, Rio de Janeiro, Brazil), Dan Marchesin (IMPA - Instituto Nacional de Matemática Pura e Aplicada, Rio de Janeiro, Brazil).

* * * -----

S15 – Multi Physics Models II – Room E, 18.50–19.20

A Discontinuous Galerkin Scheme for compressible phase field models

Mirko Kränkel Department of Applied Mathematics, University of Freiburg kraenkel@mathematik.uni-freiburg.de

The Talk will present a numerical scheme for compressible Navier-Stokes equations coupled with an Allen-Cahn Type equation. These equations are obtained from a thermodynamic consistent phase field model for two-phase flow given by a work of Gabriele Witterstein [1]. The model allows for the combination of physically different phases and control over the width of the transition layer between the faces. One application for this model

is the simulation of liquid-vapor flow, which is the main goal of our research. The governing equations are solved by a Local Discontinuous Galerkin scheme in combination with implicit and explicit Runge Kutta time stepping schemes of higher order. We will discuss in particular the treatment of the additional second order term modeling the surface tension in the momentum balance equation. Mesh adaptation and the choice of the physical parameters are addressed as well and numerical examples are presented.

References

Witterstein G., Sharp interface limit of phase change flows., Adv. Math. Sci. Appl., 20 (2010), pp. 584-629

Joint work with: Dietmar, Kröner (Department of Applied Mathematics, University of Freiburg)

6.7 Session 16 — Room A — Theory of Conservation Laws II

S16 - Theory of Conservation Laws II - Room A, 17.20-17.50

Capillarity approximation of conservation laws with discontinuous fluxes

Lorenzo di Ruvo Department of Mathematics University of Bari via E. Orabona 4, 70125 Bari, Italy diruvo@dm.uniba.it

We have studied the dynamics of flows in porous media. It is well known that small scale effects are neglected in modeling two phase flow in a porous medium. Such models including (static) capillary pressure have been studied in the context of flows in homogeneous medium. However, in reality, the porous medium is heterogeneous with possible discontinuities in the rock type. We have considered the dynamic capillary pressure. The addition of this effects results in a mixed hyberbolic-parabolic equation with a possibly discontinous dispersion term. In the other words, let $\nu > 0$. The dynamics is described by the equation

	$\partial_t u_n u + \partial_x f(k_\nu, u_\nu) = \nu \partial_x (g(l_\nu, u_\nu) \partial_x u_\nu) + \nu^2 \partial_x (h(m_\nu, u_\nu) \partial_{tx} u_\nu),$	$t>0, x\in\mathbb{R},$
	$\partial_t k_\nu = \nu \partial_{xx} k_\nu,$	$t>0, x\in \mathbb{R},$
	$\partial_t l_{\nu} = \nu \partial_{xx} l_{\nu},$	$t>0, x\in \mathbb{R},$
J	$\partial_t m_\nu = \nu \partial_{xx} m_\nu,$	$t>0, x\in \mathbb{R},$
	$u_{\nu}(0,x) = u_{0,\nu}(x),$	$x \in \mathbb{R},$
	$k_{\nu}(0,x) = k_{0,\nu}(x),$	$x \in \mathbb{R},$
	$l_{\nu}(0,x) = l_{0,\nu}(x),$	$x \in \mathbb{R},$
	$\sum m_{\nu}(0,x) = m_{0,\nu}(x),$	$x \in \mathbb{R},$

where we assume $f, g, h : \mathbb{R}^2 \to \mathbb{R}$ are smooth functions such that $g(\cdot, \cdot), h(\cdot, \cdot) \ge \alpha$, for some constant $\alpha > 0$. In addition, $f(k, \cdot)$ is assumed to be genuinely nonlinear for every $k \in \mathbb{R}$, namely the map $u \in [0, 1] \mapsto f(k, u)$ is not affine on any nontrivial interval for every $k \in \mathbb{R}$. On the functions $k_{\nu}, l, m, u_{0,\nu} : \mathbb{R} \to \mathbb{R}$ we assume that

$$k_{0,\nu}, l_{0,\nu}, m_{0,\nu} \in C^{\infty}(\mathbb{R}) \cap W^{1,1}(\mathbb{R}), \quad u_{0,\nu} \in C^{\infty}(\mathbb{R}) \cap L^{1}(\mathbb{R}) \cap L^{\infty}(\mathbb{R}), \quad 0 \le u_{0,\nu} \le 1.$$

We have that there exist u_0, k, l, m such that

$$u_{\nu,0} \to u_0, \quad k_{\nu} \to k, \quad l_{\nu} \to l, \quad m_{\nu} \to m \quad \nu \to 0.$$

Using the compansated compactness, we prove that there exist u_{ν_n} , subsequence of u_{ν} , such that $u_{\nu_n} \to u$, where u is a distributional solution of

$$\begin{cases} \partial tu + \partial_x f(k(x), u) = 0, & t > 0, x \in \mathbb{R}, \\ u(0, x) = u_0(x), & x \in \mathbb{R}. \end{cases}$$

Joint work with: G. M. Coclite (Department of Mathematics University of Bari via E. Orabona 4, 70125 Bari, Italy), S. Mishra (Seminar for Applied Mathematics (SAM) ETH Zürich, HG G 57.2, Rämistrasse 101, 8092 Zürich, Switzerland.).

S16 - Theory of Conservation Laws II - Room A, 17.50-18.20

* * * -

Conservation laws with filtered variables

 $\begin{array}{c} {\rm Franziska} \ {\rm Weber} \\ {\it ETH} \ Z\ddot{u}rich \\ {\rm frweber@student.ethz.ch} \end{array}$

The properties of the solution to the convectively filtered Burgers' equation, a regularization of Burgers' equation with the convective velocity replaced by a nonlocal averaged velocity, are examined. It is found that the limit of solutions, as the regularizing parameter α goes to zero, does not satisfy an entropy inequality owing to the reversibility of the equation and the absence of an L^1 -contraction estimate for the limit of solutions.

In an attempt to overcome the reversibility of the equation, a model with a filter depending on time is considered. The limit of solutions turns out to be a weak solution of Burgers' equation but not the entropy solution either.

Then, two possible modifications of a general scalar conservation law using filtered quantities are considered and found to be usuitable as regularizations of the conservation law since the limit function, obtained when letting the averaging parameter α go to zero, is not necessarily a weak solution of the conservation law.

References

- H. S. Bhat and R. C. Fetecau, A Hamiltonian Regularization of the Burgers Equation, Journal of Nonlinear Sciences, 16 (2006), pp. 615–638
- [2] H. S. Bhat and R. C. Fetecau, The Riemann problem for the Leray-Burgers equation, Journal of Differential Equations, 246 (2009), pp. 3957–3979
- [3] G. J. Norgard and K. Mohseni, On the Convergence of the Convectively Filtered Burgers Equation to the Entropy Solution of the Inviscid Burgers Equation, *Multiscale Modeling & Simulation*, 7 (2010), pp. 1811–1837

Joint work with: Nils Henrik Risebro (University of Oslo)

* * * -

S16 - Theory of Conservation Laws II - Room A, 18.20-18.50

On the decay property for periodic entropy solutions to scalar conservation laws

Evgeniy Yu. Panov Novgorod State University Eugeny.Panov@novsu.ru

In the half-space $\Pi = \mathbb{R}_+ \times \mathbb{R}^n$, we consider a first order multidimensional conservation law

$$u_t + \operatorname{div}_x \varphi(u) = 0, \tag{1}$$

where $\varphi(u) = (\varphi_1(u), \dots, \varphi_n(u)) \in C(\mathbb{R}, \mathbb{R}^n)$. Recall that a function $u = u(t, x) \in L^{\infty}(\Pi)$ is called an entropy solution (e.s. for short) of (1) in the sense of S.N. Kruzhkov [2] if for all $k \in \mathbb{R}$

$$|u-k|_t + \operatorname{div}_x[\operatorname{sign}(u-k)(\varphi(u) - \varphi(k))] \le 0 \text{ in } \mathcal{D}'(\Pi).$$

We assume that the requirement of space-periodicity holds: $u(t, x + e_i) = u(t, x)$ for almost all $(t, x) \in \Pi$ and all i = 1, ..., n, where $\{e_i\}_{i=1}^n$ is a fixed basis of periods in \mathbb{R}^n . Without loss of generality, we may suppose that $\{e_i\}_{i=1}^n$ is the canonical basis. We denote by $P = [0, 1)^n$ the corresponding fundamental parallelepiped (cube).

We will say that equation (1) satisfies the decay property if for every periodic e.s. u(t, x)

$$\mathop{\mathrm{ess\,lim}}_{t\to\infty} u(t,\cdot) = \mathop{\mathrm{const}} = \int_P u(0,x) dx \quad \text{in } L^1(P),$$

where u(0, x) is the trace of u(t, x) on the initial hyperspace t = 0. In the present talk we extend the known result by G.-Q. Chen and H. Frid [1]. Namely, we establish that the condition

 $\forall \xi \in \mathbb{Z}^n, \xi \neq 0$, the function $\varphi(u) \cdot \xi$ is not affine on non-empty intervals

is necessary and sufficient for the decay property. The proof of this result is based on localization principles for the H-measure corresponding to the sequence u(kt, kx), $k \in \mathbb{N}$, and contained in preprint [3].

This research was carried out with the financial support of the Russian Foundation for Basic Research (grant no. 09-01-00490-a).

References

- G.-Q. Chen and H. Frid, Decay of entropy solutions of nonlinear conservation laws, Arch. Rational Mech. Anal. 146 2 (1999), pp. 95–127.
- [2] S. N. Kruzhkov, First order quasilinear equations in several independent variables, Math. USSR-Sb. 10 (1970), pp. 217–243.
- [3] E. Yu. Panov, On decay of periodic entropy solutions to a scalar conservation law, http://www.math.ntnu.no/conservation/2011/026.html

* * *

S16 - Theory of Conservation Laws II - Room A, 18.50-19.20

Singular solutions of a fully nonlinear 2x2 system of conservation laws

Darko Mitrovic University of Montenegro matematika@t-com.me

Existence and admissibility of δ -shock solutions is discussed for the non-convex strictly hyperbolic system of equations

$$\partial_t u + \partial_x \left(\frac{u^2 + v^2}{2} \right) = 0$$

$$\partial_t v + \partial_x (v(u-1)) = 0.$$

This fully nonlinear system (i.e. nonlinear with respect to both unknowns) is considered in [1] where it is noticed that it does not admit the classical Lax-admissible solution for certain Riemann problems. By introducing complex-valued corrections in the framework of the weak asymptotic method, we show that a compressive δ -shock solution resolves such Riemann problems. By letting the approximation parameter tend to zero, the corrections become real-valued and the solutions can be seen to fit into the framework of weak singular solutions defined in [2]. Moreover, in this context, we can show that every 2×2 system of conservation laws admits δ -shock solutions.

References

- B. Hayes and P. G. LeFloch, Measure-solutions to a strictly hyperbolic system of conservation laws, Nonlinearity 9 (1996), 1547–1563.
- [2] V. G. Danilov and V. M. Shelkovich, Dynamics of propagation and interaction of δ-shock waves in conservation law system, J. Differential Equations 211 (2005), 333–381.

Joint work with: Henrik Kalisch (University of Bergen).

6.8 Session 17 — Room B — Balance Laws in Relativity

S17 – BALANCE LAWS IN RELATIVITY – ROOM B, 17.20–17.50

High Order Methods with Adaptive Mesh Refinement for the Solution of the Relativistic MHD Equations

Olindo Zanotti University of Trento olindo.zanotti@ing.unitn.it

We present a high order scheme with adaptive mesh refinement for the numerical solution of the relativistic magnetohydrodynamics equations in multiple space dimensions. The nonlinear system under consideration is purely hyperbolic and contains a source term, that for the evolution of the electric field, that can become stiff for low values of the resistivity. For the spatial discretization a high order $P_N P_M$ scheme is used [1], which combines in a single framework both finite volume methods and Galerkin methods. In addition, a high order accurate unsplit time discretization is achieved using an element-local space-time discontinuous Galerkin approach [2]. The divergence free character of the magnetic field is accounted for through the divergence cleaning procedure. The proposed method can handle equally well the resistive regime and the stiff limit of ideal relativistic MHD [3]. For these reasons it provides a powerful tool for relativistic plasma physics simulations involving the appearance of magnetic reconnection [4]. Several tests and physical applications are discussed and presented.

References

- Dumbser M., Balsara D., Eleuterio T., Munz C., A unified framework for the construction of one-step finite volume and discontinuous Galerkin schemes on unstructured meshes, *Journal of Computational Physics*, Volume no. 227 (2008), pp. 8209-8253
- [2] Dumbser M., Enaux C., Toro E., Finite volume schemes of very high order of accuracy for stiff hyperbolic balance laws, *Journal of Computational Physics*, Volume no. 227 (2008), pp. 3971-4001
- [3] Dumbser M., Zanotti O., Very high order PNPM schemes on unstructured meshes for the resistive relativistic MHD equations, *Journal of Computational Physics*, Volume no. 228 (2009), pp. 6991-7006
- [4] Zanotti O., Dumbser M., Numerical simulations of high Lundquist number relativistic magnetic reconnection, Monthly Notices of the Royal Astronomical Society, Volume no. 418 (2011), pp. 1004-1011

Joint work with: Prof. Michael Dumbser (University of Trento)

S17 – BALANCE LAWS IN RELATIVITY – ROOM B, 17.50–18.20

* * *

A front tracking analysis for the ultra relativistic Euler equations

Mahmoud Abdelrahman Otto-von-Guericke-university-Magdeburg-Germany mahmoud.abdelrahman@st.ovgu.de

In this paper we study the relativistic Euler equations in isentropic fluids with the equation of state $p = \frac{p}{3}$, which is the ultra-relativistic limit. We first analyze the single shocks and rarefaction curves. Then the Riemann problem is solved constructively. We derive sharp estimates for the strength of the waves in the Riemann solution and prove uniqueness for the Riemann problem. We study explicit examples for the non-backward uniqueness for the ultra relativistic Euler equations. We also present a new Riemann solution and a wave tracking algorithm for the relativistic Euler equations.

References

- N.H. Risebro, A. Tveito, A front tracking method for conservation laws in one dimension, *J.Comput.Phys*, 117. (1993), pp. 1125-1139,
- [2] M. Kunik, S. Qamar and G. Warnecke, Kinetic schemes for the ultra relativistic Euler equations, *J. Comput. Phys*, 187. (2003), pp. 572-596,
- [3] J. Smoller, Shock Waves and Reaction-Diffusion Equations, Springer-Verlag New York Heidelberg Berlin, (1994)
- [4] Helge Holden, Nils Henrik Risebro, Front Tracking for Conservation Laws, Springer-Verlag New York Heidelberg Berlin, (2000)
- [5] M. Kunik, Selected Initial and Boundary Value Problems for Hyperbolic Systems and Kinetic Equations. Habilitation thesis, Otto-von-GuerickeUniversity (2005).

Joint work with: Matthias Kunik (Otto-von-Guericke-university-Magdeburg-Germany), Gerald Warnecke (Otto-von-Guericke-university-Magdeburg-Germany).

* * * ------

S17 – BALANCE LAWS IN RELATIVITY – ROOM B, 18.20–18.50

Existence and stability of relativistic plasma-vacuum interfaces

Yuri Trakhinin Sobolev Institute of Mathematics, Novosibirsk, Russia trakhinin@mail.ru

The equations of relativistic magnetohydrodynamics in the Minkowski spacetime (t, x) are written as a system of conservation laws and then as the symmetric hyperbolic system

$$A_0(U)\partial_t U + A_1(U)\partial_1 U + A_2(U)\partial_2 U + A_3(U)\partial_3 U = 0$$

$$\tag{1}$$

for the vector U = (p, u, H, S) of "primitive" variables, where p is the pressure, $u = v\Gamma$, $\Gamma = (1 - |v|^2)^{-1/2}$, v is the 3-velocity, H is the magnetic field 3-vector, and S is the entropy. A concrete form of symmetric matrices A_{α} was recently found in [1]. The vacuum Maxwell equations $\partial_t \mathcal{H} + \nabla \times E = 0$, $\partial_t E - \nabla \times \mathcal{H} = 0$ for the electromagnetic field $V = (E, \mathcal{H})$ also form a symmetric system in the form of (1) with $A_0 = I$ and constant matrices A_j . Moreover, we have the divergence constraints div H = 0, div E = 0 and div $\mathcal{H} = 0$ on the initial data $(U, V)|_{t=0} = (U_0, V_0)$.

Let $\Omega^{\pm}(t) = \{x^1 \ge \varphi(t, x^2, x^3)\}$ be the domains occupied by the plasma and the vacuum respectively. Then, on the interface $\Sigma(t) = \{x^1 = \varphi(t, x^2, x^3)\}$ we have the conditions

$$\partial_t \varphi = v_N, \quad q = (|\mathcal{H}|^2 - |E|^2)/2, E_2 = \mathcal{H}_3 \partial_t \varphi - E_1 \partial_2 \varphi, \quad E_3 = -\mathcal{H}_2 \partial_t \varphi - E_1 \partial_3 \varphi,$$
(2)

where $q = p + |H|^2/(2\Gamma^2) + (v, H)^2$ is the total pressure and v_N is the normal component of the velocity. Moreover, the conditions $H_N|_{\Sigma} = 0$ and $\mathcal{H}_N|_{\Sigma} = 0$ are restrictions on the initial data, and we assume that the plasma density $\rho|_{\Sigma} > 0$.

Our final goal is to find conditions on the initial data (U_0, V_0, φ_0) providing the local-in-time existence and uniqueness of a smooth solution (U, V, φ) of the free boundary problem for system (1) in $\Omega^+(t)$ and the Maxwell equations in $\Omega^-(t)$ with the boundary conditions (2) on $\Sigma(t)$. Following [3, 4], we use the reduction to a fixed domain, the passage to Alinhac's "good unknown" (see [3]), and a suitable Nash-Moser-type iteration scheme. Since the interface is a characteristic surface, as in [3], the functional setting is provided by the anisotropic weighted Sobolev spaces H^m_* . The crucial point is finding a sufficient stability condition for a planar plasmavacuum interface. This condition was found in [6] by a so-called *secondary symmetrization* of the vacuum Maxwell equations.

Note that the secondary symmetrization of the vacuum Maxwell equations was recently used in [2] to prove the existence of solutions for the *nonrelativistic* version of the linearized plasma-vacuum interface problem [5] by a hyperbolic regularization of the elliptic div-curl system for the vacuum magnetic field.

This work was supported by RFBR (Russian Foundation for Basic Research) grant No. 10-01-00320-a.

References

[1] Freistühler H. and Trakhinin Y., Symmetrizations of RMHD equations and stability of relativistic currentvortex sheets, preprint (2012),

http://arxiv.org/pdf/1202.1946v1.pdf

- Secchi P. and Trakhinin Y., Well-posedness of the linearized plasma-vacuum interface problem, preprint (2011), http://arxiv.org/pdf/1112.3101v2.pdf
- [3] Trakhinin Y., The existence of current-vortex sheets in ideal compressible magnetohydrodynamics, Arch. Ration. Mech. Anal., 191 (2009), pp. 245-310
- [4] Trakhinin Y., Local existence for the free boundary problem for nonrelativistic and relativistic compressible Euler equations with a vacuum boundary condition, *Comm. Pure Appl. Math.*, **62** (2009), pp. 1551-1594
- [5] Trakhinin Y., On the well-posedness of a linearized plasma-vacuum interface problem in ideal compressible MHD, J. Differential Equations, 249 (2010), pp. 2577-2599
- [6] Trakhinin Y., Stability of relativistic plasma-vacuum interfaces, preprint (2011), http://arxiv.org/pdf/1006.1089v3.pdf

S17 – BALANCE LAWS IN RELATIVITY – ROOM B, 18.50–19.20

* * *

Relativistic Burgers equations on a curved spacetime

Baver Okutmustur Middle East Technical University (METU) baver@metu.edu.tr

Within the class of nonlinear hyperbolic balance laws posed on a curved spacetime (endowed with a volume form), we identify a hyperbolic balance law that enjoys the same Lorentz invariance property as the one satisfied by the Euler equations of relativistic compressible fluids. This model is unique up to normalization and converges to the standard inviscid Burgers equation in the limit of infinite light speed. Furthermore, from the Euler system of relativistic compressible flows on a curved background, we derive, both, the standard inviscid Burgers equation and our relativistic generalizations. The proposed models are referred to as relativistic Burgers equations on curved spacetimes and provide us with simple models on which numerical methods can be developed and analyzed. Next, we introduce a finite volume scheme for the approximation of discontinuous solutions to these relativistic Burgers equations. Our scheme is formulated geometrically and is consistent with the natural divergence form of the balance laws under consideration. It applies to weak solutions containing shock waves and, most importantly, is well-balanced in the sense that it preserves static equilibrium solutions. Numerical experiments are presented which demonstrate the convergence of the proposed finite volume scheme and its relevance for computing entropy solutions on a curved background.

This presentation is based on the joint paper [2].

References

- P. Amorim, P.G. LeFloch, and B. Okutmustur, Finite volume schemes on Lorentzian manifolds, Comm. Math. Sc., 6. (2008), pp. 1059–1086.
- [2] P.G. LeFloch, H. Makhlof, and B. Okutmustur, Relativistic Burgers equations on a curved spacetime. Derivation and finite volume approximation, in preparation.
- [3] P.G. LeFloch and B. Okutmustur, Hyperbolic conservation laws on spacetimes. A finite volume scheme based on differential forms, *Far East J. Math. Sci.* **31.** (2008), pp. 49–83.

Joint work with: Philippe LeFloch and Hasan Makhlof (Université Pierre et Marie Curie)

6.9 Session 18 — Room C — Control Problems for Hyperbolic Equations II

S18 - Control Problems for Hyperbolic Equations II - Room C, 17.20-17.50

Optimal control of cell mass and maturity in a model of follicular ovulation

Peipei Shang Inria Paris-Rocquencourt Laboratoire Jacques-Louis Lions peipeishang@hotmail.com

In this work, we study optimal control problems associated with a scalar hyperbolic conservation law modeling the development of ovarian follicles. Changes in the age and maturity of follicular cells are described by a 2D conservation law, where the control terms act on the velocities. The control problem consists in optimizing the follicular cell resources so that the follicular maturity reaches a maximal value in fixed time. Using an approximation method, we prove necessary optimality conditions in the form of Pontryagin Maximum Principle. Then we derive the optimal strategy and show that there exists at least one optimal bang-bang control with one single switching time.

References

- N. Echenim, D. Monniaux, M. Sorine, and F. Clément, Multi-scale modeling of the follicle selection process in the ovary, *Math. Biosci.*, **198** (2005), pp. 57-79
- [2] J.-M. Coron, Control and nonlinearity, Mathematical Surveys and Monographs, American Mathematical Society, Providence, RI, (2007)
- [3] J.-M. Coron, M. Kawski, and Z. Wang, Analysis of a conservation law modeling a highly re-entrant manufacturing system, *Discrete Contin. Dyn. Syst.-B*, 14 (4) (2010), pp. 1337-1359
- [4] F. Clément. Optimal control of the cell dynamics in the granulosa of ovulatory follicles, Math. Biosci., 152 (1998), pp. 123-142
- [5] P. Shang and Z. Wang, Analysis and control of a scalar conservation law modeling a highly re-entrant manufacturing system, J. Differential Equations., 250 (2011), pp. 949-982
- [6] E. B. Lee and L. Markus, Foundations of Optimal Control Theory, Wiley, (1986)
- [7] A. I. Smirnov, Necessary Optimality Conditions for a Class of Optimal Control Problems with Discontinuous Integrand, Proceedings of the Steklov Institute of Mathematics, 262 (2008), pp. 213-230
- [8] P. Shang, Cauchy problem for multiscale conservation laws: Application to stuctured cell populations, submitted.

Joint work with: Frédérique Clément (Inria Paris-Rocquencourt), Jean-Michel Coron (Laboratoire Jacques-Louis Lions)

* *

S18 - Control Problems for Hyperbolic Equations II - Room C, 17.50-18.20

Optimal Boundary Control for Nonlinear Hyperbolic Balance Laws

Sebastian Pfaff $TU \ Darmstadt$ pfaff@mathematik.tu-darmstadt.de

Consider the optimal control problem

 $\min J(y, u)$ subject to $u \in U_{ad}$, y = y(u)

where the control u consists of an initial datum u_0 , a boundary datum u_B and an additional control u_1 of the source term. The state y is the unique entropy solution of a nonlinear balance law on $[0, \infty)$:

$$y_t + f(y)_x = g(t, x, y(t, x), u_1(t, x))$$
(1)

$$y(0,\cdot) = u_0 \tag{2}$$

$$y(\cdot,0) = u_B. \tag{3}$$

The boundary condition (3) is understood in the sense of Bardos, LeRoux and Nédélec.

In order to apply fast optimization methods we need the differentiability of the reduced objective functional $u \mapsto \hat{J}(u) := J(u, y(u))$. As it is well known the mapping $u \mapsto y(u)$ is not differentiable in the usual sense. We show that the state depends shift-differentiable on the control by extending previous results of [1] for the control of Cauchy problems which in turn implies the Fréchet-differentiability for tracking-type functionals \hat{J} . Furthermore we present an adjoint-based gradient representation for this class of cost functionals. The adjoint equation is a linear transport equation with discontinuous coefficients on a bounded domain which requires a proper extension of the notion of a reversible solution.

References

 Stefan Ulbrich, A Sensitivity and Adjoint Calculus for Discontinuous Solutions of Hyperbolic Conservation Laws with Source Terms, SIAM Journal on Control and Optimization, Volume no. 41 (2002), pp. 740-797

Joint work with: Stefan Ulbrich (TU Darmstadt)

S18 - Control Problems for Hyperbolic Equations II - Room C, 18.20-18.50

On the attainable set for Temple class systems with characteristic boundary

Fabio S. Priuli University of Padua, Italy priuli@math.unipd.it

95

We study the exact controllability problem for entropy weak solutions to strictly hyperbolic, genuinely nonlinear, Temple class systems of balance laws

$$u_t + f(u)_x = g(t) \qquad 0 \le x \le 1, \qquad u \in \mathbb{R}^N, \tag{1}$$

on a bounded interval [0, 1], with possibly characteristic boundaries. Namely, in the same spirit of [3], we consider the mixed initial-boundary value problem for (1), where the initial data $u(0, x) = \overline{u}(x)$ is fixed, and we regard both the source term g and the boundary data $\alpha^0(t), \alpha^1(t)$ at x = 0, x = 1, as control functions. We show that, for every given profile $\Phi \in BV(0, 1)$, whose components Φ_i in Riemann coordinates satisfy $D\Phi_i \leq C$ (in the sense of measures) for some C > 0, one can choose a source term g and boundary controls α^0, α^1 , so that the corresponding solution to (1) attains the value Φ , at a sufficiently large time time T > 0. This result in particular extends exact boundary controllability properties previously obtained (with no source term acting as a control) in [1] for Temple class systems with non characteristic boundary, and in [2] for quasilinear hyperbolic systems with characteristic boundary when the initial and terminal data \overline{u}, Φ are smooth and sufficiently close in C^1 -norm to an equilibrium state.

References

- F. Ancona and G.M. Coclite, On the attainable set for Temple class systems with boundary controls, SIAM J. Control Optim. 43 (6) (2005), 2166–2190.
- [2] J.-M. Coron, O. Glass and Z.Q. Wang, Exact boundary controllability for 1-D quasilinear hyperbolic systems with a vanishing characteristic speed, SIAM J. Control Optim. 48 (5) (2010), 3105–3122.
- [3] V. Perrollaz, Exact controllability of scalar conservation laws with an additional control and in the context of entropy solutions, preprint.

Joint work with: Fabio Ancona (University of Padua, Italy), Khai T. Nguyen (University of Padua, Italy)

S18 - Control Problems for Hyperbolic Equations II - Room C, 18.50-19.20

- * * * -

Global Small Solutions of the 3D Kerr-Debye Model

Mohamed Kanso Univ. Bordeaux, IMB Mohamed.Kanso@math.u-bordeaux1.fr

In this talk, we deal with the following Kerr-Debye system modelling the electromagnetic wave propagation in a nonlinear medium exhibiting a finite response time (see [6] for details):

$$\begin{cases} \partial_t D - \operatorname{curl} H = 0, \\ \partial_t H + \operatorname{curl} E = 0, \\ \partial_t \chi + \frac{1}{\tau} \chi = \frac{1}{\tau} |E|^2, \end{cases}$$
(1)

with the constitutive relation

$$D = (1 + \chi)E.$$

This system is quasilinear hyperbolic, so it may develop singularities in finite time. Nevertheless the global existence results for a one dimensional analogous model (see [1]) and the numerical simulations let us think that the smooth solutions of (1) are global in time. In addition, this system is partially dissipative but the results in [2] and [5] cannot be applied here because the crucial Shizuta-Kawashima stability condition is not true in our case. By combining dispersive properties of Maxwell equations (see [4]) and the partial dissipative character of (1), we establish the global existence with small data of smooth solutions for the Kerr-Debye system. **References**

- [1] G. Carbou, B. Hanouzet and R. Natalini, Semilinear behavior for totally linearly degenerate hyperbolic systems with relaxation. J. Differential Equations, 246 (2009), pp. 291-319.
- [2] B. Hanouzet and R. Natalini, Global existence of smooth solutions for partially dissipative hyperbolic systems with a convex entropy. Arch. Ration. Mech. Anal., 169 (2003), pp. 89-117.
- [3] M. Kanso, Global existence of small solutions to the KerrDebye model for the three-dimensional Cauchy problem, *Port. Math.*, 68 (2011), pp. 389-409
- [4] R. Racke, Lectures on nonlinear evolution equations, Initial Value Problems, Vieweg, Wiesbaden, (1992)
- [5] W. A. Yong, Entropy and global existence for hyperbolic balance laws. Arch. Ration. Mech. Anal., 172 (2004), pp. 247-266.
- [6] R. W. Ziolkowski, The incorporation of microscopic material models into FDTD approach for ultrafast optical pulses simulations, *IEEE Trans. Antenn. Propag.*, 45 (1997), pp. 375-391

7 Abstracts of contributed lectures — Tuesday 15.15–16.15

7.1 Session 19 — Room E — Numerical Methods VII

S19 – Numerical Methods VII – Room E, 15.15–15.45

Amelioration of Shock-capturing Anomalies

Philip Lawrence Roe Department of Aerospace Engineering University of Michigan, Ann Arbor, USA philroe@umich.edu

When a capturing strategy is applied to the computational treatment of shockwaves, it is well-known that a variety of anomalies result. These include the shedding of spurious waves from slowly-moving shocks, instability of certain stationary solutions, the "wall-heating" and "carbuncle" phenomena. We contend that all of these stem, at least in part, from an ambiguity in position.

Consider a stationary shock captured by the Godunov scheme. It should be possible to deduce the location of it from the integrals of the conserved variables, but it can be shown that these will conflict because the intermediate state lies on the Hugoniot locus. There is ambiguity unless this locus lies along a straight line in state space, and most systems of practical interest do not have this property. This ambiguity can be used to explain the anomalies.

However, we have devised a shock-capturing strategy that forces the shock profile, in the limiting case of a stationary shock, to follow a straight path. It involves smoothing the values of the fluxes within the cells according to a procedure having the following properties.

- 1. The procedure has no effect if the system being solved is linear, that is, if the flux Jacobian is constant.
- 2. For nonlinear systems, the effect is $\mathcal{O}(h^2)$ in smooth regions.
- 3. For nonlinear systems, the effect is $\mathcal{O}(1)$ at shocks.
- 4. For isolated stationary shocks, the flux becomes constant in all cells

The procedure has two consequences that make physical sense. In the exact solution the flux varies only slightly across a slowly-moving shock, but by contrast varies strongly across a slowly-moving captured shock; the variation is now slight in both cases. Intermediate states are no longer in local thermodynamic equilibrium; that is true also of shocks in the physical world.

By construction this procedure eliminates any ambiguity in position for stationary shocks, and leads to a complete family of stable shock equilibria. It almost completely cures the problems with slowly-moving shocks, and reduces the wall-heating problem by at least 70%. There is virtually no loss in shock resolution.

Joint work with: Daniel Wei-Ming Zaide (University of Michigan)

S19 – Numerical Methods VII – Room E, 15.45–16.15

Asymptotic-preserving schemes for unusual long-time asymptotics

Rodolphe Turpault Université de Nantes, Laboratoire de Mathématiques Jean Leray, 2, rue de la Houssinière, 44322 Nantes (France) rodolphe.turpault@univ-nantes.fr

We consider the long-time behaviour of the following hyperbolic systems supplemented by stiff source terms:

$$\partial_t U + \partial_x F(U) = -\sigma(U)R(U),$$

where $U \in \mathbb{R}^N$ and F, $\sigma > 0$ and R are supposed to satisfy the asumptions stated in [1]. In particular, we assume the existence of a constant matrix Q such that QR(U) = 0 in order to ensure the existence of a limit regime for $u = QU \in \mathbb{R}^n$ of the form (see for instance [2,1]):

$$\partial_t u - \partial_x \big(G(u, \partial_x u) \big) = 0. \tag{1}$$

Here, G is a nonlinear function.

In many regimes of physical interest, the equation (1) ends up to be an eventually nonlinear scalar diffusion equation i.e. $G(u, \partial_x u) = h(u)\partial_x u$ for some suitable function h. In this talk however, we will focus on more challenging asymptotic regimes:

- either n > 1 and (1) cannot be written into a system of decoupled equations.
- or $\partial_x G$ is a p-Laplace-type operator.

We will give examples issuing from applications of models involving such limits, for instance the coupling between the M1-model for radiative transfer and Euler equations on the one hand and shallow-water equations with friction on the other hand.

The main purpose of this work concerns the numerical approximation of such systems through long-time asymptotic preserving schemes. During the last decade, several schemes which can deal with scalar diffusive limits were proposed. However, none of these methods can be extended to the difficult situations pointed out above. On the contrary, we will show that a suitable extension of the scheme proposed in [1] allows to tackle with these situations.

References

- C. Berthon and P. LeFloch and R. Turpault, Late-time/stiff relaxation asymptotic-preserving approximations of hyperbolic equations, *Math. of Comp.* (2012),
- [2] C. Berthon and P. LeFloch and R. Turpault, Hyperbolic Conservation Laws with Stiff Relaxation Terms and Entropy, Comm. Pure Appl. Math., Volume no. 47 (1995), pp. 787-830

Joint work with: Christophe Berthon (Université de Nantes, LMJL), Philippe LeFloch (CNRS, LJLL) and Fabien Marche (Université Montpellier II, I3M)

7.2 Session 20 — Room C — Navier-Stokes and Euler Equations III

S20 - NAVIER-STOKES AND EULER EQUATIONS III - ROOM C, 15.15-15.45

Global well-posedness of 2D compressible Navier-Stokes equations with large data and vacuum

Quansen Jiu

School of Mathematical Sciences, Capital Normal University, Beijing 100048,PRC jiuqs@mail.cnu.edu.cn

In this talk, we will present some recent results on the global well-posedness of the 2D compressible Navier-Stokes equations with large initial data and vacuum. It is proved that if the shear viscosity μ is a positive constant and the bulk viscosity λ is the power function of the density, that is, $\lambda(\rho) = \rho^{\beta}$ with $\beta > 3$, then the 2D compressible Navier-Stokes equations with the periodic boundary conditions on the torus T^2 admit a unique global classical solution (ρ, u) which may contain vacuums in an open set of T^2 . Note that the initial data can be arbitrarily large to contain vacuum states. The Cauchy problem will also be discussed. This is joint with Yi Wang and Zhouping Xin.

Joint work with: Yi Wang(Institute of Applied Mathematics, AMSS, and Hua Loo-Keng Key Laboratory of Mathematics, CAS, Beijing 100190, P. R. China), Zhouping Xin (The Institute of Mathematical Sciences, Chinese University of HongKong, HongKong)

S20 - NAVIER-STOKES AND EULER EQUATIONS III - ROOM C, 15.45-16.15

Well-posedness of the linearized plasma-vacuum interface problem in ideal incompressible MHD

Paola Trebeschi Department of Mathematics, University of Brescia paola.trebeschi@ing.unibs.it

We consider the free boundary problem for the plasma vacuum interface model in ideal incompressible magnetohydrodynamics. Under a suitable stability condition on the initial discontinuity, the well-posedness of the linearized problem, around a non constant basic state sufficiently smooth, is investigated. Since the latter amounts to be a non standard initial-boundary value problem of mixed hyperbolic-elliptic type, for its resolution we introduce a fully "hyperbolic" regularized problem. For the regularized problem, a suitable a priori estimate, uniform with respect to the small parameter of the regularization, is derived in the anisotropic Sobolev space H_*^1 .

References

- Alessandro Morando, Paola Trebeschi and Yuri Trakhinin, Stability of incompressible current-vortex sheets, J. Math. Anal. Appl., 347 no. 2 (2008), pp. 502-520
- [2] Paolo Secchi and Yuri Trakhinin, Well-posedness of the linearized plasma-vacuum interface problem, preprint (2011), submitted on *J. Math. Pures Appl.*

- [3] Yuri Trakhinin, Existence of compressible current-vortex sheets: variable coefficients linear analysis, Arch. Ration. Mech. Anal., 177 no. 3 (2005), pp. 331-366
- [4] Yuri Trakhinin, The existence of current-vortex sheets in ideal compressible magnetohydrodynamics, Arch. Ration. Mech. Anal., **191 no. 2** (2009), pp. 245-310

Joint work with: Alessandro Morando (Department of Mathematics, University of Brescia, Italy), Yuri Trakhinin (Sobolev Institute of Mathematics, Novosibirsk, Russia)

Session 21 — Room G — Numerical Methods VIII 7.3

S21 – Numerical Methods VIII – Room G, 15.15–15.45

Entropy dissipation property of adaptive mesh reconstruction techniques

Nikolaos Sfakianakis Johannes Gutenberg University, Mainz sfakiana@uni-mainz.de

Introduction. In this work we analyze the entropy dissipation property of the mesh adaptation procedures when resolving hyperbolic conservation laws. Numerical evidence in [1, 3, 4, 6] have exhibited that mesh adaptation can improve the stability properties of the numerical schemes been used and more specifically it can provide with enough entropy dissipation to stabilize entropy unstable schemes [2].

In this work we provide analytical justification of this phenomenon, and postulate conditions on the mesh adaptation and prove their sufficiency to enforce entropy stability.

Background material. We resolve the scalar CL

$$u_t + f(u)_x = 0,$$

using finite difference and finite volume schemes. The spatial disretization takes place over non-uniform adaptively redefined meshes of constant cardinality. The time steps are also adaptively redefined after the CFL condition.

To exhibit the entropy dissipation of the mesh adaptation techniques, we consider the FTCS (Forward in Time Centered in Space) scheme, with flux function that reads $F_{i+1/2}^n = \frac{f(u_{i+1}^n) - f(u_{i-1}^n)}{2}$, where u_i^n are the numerical approximation of the solution of the CL.

We study the FTCS, using its Q-form:

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{2h_i^n} \left(f_{i+1}^n - f_{i-1}^n \right) + \frac{\Delta t}{2h_i^n} \left(Q_{i+1/2}^n (u_{i+1} - u_i) - Q_{i-1/2}^n (u_i^n - u_{i-1}^n) \right)$$

where h_i^n is the cell size, and with the viscosity coefficient being $Q_{i+1/2}^{FTCS} = Q_{i+1/2}^n = \frac{f_i^n + f_{i+1}^n - 2F_{i+1/2}^n}{u_{i+1}^n - u_i^n} = 0.$ Following [8] and [7], we deduce by means of comparison that the EUCC

Following [8] and [7], we deduce by means of comparison that the FTCS is entropy stable as long as

$$Q_{i+1/2}^{FTCS} > Q_{i+1/2}^*$$

where $Q_{i+1/2}^*$ is the viscosity coefficient of an entropy stable scheme as defined in [7, 8]. More specifically, using the entropy function $U(u) = \frac{1}{2}u^2$ and the respective entropy variables as they were introduced in [5] we prove the following proposition: **Proposition.** The viscosity coefficient of the FTCS scheme satisfies $Q_{\nu+1/2}^* < 0 = Q_{\nu+1/2}^{FTCS}$ along shock waves, and $Q_{\nu+1/2}^{FTCS} = 0 < Q_{\nu+1/2}^*$ along rarefaction waves. **Remark.** This proposition point out that the FTCS is not entropy stable, i.e does not dissipate enough entropy along rarefactions. At the same time, numerical tests exhibit that FTCS dissipates enough entropy when when used on non-uniform adaptively redefined meshes (under conditions on the mesh adaptation).

So, the following question are posed:

Question. What are the mechanisms that dissipate entropy when FTCS is used over adaptively redefined meshes?

Question. Can the mesh be adapted in a way that guarantees entropy dissipation?

In this work. To answer the previous questions we focus on the case of rarefaction waves and consider a uniform mesh of the same cardinality as the non-uniform one. We define a discrete function over the uniform mesh via a per cell mass conservation between the uniform and the non-uniform mesh; justifiable by a suitable geometric conservation law. These relation are exhibited, with respect to the main steps of a mesh adaptation procedure, in the following schemes:

Non-uniform ::
$$\{h_i^n, u_i^n\} \xrightarrow{\text{mesh adapt.}} \{h_i^{n+1}, \hat{u}_i^n\} \xrightarrow{\text{num. scheme}} \{h_i^{n+1}, u_i^{n+1}\}$$
 (1)

Uniform ::
$$\{\Delta x, v_i^n\} \xrightarrow{\text{mesh adapt.}} \{\Delta x, \hat{v}_i^n\} \xrightarrow{\text{num. scheme}} \{\Delta x, v_i^{n+1}\}$$
 (2)

and the per cell mass conservation means that:

$$h_i^n u_i^n = \Delta x \, v_i^n \xrightarrow{\text{mesh adapt.}} h_i^{n+1} \, \hat{u}_i^n = \Delta x \, \hat{v}_i^n \xrightarrow{\text{num. scheme}} h_i^{n+1} \, u_i^{n+1} = \Delta x \, v_i^{n+1}. \tag{3}$$

Effectively, the numerical scheme can be written over the uniform mesh as follows

$$v_i^{n+1} = \hat{v}_i^n - \frac{\Delta t}{\Delta x} \left(\hat{F}_{i+1/2}^n - \hat{F}_{i-1/2}^n \right)$$

or in viscosity form,

$$v_{i}^{n+1} - v_{i}^{n} = \hat{v}_{i}^{n} - v_{i}^{n} + \frac{\Delta t}{2\Delta x} \left(\left(B_{i+1/2}^{n} + Q_{i+1/2}^{n} \right) \Delta v_{i+1/2}^{n} - \left(B_{i-1/2}^{n} + Q_{i-1/2}^{n} \right) \Delta v_{i-1/2}^{n} \right)$$
(4)

where $B_{\nu+1/2} = \frac{g(v_{\nu+1}^n) - g(v_{\nu}^n)}{\Delta v_{\nu+1/2}^n}$, $\Delta v_{i-1/2}^n = v_i^n - v_{i-1}^n$, and where $\hat{v}_i^n - v_i^n$ accounts for the change of the mesh change.

We follow now the works of [5, 8] to write the scheme in the corresponding entropy variables

$$U(v_i^{n+1}) - U(v_i^n) + \frac{\Delta t}{\Delta x} \left(G_{i+1/2}^n - G_{i-1/2}^n \right) = M_i^n - \frac{\Delta t}{\Delta x} \mathcal{E}_i^{(x)} + \mathcal{E}_i^{(FE)} (\Delta v^{n+1/2})$$
(5)

where $\mathcal{E}_i^{(x)}$, $\mathcal{E}_i^{(FE)}$ account for the entropy dissipation/production due to the spatial and temporal discretization, and the new term M_i^n stems from the term $\hat{v}_i^n - v_i^n$ that appears in (4) and accounts for the entropy dissipation/production due to the mesh adaptation.

We estimate the terms $\mathcal{E}_i^{(x)}$, $\mathcal{E}_i^{(FE)}$ following [8] and close by deducing that the following condition as being a sufficient one for the entropy dissipation of the mesh adaptation technique:

$$M_{i}^{n} \leq \frac{\Delta t}{\Delta x} \left\{ \left(D_{i-1/2}^{n} - \frac{K^{3}}{2} \frac{\Delta t}{\Delta x} (\tilde{B}_{i-1/2} + D_{i-1/2}^{n})^{2} \right) (\Delta v_{i-1/2}^{n})^{2} + \left(D_{i+1/2}^{n} - \frac{K^{3}}{2} \frac{\Delta t}{\Delta x} (\tilde{B}_{i+1/2} + D_{i+1/2}^{n})^{2} \right) (\Delta v_{i+1/2}^{n})^{2} \right\}$$

$$(6)$$

where $\tilde{B}_{\nu+1/2} = B_{\nu+1/2} + Q^*_{\nu+1/2}, Q^*_{\nu+1/2} = Q_{\nu+1/2} - D_{\nu+1/2}.$

References

- Ch. Arvanitis, Th. Katsaounis, Ch. Makridakis, Adaptive finite element relaxation schemes for hyperbolic conservation laws, Math. Model. Anal. Numer. 35 (2001), 17–33.
- [2] Ch. Arvanitis, Ch. Makridakis, N. Sfakianakis, Entropy conservative schemes and adaptive mesh selection for hyperbolic conservation laws, JHDE 7 (2010), 383-404.
- [3] Ch. Arvanitis, Ch. Makridakis, A. Tzavaras, Stability and convergence of a class of finite element schemes for hyperbolic systems of conservation laws, SIAM J. Numer. Anal. 42 (2004), 1357–1393.
- [4] Ch. Arvanitis, Mesh redistribution strategies and finite element method schemes for hyperbolic conservation laws, J. Sci. Computing 34 (2008), 1–25.
- [5] M.S. Mock, Systems of Conservation laws of mixed type, J. Differential Equations 37 (1980) 70-88
- [6] N. Sfakianakis, TVB property for central oscillatory Finite Difference schemes under enhanced Adaptive Mesh selection, Mathematics of Computation (to appear)
- [7] E. Tadmor, The numerical viscosity of entropy stable schemes for systems of conservation laws, Mathematics of Computations (1987), 91–103.
- [8] E. Tadmor, Entropy stability theory for difference approximations of nonlinear conservation laws and related time dependent problems, Acta Numerica (2003), 451–512.

Joint work with: Maria Lukacova, (Johannes Gutenberg University Mainz)

S21 – Numerical Methods VIII – Room G, 15.45–16.15

Nonlinear fractional equations of mixed hyperbolic parabolic type: Initial theory and numerics.

Espen Robstad Jakobsen Department of Mathematical Sciences, NTNU, Norway erj@math.ntnu.no

In this talk we consider the Cauchy problem for a class of equations with nonlinear convection and fractional nonlinear diffusion:

$$\partial_t u + \nabla \cdot f(u) = \mathcal{L}[A(u)],\tag{1}$$

where f, A are locally Lipschitz, A is nondecreasing, and \mathcal{L} is a fractional diffusion operator. \mathcal{L} can be the generator of any pure jump Levy process and the typical example is $\mathcal{L} = -(-\Delta)^{\frac{\alpha}{2}}$, the fractional Laplacian of order $\alpha \in (0, 2)$. This equation might be strongly degenerate and of mixed type. It can be seen as a generalization of fractional conservation laws [1] to the nonlinear diffusion setting, or of convection-diffusion equations to the fractional diffusion setting. Equation (1) include as special cases previous fractional/Levy conservation laws, radiation hydrodynamic models, recent fractional porous medium equations (see [5] where $A(u) = u^m, m \geq 1$), and new strongly degenerate equations.

We will introduce Kruzkov type entropy solutions for (1) and present wellposedness, continuous dependence, and some regularity results when the initial data is at least in $L^1 \cap L^{\infty}$. Then we introduce and analyze a monotone numerical method. Finally we present a new Kuznetsov type theory giving general and possibly optimal error estimates for our numerical methods even when the principal derivatives have any fractional order between 1 and 2!

The results are mainly taken from papers [3,2,4].

References

- [1] N. Alibaud. Entropy formulation for fractal conservation laws. J. Evol. Equ., 7(1):145–175, 2007.
- [2] N. Alibaud, S. Cifani, and E. R. Jakobsen. Continuous dependence estimates for nonlinear fractional convection-diffusion equations. To appear in SIAM J. Math. Anal. (http://arxiv.org/abs/1105.2288)
- [3] S. Cifani and E. R. Jakobsen. Entropy solution theory for fractional degenerate convection-diffusion equations. Ann. Inst. H. Poincare Anal. Non Lineaire 28(3):413–441, 2011.
- S. Cifani and E. R. Jakobsen. On numerical methods and error estimates for degenerate fractional convection-diffusion equations. http://arxiv.org/abs/1201.6079
- [5] A. de Pablo, F. Quiros, A. Rodriguez and and J. L. Vazquez. A fractional porous medium equation. Adv. Math. 226 (2011), no. 2, 1378–1409.

Joint work with: Simone Cifani (NTNU), Nathael Alibaud (Besancon)

7.4 Session 22 — Room H — Numerical Methods IX

S22 – Numerical Methods IX – Room H, 15.15–15.45

Second-order MUSCL schemes based on Dual Mesh Gradient Reconstruction (DMGR)

Vivien Desveaux Université de Nantes, Laboratoire de Mathématiques Jean Leray vivien.desveaux@univ-nantes.fr

We are interested in the approximation of the weak solutions of nonlinear hyperbolic systems of conservation laws on unstructured meshes. In order to achieve high-order approximation, we discuss extensions of the wellknown MUSCL scheme. This scheme is a finite volume method where the fluxes at the interface are approximated using a linear reconstruction on each cell. The gradient reconstructions are easily defined for 1D problems and naturally extend on 2D Cartesian meshes. However it becomes difficult to compute accurately the gradients as soon as unstructured meshes are considered.

To deal with such an issue, we propose to use a technique originally introduced in the field of elliptic equations (for instance, see [1]). We combine two distinct MUSCL schemes on two overlapping meshes (primal and dual). This process increases the number of numerical unknowns, but it allows to reconstruct very accurate gradients on diamond cells.

In order to avoid spurious oscillations, we have to complete the linear reconstruction by a limitation procedure. The limitation is also a key-point to ensure the invariance of the set of physical states. In general, enforcing the MUSCL scheme to satisfy such a robustness property can be made at the expense of a restrictive CFL condition, which leads to a larger computational cost.

The second-order CFL number is a fraction of the first-order one. One objective of this work is to optimize the CFL number associated to the first-order method. In [2], the authors evaluate the first-order CFL as follows:

$$\Delta t \frac{\mathcal{P}}{|K|} \lambda \le \frac{1}{2},$$

where |K| is the area and \mathcal{P} the perimeter of the control volume. Here we exhibit an optimal CFL condition given by

$$\Delta t \frac{\mathcal{P}}{|K|} \lambda \le 1.$$

Let us emphasize that this condition generalizes the usual one on Cartesian grids with uniform length Δx given as follows:

$$\frac{\Delta t}{\Delta x}\lambda \leq \frac{1}{4}.$$

This time increment evaluation is next adopted to propose a relevant second-order CFL condition to ensure the robustness of MUSCL schemes on 2D unstructured meshes.

Finally, we show numerous numerical experiments approximating the 2D Euler equations. Several simulations involving low density and strong shock waves will illustrate the good behaviour of the method (see Figure 1).



Figure 1: Double Mach reflection problem on a mesh made of 2.10^6 cells – zoom on the wave interaction – Left: first-order density approximation – Right: DMGR density approximation

References

- Domelevo, K. and Omnes, P., A finite volume method for the Laplace equation on almost arbitrary two-dimensional grids, *Mathematical Modelling and Numerical Analysis*, **39** (2005), pp. 1203–1249
- [2] Perthame, B. and Shu, C.W., On positivity preserving finite volume schemes for Euler equations, Numerische Mathematik, 73 (1996), pp. 119–130

Joint work with: Christophe Berthon (Université de Nantes, Laboratoire de Mathématiques Jean Leray), Yves Coudière (Université de Nantes, Laboratoire de Mathématiques Jean Leray and INRIA Sud-Ouest)

* * * *

S22 – Numerical Methods IX – Room H, 15.45–16.15

A mixture-energy-consistent numerical approximation of a single-velocity compressible two-phase flow model for fluids with interfaces and cavitation

> Marica Pelanti ENSTA ParisTech marica.pelanti@ensta-paristech.fr

Session 22 — Room H — Numerical Methods IX

We are interested in the simulation of cavitating flow processes by means of diffuse interface compressible multi-phase flow models. This class of models has shown to be very effective in describing complex wave propagation phenomena and interface dynamics in compressible multi-material fluids.

In the present work we begin by considering the hyperbolic 6-equation single-velocity two-phase model with instantaneous pressure relaxation of Saurel–Petitpas–Berry [1]. We adopt a variant of this model by using phasic total energy equations instead of phasic internal energy equations, in contrast with the more classical approach [1, 2]. We numerically solve the adopted two-phase model by a high-resolution multi-dimensional wave-propagation scheme based on a HLLC-type Riemann solver. The alternative formulation with phasic total energies allows us to write discrete non-conservative phasic energy equations whose sum exactly recovers the conservative discrete form of the total energy equation for the mixture. A first advantage of this method is that there is no need to augment the 6-equation model with an extra equation for the mixture total energy as done in [1, 2] to correct the thermodynamic state resulting from the non-conservative energy equations. Moreover, the consistence of the computed phasic energies with conservation of the mixture total energy enables us to ensure agreement of the relaxed pressure with the mixture equation of state, in combination with a simple non-iterative pressure relaxation procedure. Temperature and Gibbs free energy relaxation terms can be also included (see e.g. [2]) in a way that preserves consistency with the mixture energy at the discrete level.

Several numerical experiments of cavitation appearance and dynamic creation of interfaces are presented to show the efficiency of the numerical model.

References

- R. Saurel, F. Petitpas, and R. A. Berry, Simple and efficient relaxation methods for interfaces separating compressible fluids, cavitating flows and shocks in multiphase mixture, *J. Comput. Physics*, Volume no. 228 (2009), pp. 1678–1712.
- [2] A. Zein, M. Hantke, and G. Warnecke, Modeling phase transition for compressible two-phase flows applied to metastable liquids, J. Comput. Physics, Volume no. 229 (2010), pp. 2964–2998.

Joint work with: Keh-Ming Shyue (National Taiwan University)

7.5 Session 23 — Room F — Complex and Social Models

S23 - Complex and Social Models - Room F, 15.15-15.45

Hyperbolic Nets: Modeling, Analysis, Numerical Simulation, and Numerics

Sunčica Čanić University of Houston canic@math.uh.edu

From local to global, and from simple to complex, hyperbolic nets can be used to capture the structural properties of various multi-component, net-like objects whose global properties emerge from complex combinations of local components modeled by 1D conservation laws. Examples include emerging new constructs such as tissue scaffolds, carbon nano-tubes, and endovascular stents, or classical structures such as bridges and buildings made of metallic frames, which have been modeled using simplified net-based truss theory.

This talk will present our first steps in the development of a general theory, modeling, numerical simulation, and applications of nonlinear hyperbolic nets. As a prototypical example, we will focus on studying the structural properties of endovascular stents modeled as hyperbolic nets in 3D. The speaker will talk about a novel modeling approach to studying mechanical properties of these cardiovascular devices, and about the consequences of the numerical results in cardiovascular applications. The new modeling approach based on dimension reduction and hyperbolic net ideas, provides substantial computational savings, it provides new information about the emergent mechanic behavior of stents, and it provides a novel framework for the development of general mathematical hyperbolic net theory.

While the classical 1D nonlinear hyperbolic conservation laws theory is well developed, mathematical theory of 1D nonlinear hyperbolic systems defined on nets and networks is in its infant stages. The main challenges stem from the complex nonlinear wave interactions that occur at net's vertices, exhibiting mathematical complexity of nonlinear moving-boundary problems, and from the fact that local solution depends on the global properties of the entire hyperbolic net. This talk will address the main difficulties associated with the development of general hyperbolic net theory, the modeling strategy, and numerical method development for studying multi-component, net-like structures such as stents as nonlinear hyperbolic nets.

Collaborators include: Prof. J. Tambaca (University of Zagreb), Prof. B. Picoli, (Rutgers, Camden), graduate student M. Kosor (University of Houston and U of Zagreb), Dr. D. Paniagua (Texas Heart Institute), and Dr. D. Fish (Texas Heart Institute).

Conservation Laws in the Modeling of Moving Populations

Rinaldo M. Colombo Dept. of Mathematics - Brescia University rinaldo@ing.unibs.it

Conservation laws provide tools to describe the movement of a population, be it a crowd of pedestrians or a flock of birds, for instance. This presentation overviews analytical results of the resulting models.

First, the case of a single population is recalled, see [1]. Then, we present well posedness theorems for systems of nonlocal conservation laws in several space dimensions that describe the interactions among different populations, see [2]. Confinement problems where few agents aim at steering a population within a certain region are discussed, see [3]. Finally, this latter situation is addressed also with other analytical tools, see [4], and the different models and results are compared.

References

- Rinaldo M. Colombo, Mauro Garavello, Magali Lécureux-Mercier: A Class of Non-Local Models for Pedestrian Traffic. Mathematical Models and Methods in the Applied Sciences, 22, 4, 2012, see also Comptes Rendus Mathematique, 349, 769-772, 2011.
- [2] Rinaldo M. Colombo, Magali Lécureux-Mercier: Nonlocal Crowd Dynamics Models for Several Populations. Acta Mathematica Scientia, 32, 1, 177-196, 2011.
- [3] Rinaldo M. Colombo, Magali Lécureux-Mercier: An Analytical Framework to Describe the Interactions Between Individuals and a Continuum. *Journal of Nonlinear Science*, 22, 39-61, 2012.
- [4] Rinaldo M. Colombo, Nikolay Pogodaev: Confinement Strategies in a Model for the Interaction between Individuals and a Continuum. To appear on *SIAM Journal on Applied Dynamical Systems*.

Joint work with: Mauro Garavello (Università di Milano–Bicocca), Magali Lécureux-Mercier (Technion, Haifa), Nikolay Pogodaev (Università di Brescia)

S23 - Complex and Social Models - Room F, 15.45-16.15
7.6 Session 24 — Room I — Multi Physics Models III

S24 – Multi Physics Models III – Room I, 15.15–15.45

Harten's Artificial Compression Method applied to a Multiphase Flow for Interface Sharpening

Olivier Rouch Université de Montréal rouch@dms.umontreal.ca

In 2009 and 2010, Harten's artificial compression method (ACM - see [1]) was extended to two-dimensional problems (see [2,3]). One of the main interests of ACM is that it transforms a linearly degenerated contact discontinuity into a shock, with a constant viscous profile. One can take advantage of this behaviour in the sharpening of the interface between two immiscible fluids. The performance of Harten's ACM is dependent upon the choice of a reliable detector of discontinuities. Fortunately, the mass fraction of one of the two fluids can be directly used to fulfill this function. In these conditions, the ACM is used as a supplementary module, independent from the scheme used for solving the main conservation equation, and computationaly quite inexpensive.

In this talk, we will begin with a quick review of Harten's ACM and its two-dimensional extensions. Then we shall see the details of its application to a simple two-phase flow. One- and two-dimensional numerical results will be presented.

References

- Harten A., The Artificial Compression Method for Computation of Shocks and Contact Discontinuities: III. Self-Adjusting Hybrid Schemes, *Math. of Comput.*, Vol. 32, Num. 142, (1978) pp. 363-389
- [2] Rouch O., Enhancing the Capture of Two-dimensional, Shock induced Detonation Fronts using Harten's Artificial Compression Method on Underresolved Cartesian Grids, in Proc. of the 2009 ICNAAM International Conference on Numerical Analysis and Applied Mathemathics, American Institute of Physics (2009)
- [3] Rouch O. and Arminjon P., Extension of Harten's Artificial Compression Method and of an Entropy-based Detector of Discontinuity to Unstructured Triangular Grids, using Arminjon-Viallon's Staggered Central Scheme, in Proc. of the 12th International Conference on Hyperbolic Problems (2010)

Dedicated to the memory of Pr. Paul Arminjon

* * * ------

S24 – Multi Physics Models III – Room I, 15.45–16.15

An entropy-satisfying relaxation approximation for the isentropic Baer & Nunziato model with vanishing phases

Khaled Saleh Université Pierre et Marie Curie, LJLL and EDF R&D, FRANCE saleh@ann.jussieu.fr Assuming one-dimensional flow, the governing equations of the isentropic Baer-Nunziato two-phase flow model, without exchange terms, is given by

$$\begin{cases} \partial_t \alpha_1 + u_2 \partial_x \alpha_1 = 0, \\ \partial_t (\alpha_1 \rho_1) + \partial_x (\alpha_1 \rho_1 u_1) = 0, \\ \partial_t (\alpha_1 \rho_1 u_1) + \partial_x (\alpha_1 \rho_1 u_1^2 + \alpha_1 p_1(\rho_1)) - p_1(\rho_1) \partial_x \alpha_1 = 0, \\ \partial_t (\alpha_2 \rho_2) + \partial_x (\alpha_2 \rho_2 u_2) = 0, \\ \partial_t (\alpha_2 \rho_2 u_2) + \partial_x (\alpha_2 \rho_2 u_2^2 + \alpha_2 p_2(\rho_2)) - p_1(\rho_1) \partial_x \alpha_2 = 0. \end{cases}$$
(1)

Here, α_1 , ρ_1 and u_1 denote the volume fraction, density and velocity of phase 1 respectively, while $\alpha_2 = 1 - \alpha_1$, ρ_2 and u_2 are the analogous quantities for phase 2. We assume barotropic pressure laws for each phase $\rho_i \mapsto p_i(\rho_i), i \in \{1, 2\}$.

In the spirit of [2], we propose a suitable Siliciu-type relaxation approximation for system (1) which relies on a linearization of the pressure laws. In the present work however, a specific emphasis is put on the satisfaction of an energy inequality and on the stability of the approximation in the regime of a vanishing phase. As opposed to system (1), the homogeneous part of the relaxation system has only linearly degenerate characteristic fields, which provides the helpful property that jump relations can be easily derived for each wave.

In many applications, the relevant solutions that must be considered are the solutions with *subsonic relative* speeds:

$$|u_1 - u_2| < c_1, (2)$$

where c_1 is the speed of sound of phase 1. The equality case $|u_1 - u_2| = c_1$ is related to a resonance phenomemon which corresponds to a loss of hyperbolicity of the system.

The new and remarkable contributions of this work are the following:

- 1. Considering the Riemann problem for the relaxation system, we prove an existence theorem for solutions with subsonic relative speeds. In particular, for each subsonic ordering of the characteristic eigenvalues, we formulate **explicit conditions on the left and right initial data** of the Riemann problem, that imply the existence of a solution with such a wave ordering. In addition, the proof is constructive and the expressions of the intermediate states are known.
- 2. We prove that, in the neighbourhood of the resonance, $|u_1 u_2| \leq c_1$, the total energy associated with the relaxation system is dissipated, which is in accordance with the equilibrium system (1) in the context of resonance.
- 3. We also derive a relaxation numerical scheme relying on this Riemann solver. The relaxation scheme appears to compare very favourably with Lax-Friedrichs type solvers. In particular, it handles some difficult cases as the vanishing phase case (either $\alpha_1 \rightarrow 0$ or $\alpha_1 \rightarrow 1$). To that purpose, energy dissipation is proved to be necessary in some instance.
- 4. The overall method is shown to be entropy-satisfying under some natural sub-characteristic condition.

References

- M.R. Baer and J.W. Nunziato, A two-phase mixture theory for the deflagration -to-detonation transition (ddt) in reactive granular materials, *International Journal of Multiphase Flow*, **12** (1986), pp. 861-889
- [2] A. Ambroso, C. Chalons, F. Coquel and T. Galié, Relaxation and numerical approximation of a two-fluid two-pressure diphasic model, M2AN Math. Model. Numer. Anal., 43 (2009), pp. 1063-1097

Joint work with: Frédéric Coquel (*Ecole Polytechnique, CMAP*), Jean-Marc Hérard (*EDF R&D, Département MFEE*) and Nicolas Seguin (*UMPC, LJLL*).

7.7 Session 25 — Room A — Theory of Conservation Laws III

S25 - Theory of Conservation Laws III - Room A, 15.15-15.45

Measure-valued coupling of non-linear hyperbolic PDEs

Florent Renac ONERA The French Aerospace Lab, 92320 Châtillon Cedex, France florent.renac@onera.fr

The present work treats the mathematical and numerical coupling of nonlinear hyperbolic PDEs at given fixed interfaces. The Cauchy problem under consideration typically writes:

$$\begin{cases} \partial_t \mathbf{u} + \partial_x \mathbf{f}(\mathbf{u}, x) = 0, & x \in \mathbb{R} - \{0\}, \ t > 0, \\ \mathbf{u}(., 0) = \mathbf{u}_0, \end{cases}$$
(1)

for some unknown $\mathbf{u} = \mathbf{u}(x,t)$ in an open subset $\Omega \subset \mathbb{R}^n$. Here, the flux function is discontinuous at the coupling interface $\{x = 0\}$ and reads:

$$\mathbf{f}(\mathbf{u}, x) = \mathbf{f}_{\pm}(\mathbf{u}), \quad \pm x > 0, \tag{2}$$

where \mathbf{f}_{\pm} are given smooth maps defined on Ω . Obviously an additional information, the so-called coupling condition, must be supplemented to model the transient exchange of informations at x = 0. Complementary frameworks ([1], [2] and the references therein) have been recently proposed to handle this issue. Generically, the coupling condition expresses a continuity property at x = 0 either for the unknown \mathbf{u} or some non-linear transformation of it. As a consequence of the prescribed continuity property, distinct conservation properties are inherited and may range from full to partial conservation of the components of the unknown \mathbf{u} .

To reflect this very general property, we propose here to model the mathematical coupling thanks to a bounded vector-valued Borel measure concentrated at x = 0:

$$\begin{cases} \partial_t \mathbf{u} + \partial_x \mathbf{f}(\mathbf{u}, x) = \mathcal{M}(\mathbf{u}(0^+, t), \mathbf{u}(0^-, t)) \ \delta_{x=0}, \quad x \in \mathbb{R}, \ t > 0, \\ \mathbf{u}(., 0) = \mathbf{u}_0, \end{cases}$$
(3)

where the precise definition of the map $\mathcal{M} : \Omega \times \Omega \to \mathbb{R}^n$, *i.e.* the mass of the measure, prescribes the coupling condition. The proposed framework is proved to be fairly flexible. In particular, it allows to handle coupling problems constrained by the property that coupled Riemann solutions should keep their values in a given convex subset of Ω while minimizing some convex non-linear cost function built on the mass \mathcal{M} at the interface. This opens the gates to prediction-correction strategies in the design of coupling procedures. To that purpose, we will show that existing relaxation frameworks provide convenient numerical tools. Special attention is paid to Suliciu's relaxation procedures in the gas dynamics framework.

References

- [1] E. Godlewski, K.-C. Le Thanh and P.-A. Raviart, The numerical interface coupling of nonlinear systems of conservation laws. II. The case of systems, *M2AN*, **39** (2005), pp. 649–692.
- [2] A. Ambroso, C. Chalons, F. Coquel, E. Godlewski, F. Lagoutière, P.-A. Raviart and N. Seguin, Relaxation methods and coupling procedures, Int. J. Numer. Meth. Fluids, 56 (2008), pp. 1123–1129.

Joint work with: Frédéric Coquel (CMAP, Ecole Polytechnique, 91128 Palaiseau Cedex, France), Claude Marmignon (ONERA The French Aerospace Lab, 92320 Châtillon Cedex, France).

S25 - Theory of Conservation Laws III - Room A, 15.45-16.15

Almost global existence of classical discontinuous solutions to general quasilinear hyperbolic systems of conservation laws with small BV initial data

Zhi-Qiang Shao Department of Mathematics, Fuzhou University, Fuzhou 350002, China E-mail:zqshao_fzu@yahoo.com.cn

In the present paper the author investigates the global structure stability of Riemann solutions for general quasilinear hyperbolic systems of conservation laws under small BV perturbations of the initial data, where the Riemann solution only contains shocks and contact discontinuities, the perturbations are in BV but they are assumed to be C^1 -smooth, with bounded and possibly large C^1 -norms. The author obtains the almost global existence and lifespan of classical discontinuous solutions to a class of the generalized Riemann problem, which can be regarded as a small BV perturbation of the corresponding Riemann problem. Some applications to quasilinear hyperbolic systems of conservation laws arising in physics, particularly to one-dimensional compressible Euler equations in Eulerian coordinates, are also given.

References

- A. Bressan, Hyperbolic Systems of Conservation Laws: The one-dimensional Cauchy Problem, Oxford University Press, (2000)
- [2] A. Bressan, Contractive metrics for nonlinear hyperbolic systems, Indiana Univ. Math. J., 37 (1988), pp. 409-421
- [3] D.X. Kong, Global structure stability of Riemann solutions of quasilinear hyperbolic systems of conservation laws: shocks and contact discontinuities, J. Differential Equations, 188 (2003), pp. 242-271
- [4] P. D. Lax, Hyperbolic systems of conservation laws II, Comm. Pure Appl. Math., 10 (1957), pp. 537-556
- [5] T.T. Li and D.X. Kong, Global classical discontinuous solutions to a class of generalized Riemann problem for general quasilinear hyperbolic systems of conservations laws, *Comm. Partial Differential Equations*, 24 (1999), pp. 801-820
- [6] T.T. Li and W.C. Yu, Boundary Value Problems for Quasilinear Hyperbolic Systems, Duke University, Mathematics Series V, Duke University, Durham, (1985)
- [7] Z.Q. Shao, Global structure stability of Riemann solutions for linearly degenerate hyperbolic conservation laws under small BV perturbations of the initial data, *Nonlinear Anal. Real World Appl.*, **11** (2010), pp. 3791-3808

7.8 Session 26 — Room D — Kinetic Models I

S26 - KINETIC MODELS I - ROOM D, 15.15-15.45

Sticky particles with interactions

Giuseppe Savaré Department of Mathematics, University of Pavia giuseppe.savare@unipv.it We discuss a simple one-dimensional model for a sticky-particle dynamics in Lagrangian coordinates. Assuming that the particles self-interact through a force field generated by themselves, we explain how the flow can be described by a differential inclusion on the space of transport maps. Starting from a discrete particle approximation, we prove global existence and stability results, obtaining a well defined semigroup. In the particular case of the Euler-Poisson system in the attractive regime this approach yields an explicit representation formula for the solutions.

References

- Y. Brenier, W. Gangbo, G. Savaré and M. Westdickenberg, Sticky particle dynamics with interactions, preprint (2011), Arxiv: 1201.2350, pp. 1-42.
- [2] L. Natile and G. Savaré, A Wasserstein approach to the one-dimensional sticky particle system, SIAM J. Math. Anal. 41 (2009), pp. 1340-1365.

Joint work with: Y. Brenier (Université de Nice), W. Gangbo (Georgia Institute of Technology), and M. Westdickenberg (Aachen University)

S26 – Kinetic Models I – Room D, 15.45–16.15

* * * -

Consensus and clustering in kinetic and hydrodynamic descriptions of self-alignment

Eitan Tadmor University of Maryland tadmor@cscamm.umd.edu

Self-alignment dynamics is driven by the collective interaction of agents who are influenced by their neighbors. Examples range from consensus of voters and traffic flows to the formation of flocks of birds, tumor growth etc. Motivated by particle-based models for self-alignment, we discuss the large time behavior of kinetic and hydro-dynamic models of self-alignment. We are interested in the emergence of one or more clusters, and in particular, consensus and unconditional flocking. The dynamics is dictated by a balance between nonlinear convection and non-symmetric interactions, governed by a convolution-based mid-range alignment. The large time behavior depends on the keeping the connectivity of non-vacuous clusters.

References

- H. Liu & E. Tadmor, Critical thresholds in a convolution model for nonlinear conservation laws SIMA 33 (2001), 930-945.
- [2] S. Motsch & E. Tadmor, A new model for self-organized dynamics and its flocking behavior, J. Stat. Physics 144(5) (2011) 923-947.
- [3] S. Motsch & E. Tadmor, Consensus and flocking in self-organized dynamics, preprint.
- [4] E. Tadmor & C. Tan, Critical threshold in convolution-based models for self-alignment, preprint

Joint work with: Sebastien Motsch and Changhui Tan (Center for Sceintific Computation and Mathematical Modeling (CSCAMM), University of Maryland)

7.9 Session 27 — Room B — Control Problems for Hyperbolic Equations III

S27 - Control Problems for Hyperbolic Equations III - Room B, 15.15-15.45

Controllability of a scalar conservation law with nonlocal velocity

Zhiqiang Wang Fudan University wzq@fudan.edu.cn

We consider some control problems of a conservation law with nonlocal velocity that models a highly reentrant manufacturing system as encountered in semi-conductor production:

$$\begin{cases} \rho_t(t,x) + (\rho(t,x)\lambda(W(t)))_x = 0, & t \in (0,T), x \in (0,1), \\ \rho(0,x) = \rho_0(x), & x \in (0,1), \\ \rho(t,0)\lambda(W(t)) = u(t), & t \in (0,T), \end{cases}$$

where $\lambda \in C^1(\mathbb{R}), \lambda > 0$,

$$W(t) := \int_0^1 \rho(t, x) dx$$

Here, u is the natural control imposed on the influx. The goal of control is to drive the solution of the above system either to reach a given final data

$$\rho(T, x) = \rho_1(x), \quad x \in (0, 1)$$

in the case of *state controllability* or to reach a given outflux condition

$$\rho(t,1)\lambda(W(t)) = y_d(t), \quad t \in (T_1,T)$$

in the case of nodal profile controllability.

We first prove a local state controllability result, i.e., there exists a control u that drives the solution from any given initial data ρ_0 to any desired final data ρ_1 in a certain time period [0, T], provided that ρ_0 and ρ_1 are both close to a given equilibrium $\bar{\rho}$ and $T > \frac{1}{\lambda(\bar{\rho})}$. We also obtain a global state controllability result for the same system with sufficiently large T, where there is no limitation on the distance between the initial data ρ_0 and final data ρ_1 . Finally, we prove a nodal profile controllability result, i.e., there exists a control u under which the solution starts from any initial data ρ_0 verifies exactly any given outflux y_d over a fixed time period $[T_1, T]$ with T_1 suitably large.

References

 Jean-Michel Coron and Zhiqiang Wang, Controllability for a scalar conservation law with nonlocal velocity, J. Differential Equations, 252 (2012), pp. 181-201.

- * * * -

Joint work with: Jean-Michel Coron (Université Pierre et Marie Curie)

S27 – Control Problems for Hyperbolic Equations III – Room B, 15.45–16.15

Control problems for conservation laws in the context of entropy solutions and with three controls.

Vincent Perrollaz Université Paris Dauphine perrollaz@ann.jussieu.fr

In a first part, we will consider the initial boundary value problem for a scalar conservation law with a C^2 flux f strictly convex:

$$\begin{cases} u_t + (f(u))_x = g(t) & \text{on} \quad \mathbb{R}^+ \times (0, 1), \\ "u(t, 0) = u_l(t)", \quad "u(t, 1) = u_r(t)" & \text{for} \quad t \in \mathbb{R}^+, \\ u(0, .) = u_0 & \text{on} \quad (0, 1), \end{cases}$$

where the boundary conditions are taken in the sense of Bardos & Leroux & Nedelec. We will show how we can use u_l, u_r and g as controls to obtain results on the exact controllability problem and also on the problem of asymptotic stabilization of constant states, both problems being considered in the context of entropy solutions. We will also describe how the addition of a third control allowed us to get less restraining conditions for a state to be reachable than those obtained by Ancona & Marson and Horsin.

In a second part, we will show why the isentropic Euler-Poisson system used in semiconductor physics might be considered a generalization of the scalar control system above. This system, studied in particular by Degond & Markowich, Bo Zhang, Poupaud & Rascle & Vila reads:

$$\begin{cases} \rho_t + j_x = 0, \\ j_t + \left(\frac{j^2}{\rho} + P(\rho)\right)_x = -\sigma j + \frac{q}{\mu}\phi_x, \\ \phi_{xx} = \frac{q}{\epsilon}(\rho - n). \end{cases}$$
(1)

Here ρ is the density of electrons, j the density of current, ϕ the electrostatic potential. We will explain the additional difficulties encountered in the study of this system and discuss different ways in which the methods used on the scalar equation might be adapted to this problem.

References

- Ancona F. Marson A., On the attainable set for scalar nonlinear conservation laws with boundary control, SIAM J. Control Optim., Volume no. 36 (1999), pp. 290-312.
- [2] Bardos C. Leroux G. Nedelec J.-C., First order quasilinear equations with boundary conditions, Comm. in Partial Differential Equations, Volume no. 9 (1979), pp. 1017-1034.
- [3] Degond P. Markowich P. A., On a one dimensional steady-state hydrodynamic model for semiconductors, *Appl. Math. Lett.*, Volume no. 3, no. 3 (1990), pp. 25-29.
- [4] Horsin T., On the controllability of the Burgers equation, ESAIM Control Opt. Calc. Var., Volume no. 3 (1998), pp. 83-95.
- [5] Poupaud . Rascle M. Vila J.-P., Global solutions to the isothermal Euler-Poisson system with arbitrarily large data, *Journal of Differential Equation*, Volume no. 123 (1) (1995), pp. 93-121.
- [6] Bo Zhang, Convergence of the Godunov scheme for a simplified one dimensional hydrodynamic model for semiconductor devices, *Commun. Math. Phys.*, Volume no. 157 (1993), pp. 1-22.

8 Abstracts of contributed lectures — Tuesday 17.20–19.20

8.1 Session 28 - Room E - Numerical Methods X

S28 – Numerical Methods X – Room E, 17.20–17.50

Absorbing Boundaries for Free Surface Flow

Smadar Karni Department of Mathematics, University of Michigan, USA karni@umich.edu

The evolution of water waves often involves flows in open unbounded domains. Truncating the domain in numerical simulations raises the need for non-reflecting BCs. We adopt a novel formulation for the water wave equation [Wu, 1997], in which the problem is reduced to a one dimensional nonlocal PDE for the free surface, and derive a one-way version of the linearized equation that acts as an absorbing layer near the artificial computational boundaries. The one way nature of the proposed equation prevents potential errors at the boundary from propagating back and polluting the solution in the interior of the domain. A version of the one-way equations that incorporates additional wave damping is also discussed. The equation involves a fractional derivative operator corresponding to a half-derivative, and can be viewed as a conservation law with a linear nonlocal flux involving a convolution with a singular integrable kernel. We derive a hierarchy of high order numerical methods, where the solution is approximated by conservative piecewise polynomials, and the convolution with the singular kernel is then integrated *exactly*. Time integration uses Runge-Kutta schemes to matching order. In this talk, we will discuss the one way water wave equation, the numerical method, and show numerical results. This is joint work with G.I. Jennings and J.B. Rauch.

S28 – Numerical Methods X – Room E, 17.50–18.20

* * * -

Scalar conservation laws on moving hypersurfaces

Dietmar Kroener University of Freiburg dietmar@mathematik.uni-freiburg.de

We consider conservation laws on moving hypersurfaces. In this work the velocity of the surface is prescribed. But one may think of the velocity to be given by PDEs in the bulk phase. We prove existence and uniqueness for a scalar conservation law on the moving surface. This is done via a parabolic regularization of the hyperbolic PDE. We then prove suitable estimates for the solution of the regularized PDE, that are independent of the regularization parameter. We introduce the concept of an entropy solution for a scalar conservation law on a moving hypersurface. We also present some numerical experiments. As in the Euclidean case we expect discontinuous solutions, in particular shocks. It turns out that in addition to the "Euclidean shocks" geometrically induced shocks may appear. This will be demonstrated in a video.

Joint work with: Gerhard Dziuk (University of Freiburg), Thomas Mueller (University of Freiburg)

· * * * —

S28 – Numerical Methods X – Room E, 18.20–18.50

Explicit Numerical Schemes for the Coupling of Dimensionally Heterogeneous Free-Surface Flow Models

Christoph Gersbacher Department of Applied Mathematics, University of Freiburg christoph.gersbacher@mathematik.uni-freiburg.de

In this talk, we are interested in the numerical simulation of shallow free-surface river flows. In many situations, the classical two-dimensional Shallow Water Equations (see e.g. [3]) provide a sufficient description of the physical phenomena. Sometimes, even one-dimensional models may be accurate enough to capture the hydrodynamics of interest, at much lower computational cost.

In order to reduce overall numerical cost, techniques for coupling one- and two-dimensional Shallow Water Equations have been proposed in the literature (see e.g. [1,2]). Driven by *a-priori* considerations, the computational domain is decomposed into regions where either the coarse, one-dimensional, or the fine, two-dimensional model will be solved.

Based on a *a-priori* domain decomposition, we derive an explicit scheme for coupled 1D-2D Shallow Water systems. The reliability of the scheme and the accuracy of the numerical solutions obtained are illustrated in suitable test cases.

References

- E.D. Fernández-Nieto, J. Marin and J. Monnier, Coupling superposed 1D and 2D shallow-water models: Source terms in finite volume schemes, *Computers & Fluids*, **39** (2010), pp. 1070-1082
- [2] E. Miglio, S. Perotto, and F. Saleri, Model coupling techniques for free-surface flow problems. I, Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods, 63 (2005), pp. e1885-e1896
- [3] E. F. Toro, Shock-Capturing Methods for Free-Surface Flows, Chichester: Wiley, (2001)

S28 – Numerical Methods X – Room E, 18.50–19.20

A well-balanced numerical scheme for solutions with vacuum to a 1d quasilinear hyperbolic model of chemotaxis

Monika Twarogowska INRIA Sophia Antipolis - OPALE Project-Team monika.twarogowska@inria.fr

We consider a hyperbolic system of chemotaxis introduced by Gamba et.al. in [2], which models in vitro experiments of early stages of the vasculogenesis process. It describes the evolution of the density $\rho(x,t)$ of endothelial cells, their velocity u(x,t) and is coupled with a parabolic equation for the concentration $\phi(x,t)$ of a chemical substance. In one space dimension the system writes as

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + P(\rho))_x = -\alpha \rho u + \chi \rho \phi_x, \\ \phi_t = D \phi_{xx} + a\rho - b\phi. \end{cases}$$
(1)

Chemoattractant ϕ is released by cells, whereas cells motion is directed by its gradient and is slowed down due to the adhesion with the substratum. Overcrowding of cells is prevented by the pressure law for isentropic gases that is $P(\rho) = \varepsilon \rho^{\gamma}, \gamma > 1$.

This model was developed to describe formation of capillary-like networks from randomly seeded cells. Emerging of such structured patterns corresponds to non constant stationary solutions composed of regions where the density ρ is strictly positive and regions where it vanishes. We first provide a detailed description in the case $\gamma = 2$ of the non constant stationary solutions composed of vacuum and only one interval where $\rho > 0$.

Then we propose a numerical approximation of the chemotaxis system (1) where the hyperbolic part deals with two problems, namely treating vacuum states and having an accurate approximation of the non constant stationary states of the system. We use a scheme that couples a well-balanced strategy, in the framework of the USI method (see [1]), to capture the non constant equilibria for $\gamma > 1$ with an adapted flux solver in order to treat vacuum. Moreover, this scheme preserves the non negativity of the density and shows a small numerical viscosity.

Using this scheme, we study the dependence of the steady states on the length of the domain, the chemosensitivity constant χ , the adiabatic exponent γ , and the initial mass of cells. In particular, we present some cases where the asymptotic state is different from the one obtained using the diffusive parabolic Keller-Segel type model of chemotaxis with a non linear pressure.

References

- Bouchut F., Ounaissa H., Perthame B., Upwinding of the source term at interfaces for Euler equations with high friction, *Comput. Math. Appl.*, 53(3-4) (2007), pp. 361-375
- [2] Gamba D., Ambrosi D., Coniglio A., de Candia A., Di Talia S., Giraudo E., Serini G., Preziosi L., Bussolino F., Percolation, morphogenesis, and burgers dynamics in blood vessels formation, *Phys Rev Lett*, 90(11) (2003), pp. 118101

Joint work with: Roberto Natalini (IAC-CNR), Magali Ribot (Université Nice Sophia Antipolis)

8.2 Session 29 - Room C - Navier-Stokes and Euler Equations IV

S29 - NAVIER-STOKES AND EULER EQUATIONS IV - ROOM C, 17.20-17.50

On Whitham's modulated equations for the Euler-Korteweg system

Sylvie Benzoni-Gavage Institut Camille Jordan, Université Claude Bernard Lyon 1 benzoni@math.univ-lyon1.fr

The Euler–Korteweg system is a third-order system of conservation laws that takes into account capillary effects. Written either in Eulerian coordinates (EKE) or in mass Lagrangian coordinates (EKL), its one-dimensional version takes the form of an abstract Hamiltonian equation

(H) $\partial_t \mathbf{U} = \partial_x (\mathbf{J} \mathsf{E} \mathcal{H}[\mathbf{U}]),$

where \mathbf{J} is a nonsingular symmetric matrix, and $\mathsf{E}\mathcal{H}$ denotes the variational derivative of a Hamiltonian functional $\mathcal{H} = \mathcal{H}(\mathbf{U}, \partial_x \mathbf{U})$. Some celebrated dispersive equations fall into this framework too, namely the generalized Korteweg–de Vries equation (gKdV), and also the non-linear Schrödinger equation (NLS) via the Madelung transform. Depending on the pressure law involved in the Hamiltonian \mathcal{H} , the Euler–Korteweg system admits more or less rich families of periodic traveling wave solutions. Whitham's equations for slow modulations of these waves were first addressed by Gavrilyuk and Serre [1], who derived four first order conservation laws as modulated equations for (EKL), and pointed out that the hyperbolicity of the resulting system is encoded by the convexity of the averaged energy with respect to suitably chosen dependent variables. In the present work [2], we perform a similar analysis in Eulerian coordinates, and show the connection between the two modulated systems, which we summarize in the following commutative diagram.

		mass Lagrangian	
		change of coordinates	
Whitham's averaging	(EKE)	\longrightarrow	(EKL)
	\downarrow		\downarrow
	$\langle \text{EKE} \rangle$	\longrightarrow	$\langle \mathrm{EKL} \rangle$

In particular, the hyperbolicity of the two modulated systems $\langle EKE \rangle$ and $\langle EKL \rangle$ occurs simultaneously. In addition, we extend to the abstract Hamiltonian equation (H) a result that was known for other types of PDEs (see [3] and references therein) and in the particular case of (gKdV) [4], namely that the hyperbolicity of the modulated system at some given point, corresponding to a periodic traveling wave, is necessary for the linearized stability of this wave. Finally, we investigate the hyperbolicity of Whitham's modulated equations for the Euler–Korteweg system with van der Waals type pressure laws. We find numerical evidence of unstable waves, consistently with an unpublished work of Serre reported in [5], and also regions of hyperbolicity of modulated equations, which allow stable waves as those found analytically by Gallay and Hărăgus for (NLS) [6].

References

- S. L. Gavrilyuk and D. Serre, A model of a plug-chain system near the thermodynamic critical point: connection with the Korteweg theory of capillarity and modulation equations, in *Waves in liquid/gas and liquid/vapour two-phase systems (Kyoto, 1994)*, Kluwer Acad. Publ. Dordrecht, (1995), volume 31 of *Fluid Mech. Appl.*, pages 419–428.
- [2] S. Benzoni-Gavage, P. Noble and L. M. Rodrigues, Slow modulations of periodic waves in capillary fluids, in preparation.
- [3] D. Serre, Spectral stability of periodic solutions of viscous conservation laws: large wavelength analysis, *Comm. Partial Differential Equations*, **30** (2005), pp. 259–282.
- [4] M. A. Johnson, K. Zumbrun, and J. C. Bronski, On the modulation equations and stability of periodic generalized Korteweg-de Vries waves via Bloch decompositions, *Phys. D*, 239 (2010), pp. 2057–2065.
- [5] S. Benzoni-Gavage, Planar traveling waves in capillary fluids, preprint (2012).
- [6] T. Gallay and M. Hărăgus, Orbital stability of periodic waves for the nonlinear Schrödinger equation, J. Dynam. Differential Equations, 19 (2007), pp. 825–865.

Joint work with: Pascal Noble (Institut Camille Jordan, Université Claude Bernard Lyon 1), L. Miguel Rodrigues (Institut Camille Jordan, Université Claude Bernard Lyon 1)

S29 – Navier-Stokes and Euler Equations IV – Room C, 17.50-18.20

A non-uniqueness result for entropy solutions to the compressible Euler system

Elisabetta Chiodaroli Institut für Mathematik der Universität Zürich elisabetta.chiodaroli@math.uzh.ch The deceivingly simple-looking compressible Euler equations of gas dynamics have a long history of important contributions over more than two centuries. If we allow for discontinuous solutions, uniqueness and stability are lost. In order to restore such properties, further restrictions on weak solutions have been proposed in the form of entropy inequalities. In this talk we will discuss a counterexample to the well-posedness of entropy solutions to the multi-dimensional compressible Euler equations (see [1]): we show failure of uniqueness on a finite time-interval for entropy solutions starting from any continuously differentiable initial density and suitably constructed bounded initial linear momenta. Our methods are inspired by a new analysis of the incompressible Euler equations recently carried out by De Lellis and Szkelyhidi (see [3]-[4]) and based on a revisited "h-principle".

References

- Chiodaroli, E.: A counterexample to well-posedeness of entropy solutions to the compressible Euler system, Preprint,2011
- [2] Chiodaroli, E., De Lellis, C.: Non-standard solutions of the p-system with Riemann data, In preparation
- [3] De Lellis, C., Székelyhidi, L.J.: The Euler equations as a differential inclusion, Ann. Math., 170:101-120, 2009
- [4] De Lellis, C., Székelyhidi, L.J.: On admissibility criteria for weak solutions of the Euler equations, Arch. Rational Mech. Anal., 195:225-260, 2010

S29 – Navier-Stokes and Euler Equations IV – Room C, 18.20–18.50

* * * -

On the long-time behavior of 2D dissipative Euler equations

Luigi C. Berselli Dipartimento di Matematica Applicata "U.Dini," Università degli Studi di Pisa berselli@dma.unipi.it

We study the long-time behavior of the 2D dissipative Euler equations

$\partial_t u + \chi u + (u \cdot \nabla) u + \nabla p = f$	in $\Omega \times]0, T[,$	(1a)
$ abla \cdot u = 0$	in $\Omega \times]0, T[,$	(1b)
$u \cdot n = 0$	on $\partial \Omega \times]0, T[,$	(1c)

as those considered in [1,2]. In particular, to construct certain attractors, we will study carefully the transport equation for the vorticity. This approach allows to characterize the long-time behavior in a way alternative to [2,3], since properties will be completely independent of the viscosity and on vanishing viscosity approximations. The classical solutions of the transport equation will give the way to identify, with tools similar to [4], strong attractors in the phase space.

References

- V. Barcilon, P. Constantin, and E. S. Titi, Existence of solutions to the Stommel-Charney model of the Gulf Stream, SIAM J. Math. Anal. 19 (1988), no. 6, 1355–1364.
- [2] H. Bessaih and F. Flandoli, Weak attractor for a dissipative Euler equation, J. Dynam. Differential Equations 12 (2000), pp. 713–732.

- [3] A. A. Ilyin, A. Miranville, and E. S. Titi, Small viscosity sharp estimates for the global attractor of the 2-D damped-driven Navier-Stokes equations, *Commun. Math. Sci.* 2 (2004), pp. 403–426.
- [4] H. Koch, Transport and instability for perfect fluids, Math. Ann. 323 (2002), pp. 491–523.

S29 - NAVIER-STOKES AND EULER EQUATIONS IV - ROOM C, 18.50-19.20

- * * * -

The vanishing viscosity limit for Navier-Stokes equations in bounded domain with slip boundary conditions

Stefano Spirito University of L'Aquila spiritostefano@gmail.com

In this talk we present a result regarding the vanishing viscosity problem in a bounded domain $\Omega \subset \mathbb{R}^3$ for the incompressible Navier-Stokes equations. By assuming the following particular boundary conditions of Navier's type

$$\begin{cases} \omega \times n = 0\\ u \cdot n = 0, \end{cases}$$

where ω is the vorticity and n is the unit normal to the boundary, we prove the convergence of the inviscid limit of the Leray-Hopf weak solutions to the unique local smooth solutions of the Euler equations. Moreover, we show that with a particular choice of initial datum a better rate of convergence in the energy norm is obtained.

References

[1] L. C. Berselli, S. Spirito, On inviscid limits for the Navier-Stokes equations with slip boundary conditions involving the vorticity, preprint (2011),

Joint work with: Luigi C. Berselli (University of Pisa)

8.3 Session 30 — Room G — Numerical Methods XI

S30 – Numerical Methods XI – Room G, 17.20–17.50

A posteriori estimates from approximate solutions of the Euler or Navier-Stokes equations

Livio Pizzocchero Dipartimento di Matematica, Università di Milano and INFN, Sez. di Milano livio.pizzocchero@unimi.it This communication deals with the Cauchy problem for the incompressible Euler or Navier-Stokes (NS) equations on a *d*-dimensional torus \mathbf{T}^d , in a setting based on the Sobolev spaces $H^n(\mathbf{T}^d)$ (n > d/2 + 1; typically, d = 3).

Following [1], an approach will be presented to obtain fully quantitative information on the exact solution u of the Euler or NS Cauchy problem from a posteriori analysis of any approximate solution u_a .

This approach allows to derive estimates on the interval of existence [0,T) of the exact solution u, and on the Sobolev distance between the exact and the approximate solution. The latter estimate has the form $||u(t) - u_a(t)||_n \leq R_n(t)$ where $R_n(t)$ is a real, nonnegative function of time t, obtained solving a differential "control inequality". In particular, the exact solution u of the Cauchy problem is granted to be global in time if the control inequality has a global solution $R_n : [0, +\infty) \to [0, +\infty)$.

The quantitative implementation of the above setting requires accurate estimates on the constants in a number of inequalities, in the Sobolev setting for the Euler/NS equations. For example, it is necessary to use estimates [2] on the constants in the celebrated Kato inequality for $\langle (v \cdot \nabla) w | w \rangle_n$ (with v, w two velocity fields).

The above scheme will be compared with the setting proposed by Chernyshenko *et al* [3] for the approximate solutions of the Euler or NS equations (and with other works on this subject by Morosi and Pizzocchero [4][5]).

Finally, as an application, some results will be presented on the Euler or NS equations on \mathbf{T}^3 with the Behr-Nečas-Wu initial datum [6]; such a datum was proposed by the cited authors as a candidate for finite-time blow-up of the Euler equations.

References

- C. Morosi, L. Pizzocchero, On approximate solutions of the incompressible Euler and Navier-Stokes equations, Nonlinear Analysis 75 (2012), 2209-2235.
- [2] C. Morosi, L. Pizzocchero, On the constants in a Kato inequality for the Euler and Navier-Stokes equations, Commun. Pure Appl. Analysis 11 (2012), 557-586.
- [3] S.I. Chernyshenko, P. Constantin, J.C. Robinson, E.S. Titi, A posteriori regularity of the three-dimensional Navier-Stokes equations from numerical computations, J. Math. Phys. 48 (2007), 065204.
- [4] C. Morosi, L. Pizzocchero, On approximate solutions of semilinear evolution equations II. Generalizations, and applications to Navier-Stokes equations, *Reviews in Mathematical Physics* 20 (2008), 625-706.
- [5] C. Morosi, L. Pizzocchero, An H¹ setting for the Navier-Stokes equations: quantitative estimates, Nonlinear Analysis 74 (2011), 2398-2414.
- [6] E. Behr, J. Nečas, H. Wu, On blow-up of solution for Euler equations, ESAIM: M2AN 35 (2001), 229-238.

Joint work with: Carlo Morosi (Dipartimento di Matematica, Politecnico di Milano).

S30 – Numerical Methods XI – Room G, 17.50–18.20

High-order accuracy, entropy stability and convergence for finite difference methods for hyperbolic conservation laws

Ulrik Skre Fjordholm ETH Zürich ulrikf@sam.math.ethz.ch We consider systems of hyperbolic conservation laws in one dimension,

$$u_t + f(u)_x = 0. (1)$$

As solutions of (1) develop discontinuities over time, the equation must be interpreted weakly. To single out the physically correct solution from the (large) set of weak solutions, one enforces the *entropy condition*

$$\eta(u)_t + q(u)_x \le 0 \tag{2}$$

for all entropy pairs (η, q) .

Our strategy is to mimic this procedure in a discrete setting, with the aim of designing high-order accurate finite difference methods

$$\frac{d}{dt}u_i + \frac{1}{\Delta x}\left(F_{i+1/2} - F_{i-1/2}\right) = 0 \tag{3}$$

that converge to the entropy solution. Specifically, in [1], we design *entropy stable* methods – finite difference methods that satisfy a discrete entropy inequality

$$\frac{d}{dt}\eta(u_i) + \frac{1}{\Delta x} \left(Q_{i+1/2} - Q_{i-1/2} \right) \le 0 \tag{4}$$

for any given entropy pair (η, q) . We utilize the high-order accurate entropy conservative methods of [2], along with an ENO (Essentially Non-Oscillatory) reconstruction in entropy variables $v(u) := \nabla \eta(u)$, to obtain high-order accurate, computationally effective, parameter-free FD methods.

To conclude that the method converges strongly one needs a bound on the spacial variation of the solution, in addition to an L^{∞} bound. This comes in the form of a weak TV (total variation) bound. The entropy stability of our method implies a seemingly weaker bound, formulated in terms of reconstructed values. We show that for certain orders of reconstruction, this bound is enough to obtain weak TV bounds. Through a compensated compactness argument we conclude that our method converges strongly to a weak solution of (1). **References**

- U. S. Fjordholm, S. Mishra and E. Tadmor: Arbitrarily high-order essentially non-oscillatory entropy stable schemes for systems of conservation laws. Accepted in *SINUM* (2012).
- [2] P. G. LeFloch, J. M. Mercier and C. Rohde: Fully Discrete, Entropy Conservative Schemes of Arbitrary Order. SINUM, 2003, 40, pp. 1968-1992

Joint work with: Siddhartha Mishra (ETH Zürich) and Eitan Tadmor (CSCAMM, University of Maryland).

* * * ------

S30 – Numerical Methods XI – Room G, 18.20–18.50

An adaptive moving finite volume scheme for shallow water equations with dry and complex topography

Guoxian Chen

Division of Numerical Mathematics, IGPM, RWTH Aachen, Templergraben 55, 52062 Aachen, Germany gxchen@igpm.rwth-aachem.de

An adaptive moving finite volume (AMFV) scheme with unconstructed triangular meshes is proposed for the simulation of shallow water equations with dry and complex topography. Unlike traditional schemes involving fixed or refined meshes, its iteration process from ν to ν +1 adaptively moves a fixed number of meshes according

to flow variables calculated in prior solutions and then simulating flow variables on the new meshes. At each time step of the simulation, the AMFV scheme consists of three parts: an adaptive mesh movement equation to shift the vertices from $\vec{x}_{ij}^{n,0}$ to $\vec{x}_{ij}^{n,+\infty}$, geometrical conservative interpolation to remap the flow variables from $\vec{U}_i^{n,0}$ to $\vec{U}_i^{n,+\infty}$, and Harten-Lax-van Leer-based partial differential equations discretization to update the flow variables from $\vec{U}_i^{n,+\infty}$ to $\vec{U}_i^{n,+\infty}$. Two analytical and two experimental test cases are presented to verify the advantages of the proposed scheme over nonadaptive methods. The results reveal two attractive features: this scheme enables high-accuracy, high-resolution shock-capture of dam-break inundation over dry and complex topography with minimal computational cost, while satisfying well-balanced, positivity-preserving properties, and it improves the capability of shallow water equations for handling non-hydrostatic pressure problems by realizing streamwise meshes parallel to the spatial distribution of time-variant streamlines.

S30 – Numerical Methods XI – Room G, 18.50–19.20

* * *

Grid adaptivity for systems of conservation laws

Matteo Semplice Università Statale di Torino matteo.semplice@unito.it

The approximation of solutions of conservation laws over large domains, especially when complex wave structures emerge, depends crucially on the grid size that one can work with. In this respect, the possibility of concentrating the computing power on the important features of the solution, adapting the grid to the solution itself, as the evolution of shocks and waves proceeds, can yield important computational savings.

In this talk I will describe the work I've been doing with G. Puppo on grid adaptive techniques for systems of conservation laws. I will touch upon the obvious questions that arise when using cells of different sizes in a high order finite volume scheme: how to maintain conservativity, how to select the timestep respecting the CFL condition, how to ensure that shocks or other waves will not be deformed or reflected crossing a discontinuity in the grid.

In particular, we present a numerical method for a system of conservation laws, based on a single (nonuniform) grid, stored in a tree. The depth of the nodes in the tree determines the size of the corresponding cell and visiting all the leaves of the tree means traversing all the currently active cells of the grid. Time advancement is achieved either globally (selecting a timestep satisfying a global CFL condition) or with local timestepping techniques, with a timestep that varies from cell to cell, while maintaining conservativity [1,2]. We present a comparison of the different approaches, with respect to errors and computational times.

The second step is to introduce grid adaptivity, for which we employ the entropy residual as an indicator [1]. We discuss the accuracy on smooth and nonsmooth problems, using a number of numerical fluxes and flux limiters. 2D applications developed with the DUNE library[3] will be shown.

References

- Puppo G. and Semplice M., Numerical Entropy and Adaptivity for Finite Volume Schemes, Commun. Comput. Phys., 10 (2011), pp. 1132–1160
- [2] Puppo G. and Semplice M., Adaptive grids and the entropy error indicator, Proceedings of HYP2010
- [3] Dune (Distributed and Unified Numerics Environment), http://www.dune-project.org/index.html

Joint work with: Gabriella Puppo (Politecnico di Torino)

8.4 Session 31 — Room H — Numerical Methods XII

S31 – Numerical Methods XII – Room H, 17.20–17.50

Multi-Level Monte Carlo finite volume methods for nonlinear systems of stochastic conservation laws in multi-dimensions

Jonas Šukys ETH, Zürich jonas.sukys@sam.math.ethz.ch

We extend the Multi-Level Monte Carlo (MLMC) algorithm in order to quantify uncertainty in the solutions of multi-dimensional hyperbolic systems of conservation laws with uncertain initial data, sources and coefficients. The algorithm together with the novel load balancing procedure is presented in a software named ALSVID-UQ and the scalability on the massively parallel hardware is verified. Numerical simulation results of uncertain solutions of the Euler equations, ideal magnetohydrodynamics (MHD) equations and shallow water equations are reported; furthermore, new simulations of acoustic and linear elasticity equations with uncertain log-Gaussian material coefficients are investigated. In particular, numerical experiments showing the robustness, efficiency and scalability of the proposed algorithm are presented.

- S. Mishra, Ch. Schwab and J. Šukys, Multi-level Monte Carlo finite volume methods for nonlinear systems of conservation laws in multi-dimensions, *J. Comp. Phys.*, 2011 (to appear). Available from: http://www.sam.math.ethz.ch/reports/2011/02.
- [2] S. Mishra and Ch. Schwab, Sparse tensor multi-level Monte Carlo Finite Volume Methods for hyperbolic conservation laws with random initial data, *Math. Comp.* (to appear).

Available from: http://www.sam.math.ethz.ch/reports/2010/24.

[3] J. Šukys, S. Mishra, and Ch. Schwab, Static load balancing for multi-level Monte Carlo finite volume solvers, *Parallel Processing and Applied Mathematics 9th International Conference*, PPAM 2011, Torun, Poland, 2011 (to appear).

Available from: http://www.sam.math.ethz.ch/reports/2011/32.

[4] ALSVID-UQ: http://mlmc.origo.ethz.ch/.

Joint work with: Siddhartha Mishra (ETH, Zürich) and Christoph Schwab (ETH, Zürich).

* * * -----

S31 – Numerical Methods XII – Room H, 17.50-18.20

Time Asymptotic High Order Schemes for Dissipative BGK Hyperbolic Systems

Maya Briani Istituto per le Applicazioni del Calcolo "Mauro Picone", Consiglio Nazionale delle Ricerche m.briani@iac.cnr.it We introduce new finite differences schemes to approximate one dimensional dissipative semilinear hyperbolic systems with a BGK structure. Using accurate analytical time-decay properties of the local truncation error, it is possible to design schemes based on standard upwinding schemes, which are increasingly accurate for large times when computing small perturbations of constants asymptotic states.

Consider the class of one dimensional BGK systems, which are given for $f^i \in \mathbb{R}^k$, i = 1, ..., m, by the equations

$$\begin{cases} \partial_t f^i(x) + \lambda_i \partial_x f^i = M_i(u) - f^i, \\ u := \sum_{i=1}^m f^i. \end{cases}$$

Here $x \in \mathbb{R}$ and t > 0, and the functions $M_i = M_i(u) \in \mathbb{R}^k$ are smooth functions of u such that: $\sum_{i=1}^m M_i(u) = u$.

To obtain the time decay rates of these solutions, we need to rewrite the problem in more suitable coordinates. Following [1], we rewrite the BGK systems in its *conservative-dissipative* form for the new unknowns

$$Z = (u, \tilde{Z})^T.$$

It is proved in [1] that, under some dissipativity conditions and for initial data which are small and smooth in some suitable norms, the time decay of the global solutions, for large times and in the L^{∞} -norm, is given by

$$\partial_x^l u = O(t^{-1/2 - l/2}), \quad \partial_x^l \tilde{Z} = O(t^{-1 - l/2}),$$

and similar estimates are available for their time derivatives. Notice that the improved estimate for \tilde{Z} can only be obtained in these new coordinates.

The aim of this work it to take advantage of these precise decay estimates to build up more accurate numerical schemes. To be more specific, we shall show that for standard numerical schemes, for instance upwind schemes with the source term approximated pointwise by the standard Euler scheme, the truncation error has the following decay as $t \to +\infty$:

$$\mathcal{T}_u(x,t) = O(\Delta x \ t^{-3/2}) + O(\Delta t \ t^{-3/2}), \quad \mathcal{T}_{\tilde{z}}(x,t)) = O(\Delta x \ t^{-3/2}) + O(\Delta t \ t^{-3/2}).$$

It can be seen numerically that the corresponding absolute error, for a fixed space step, decays as

$$e_u(t) = O(t^{-1/2}), \ e_z(t) = O(t^{-1}),$$

which implies that the relative error is essentially constant in time.

Here, our main goal is to improve the decay estimates on the truncation order to achieve an effective decay in time of the relative error, both in u and \tilde{Z} . To obtain this result, using the estimates in [1], we perform a detailed analysis of the behavior of the truncation error for a general class of schemes, which generalize those introduced in [2]. Thanks to this analysis, we are able to select some schemes such that the truncation order behaves as

$$\mathcal{T}_u(x,t) = O(\Delta x \ t^{-2}), \quad \mathcal{T}_{\tilde{Z}}(x,t)) = O(\Delta x \ t^{-2}),$$

for a fixed CFL ratio and such that the numerical error observed in the practical tests improves of $t^{-1/2}$ on other schemes.

References

- [1] Bianchini, B. Hanouzet and R. Natalini, Asymptotic behavior of smooth solutions for partially Dissipative hyperbolic systems with a convex entropy *Communications Pure Appl. Math.*, **60** (2007), pp. 1559-1622
- [2] D. Aregba-Driollet, M. Briani, R. Natalini, Asymptotic high-order schemes for 2X2 dissipative hyperbolic systems SIAM J. Numer. Anal., 46 (2008), pp. 869-894

Joint work with: D. Aregba-Driollet (IMB, Mathématiques Appliquées de Bordeaux, Université Bordeaux 1), Roberto Natalini (Istituto per le Applicazioni del Calcolo "Mauro Picone", Consiglio Nazionale delle Ricerche) ------ * * * -----

S31 – Numerical Methods XII – Room H, 18.20–18.50

A multiscale method for compressible liquid-vapor flow with surface tension

Christoph Zeiler

University of Stuttgart christoph.zeiler@mathematik.uni-stuttgart.de

We consider a compressible fluid in an open bounded domain $\Omega \subset \mathbb{R}^d$ with $d \in \{1, 2, 3\}$. It can appear either in a liquid or in a vapor phase. For any time t > 0, we assume that Ω splits up into the union of two open domains $\Omega_{\text{vap}}(t)$, $\Omega_{\text{liq}}(t)$, which contain the two bulk phases, and a phase boundary $\Gamma(t)$ that separates the two bulks. In the two bulk phases we assume that the dynamics of the fluid is governed by the isothermal Euler equations

$$\begin{array}{rcl}
\rho_t & + & \operatorname{div}(\rho \, \mathbf{v}) & = & 0, \\
(\rho \, \mathbf{v})_t & + & \operatorname{div}(\rho \, \mathbf{v} \otimes \mathbf{v} + p(\rho) \, \mathbf{I}) & = & \mathbf{0},
\end{array} \tag{1}$$

for t > 0 and $\mathbf{x} \in \mathbf{\Omega}_{vap}(\mathbf{t}) \cup \mathbf{\Omega}_{liq}(\mathbf{t})$. Here $\rho = \rho(\mathbf{x}, \mathbf{t}) > \mathbf{0}$ denotes the unknown density field and $\mathbf{v} = \mathbf{v}(\mathbf{x}, \mathbf{t}) \in \mathbb{R}^{\mathbf{d}}$ the unknown velocity field. The given function $p = p(\rho)$ is a non-monotone equation of state, e.g. the Van-der-Waals pressure. The phase boundary between liquid and vapor phase is represented by the (dynamic) sharp interface $\Gamma(t) \subset \mathbb{R}^d$ at time t > 0. Let σ denote the speed of the phase boundary in normal direction \mathbf{n} . Across the interface the trace conditions

$$\llbracket \rho (\mathbf{v} \cdot \mathbf{n} - \sigma) \rrbracket = 0, \tag{2}$$

$$\llbracket \rho(\mathbf{v} \cdot \mathbf{n} - \sigma) \, \mathbf{v} + \mathbf{p}(\rho) \, \mathbf{n} \rrbracket = (d - 1) \gamma \kappa \, \mathbf{n},\tag{3}$$

have to be satisfied. Condition (2) ensures local conservation of mass across $\Gamma(t)$. Relation (3) corresponds to a dynamical version of the Young-Laplace law for static phase boundaries. The surface tension coefficient $\gamma > 0$ is assumed to be constant and κ denotes the mean curvature of the phase boundary $\Gamma(t)$ associated with orientation given through the choice of the normal **n**. We treat this free boundary value problem with a heterogeneous multiscale method in the sense of [1]. Further results has been published in [3]

On the microscale we consider the fluid dynamics at the phase boundary, more precisely around a segment of the phase boundary. The governing equations are (1), (2) and (3). The local view on the interface allows us to reduce microscale problems to Riemann type problems. We will present a new microscale solver where effects of surface tension are included via relation (3). A modified pressure function together with Liu's entropy criterion (cf. [4]) will determine a unique wave solution, similar to the approach in [6].

The macroscale domain is divided in time-dependent liquid and vapor domains $\Omega_{vap}(t)$, $\Omega_{liq}(t)$ and we use standard fluid solvers for (1) in the single phase areas. More complicated is the coupling of the scales and the correct treatment of the interface. For communication of the fluid variables ρ and \mathbf{v} between macro- and microscale we use a ghost fluid like approach, cf. [5]. The dynamics of the phase boundary is treated with an additional level set equation

$$\phi_t + (\sigma \mathbf{n}) \cdot \nabla \phi = \mathbf{0}$$

for t > 0 and $\mathbf{x} \in \mathbf{\Omega}$. In particular we, drive the level set function $\phi = \phi(\mathbf{x}, \mathbf{t}) \in \mathbb{R}$ with the interface speed σ , available from the microscale solver. The curvature κ in (3) is estimated applying the zero level of ϕ .

Multidimensional numerical examples will show how surface tension affects the behavior of bubbles respectively droplets of real fluids. We validate on stationary two phase solutions (cf. [2]) and show experimentally the order of convergence.

References

 W. E, B. Engquist, X. Li, W. Ren, and E. Vanden-Eijnden. Heterogeneous multiscale methods: a review. Commun. Comput. Phys., 2(3):367–450, 2007.

- [2] M. E. Gurtin. On a theory of phase transitions with interfacial energy. Arch. Rational Mech. Anal., 87(3):187-212, 1985.
- [3] F. Jaegle, C. Rohde, and C. Zeiler. A multiscale method for compressible liquid-vapor flow with surface tension. Preprint, 2012. www.simtech.uni-stuttgart.de/forschung/publikationen/publ/index.html
- [4] T. P. Liu. The Riemann problem for general systems of conservation laws. J. Differential Equations, 18:218–234, 1975.
- [5] C. Merkle and C. Rohde. The sharp-interface approach for fluids with phase change: Riemann problems and ghost fluid techniques. M2AN Math. Model. Numer. Anal., 41(6):1089–1123, 2007.
- [6] S. Müller and A. Voß. The Riemann problem for the Euler equations with nonconvex and nonsmooth equation of state: construction of wave curves. SIAM J. Sci. Comput., 28(2):651–681, 2006.

S31 – Numerical Methods XII – Room H, 18.50–19.20

* * *

A model for shock wave chaos

Aslan Kasimov King Abdullah University of Science and Technology (KAUST), Thuwal, Saudi Arabia aslan.kasimov@kaust.edu.sa

We propose the following simple model equation that describes chaotic shock waves:

$$u_t + \frac{1}{2} \left(u^2 - u u_s \right)_x = f\left(x, u_s \right).$$

It is given on the half-line x < 0 and the shock is located at x = 0 for any $t \ge 0$. Here $u_s(t)$ is the shock state and f is a given source term [1]. The equation is a modification of the Burgers equation that includes non-locality via the presence of the shock-state value of the solution in the equation itself. The model predicts steady-state solutions, their instability through a Hopf bifurcation, and a sequence of period-doubling bifurcations leading to chaos. This dynamics is similar to that observed in the one-dimensional reactive Euler equations that describe detonations. We present nonlinear numerical simulations as well as a complete linear stability theory for the equation.

References

[1] A. Kasimov, L. Faria, R.R. Rosales, A model for shock wave chaos, preprint (2012), arXiv:1202.2989v1.

Joint work with: Luiz Faria (KAUST, Thuwal, Saudi Arabia), Rodolfo R. Rosales (MIT, Cambridge, MA)

8.5 Session 32 — Room F — Relaxation Processes and Complex Models

S32 – Relaxation Processes and Complex Models – Room F, 17.20–17.50

Metastable and interface dynamics for the parabolic Burgers equation and for the hyperbolic Jin-Xin system

Marta Strani Dipartimento di Matematica Guido Castelnuovo, La Sapienza, Roma strani@mat.uniroma1.it

This study concerns the slow motion of internal shock layer of the initial boundary value problem for the scalar viscous Burgers equation

$$\partial_t u + \partial_x f(u) = \varepsilon \,\partial_x^2 u$$

and for the scalar hyperbolic-parabolic Jin-Xin system

$$\begin{cases} \partial_t u + \partial_x v = 0\\ \partial_t v + \partial_x u = \frac{1}{\varepsilon} (f(u) - v) \end{cases}$$

where the space variable x belongs to a bounded interval $I = (-\ell, \ell)$. We are interested on *metastable dynamics*, whereby the time-dependent solution approaches its steady state in an asymptotically exponentially long time interval as the viscosity coefficient $\varepsilon > 0$ goes to zero. To study such behavior, we construct a one-parameter family of approximate stationary solutions $\{U^{\varepsilon}(\cdot;\xi)\}_{\xi}$, where the parameter ξ describes the position of an internal shock layer. By linearizing around these family, we derive an ODE for the location of the interface, and we estimate rigorously the size of the layer location ξ .

References

- de Groen, P. P. N.; Karadzhov, G. E., Exponentially slow traveling waves on a finite interval for Burgers' type equation, *Electron. J. Differential Equations*, Volume no. 30 (1998)
- [2] Kreiss, G., Kreiss, H. O., Convergence to steady state of solutions of Burgers' equation, Appl. Numer. Math. 2 Volume no. 3-5 (1986), 161–179.
- [3] Reyna, L.G.; Ward, M.J., On the exponentially slow motion of a viscous shock, Comm. Pure Appl. Math. 48 Volume no. 2 (1995), 79–120.
- [4] Mascia, C., Strani, M., Metastability for scalar conservation laws in a bounded domain, in preparation

Joint work with: Corrado Mascia (Dipartimento di Matematica Guido Castelnuovo, La Sapienza, Roma).

______ * * * _____

S32 – Relaxation Processes and Complex Models – Room F, 17.50–18.20

Relaxation schemes for modelling erosion processes

Emmanuel Audusse LAGA, University Paris 13, France & BANG project, INRIA, France audusse@math.univ-paris13.fr In rivers, mean sediment discharge may represent several hundred cubic meters of gravels or silt per year. Therefore, the sediments must be taken into account in order to predict the river bed evolutions and have huge environmental and industrial impacts. This work focuses on the modelling of bedload transport which refers to gravel transport and pushes aside the transport of fine sediments by suspension. Up to now, one very classical approach is to consider the shallow water equations for the fluid and to approximate the solid phase equation by a simplified one, the well-known Exner equation, that is obtained by writing a mass conservation on the solid phase in interaction with the fluid, without considering dynamic effect in the solid phase. The coupled model stands in 1d :

$$\frac{\partial H}{\partial t} + \frac{\partial Q}{\partial r} = 0, \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{Q^2}{H} + \frac{g}{2} H^2 \right) = -g H \frac{\partial z_b}{\partial x}, \tag{2}$$

$$\frac{\partial z_b}{\partial t} + \frac{1}{1-p} \frac{\partial Q_s}{\partial x} = 0, \tag{3}$$

where Q = Hu is the water discharge, H is the water height, z_b is the bed elevation, Q_s is the bed load and p is the porosity of the gravel bed. The bed load may be expressed by empirical formula of the form $Q_s = A_g(u)|u|^{m-1}u$ where u is the velocity in the fluid (Grass formula) or $Q_s = f(\tau_b)$ where τ_b is the boundary shear stress (see for example Meyer-Peter and Muller, Einstein or Engelund and Fredsoe formulas).

Robust and accurate numerical schemes are now available for the fluid part, see for example [1]. But the numerical simulation of the coupled model is still an open question. In most of the industrial codes, the system is solved by coupling two distinguished numerical codes, one for the fluid phase and one for the solid phase. This approach allows the user to use existing works but is not stable [2] and some recent works have been devoted to the derivation of robust coupled approach [2,3,4]. In this note, we would like to present a new relaxation solver that allows us to give a unified framework to handle the problem whatever the sediment flux Q_s . The heart of the method is to relax the fluid pressure and the sediment flux and to consider a larger five by five linearly degenerate hyperbolic system. We present the derivation of the relaxation model and the details of the relaxation solver. We also extend the approach to second order accuracy and to wet/dry transition. Then we test the ability of the method for classical numerical test cases. Finally we investigate two main perspectives : first, we consider a more general relaxation model that allows us to preserve some equilibrium states and second, we consider a more general coupled model [5], that includes dynamic effects for the solid phase, and we show how to use our relaxation framework to deal with this new approach.

References

- F. Bouchut, Nonlinear stability of finite volume methods for hyperbolic conservation laws, and well-balanced schemes for sources, Frontiers in Mathematics series, Birkhäuser, 2004.
- [2] S. Cordier, M.H. Le and T. Morales, Bedload transport in shallow water models: why splitting (may) fail, how hyperbolicity (can) help, Advances in Water Resources, Volume 34, Issue 8, Pages 980-989, 2011.
- [3] F. Benkhaldoun, S. Sahmim and M. Seaid, Mathematical development and verification of a finite volume model for morphodynamic flow applications, Advances in Applied Mathematics and Mechanics, Vol 3, pp:470-492, 2011.
- [4] M.J. Castro, E.D. Fernandez-Nieto, A. Ferreiro and C. Pares, Two dimensional seidment transport models in shallow water equations. A second order finite volume approach on unstructured meshes, Computer Methods in Applied Mechanics and Engineering 2520–2538 vol. 198, n. 33, 2009.
- [5] E. Audusse, C. Chalons, O. Delestre, N. Goutal, M. Jodeau, J. Sainte-Marie, J. Giesselmann and G. Sadaka, Sediment transport modelling : relaxation schemes for Saint-Venant-Exner and three layer models, submitted to ESAIM Proceedings (CEMRACS), 2011.

Joint work with: Christophe Chalons (LJLL, University Paris Diderot, France) and Olivier Delestre (Lab. J. Dieudonné, University of Nice, France).

* * *

S32 – Relaxation Processes and Complex Models – Room F, 18.20–18.50

Relative entropy for the finite volume approximation of hyperbolic systems with relaxation

Hélène Mathis Laboratoire de Mathématiques Jean Leray, Université de Nantes helene.mathis@univ-nantes.fr

We consider a system of (n + m)-dimensional balance laws with dissipative source term

$$\begin{cases} \partial_t \begin{pmatrix} u \\ v \end{pmatrix} + \partial_x \mathbf{F} \begin{pmatrix} u \\ v \end{pmatrix} = \frac{1}{\epsilon} \begin{pmatrix} 0, \\ R(u, v) \end{pmatrix}, \quad t > 0, x \in \mathbb{R}, \\ \begin{pmatrix} u \\ v \end{pmatrix}(x, 0) = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}(x), \quad x \in \mathbb{R}. \end{cases}$$
(1)

Following [5], we assume that the equilibrium manifold associated to the stiff source term R satisfies

$$R(u, v) = 0 \Leftrightarrow v = v_{eq}(u),$$

so that the vector of the conserved quantities $u \in \mathbb{R}^n$ is close to the solution \overline{u} of the so-called equilibrium problem

$$\begin{cases} \partial_t \overline{u} + \partial_x \mathbb{P} \mathbf{F} \begin{pmatrix} u \\ v \end{pmatrix} eq = 0, \\ \overline{u}(x,0) = u_0(x), \end{cases}$$
(2)

where $\mathbb{P}: \mathbb{R}^{n+m} \to \mathbb{R}^n$ is defined by $\mathbb{P} \begin{pmatrix} u \\ v \end{pmatrix} = u$.

The purpose of this work is to determine error estimates between the smooth solution of (1) and the approximate solution of the equilibrium model (2) given by finite volume schemes. Estimates in ϵ between the smooth solutions of (1) and of (2) are given in [9], using the relative entropy [4]. Error estimates for finite volume schemes are derived in [3,6,7], but in the scalar case. Therefore, we adapt these estimates to the case of the approximation of systems (1) and (2) using again the relative entropy, in order to merge in a same framework the errors in ϵ and in Δx .

The aim is to use such local error estimates to carry out model adaptation as it is presented in [1,2,8]. It consists in automatically detecting the part of a computational domain where the model (1) can be replaced by the simpler model (2), at the numerical level.

References

- [1] A.-C. Boulanger, C. Cancès, H. Mathis, K. Saleh, N. Seguin, OSAMOAL: optimized simulations by adapted models using asymptotic limits, submitted.
- [2] C. Cancès, F. Coquel, E. Godlewski, H. Mathis, N. Seguin, Model adaptation for hyperbolic systems with relaxation, in preparation.

- [3] C. Chainais, S. Champier, Finite volume schemes for nonhomogeneous scalar conservation laws: error estimate, *Numer. Math*, 88 (2001).
- [4] C. Dafermos, Hyperbolic conservation laws in continuum physics, Springer-Verlag, (2005).
- [5] G. Q. Chen, C. D. Levermore, and T. P. Liu, Hyperbolic conservation laws with stiff relaxation terms and entropy, *Comm. Pure Appl. Math.*, 47 (1994), pp. 787-830.
- [6] R. Eymard, T. Gallouët, R. Herbin, *Finite volume methods, Handbook of numerical analysis, Vol. VII*, North-Holland, Amsterdam, (2000).
- [7] N. N. Kuznetsov, Accuracy of some approximate methods for computing the weak solutions of a first-order quasi-linear equation, USSR Comput. Math. and Math. Phys., 16 (1976), pp. 105-119.
- [8] H. Mathis, N. Seguin, Model adaptation for hyperbolic systems with relaxation, in *Finite volumes for complex applications VI*, Springer (2011), pp. 673-681.
- [9] A. Tzavaras, Relative entropy in hyperbolic relxation, Comm. Math. Sci, 3 (2005), pp. 119-132.

Joint work with: Clément Cancès, UPMC Univ Paris 06, UMR 7598, Laboratoire Jacques-Louis Lions, F-75005, Paris (cances@ann.jussieu.fr); Frédéric Coquel, INSMI CNRS, École Polytechnique, F-91128, Palaiseau (frederic.coquel@cmap.polytechnique.fr); Edwige Godlewski, UPMC Univ Paris 06, UMR 7598, Laboratoire Jacques-Louis Lions, F-75005, Paris (godlewski@ann.jussieu.fr); Nicolas Seguin, UPMC Univ Paris 06, UMR 7598, Laboratoire Jacques-Louis Lions, F-75005, Paris (nicolas.seguin@upmc.fr)

S32 – Relaxation Processes and Complex Models – Room F, 18.50–19.20

New Entropy Satisfying and Accurate Approximate Riemann Solvers based on the Suliciu Relaxation Approach

Christophe Chalons

Université Paris Diderot-Paris 7 & Laboratoire J.-L. Lions, U.M.R. 7598, UMPC, Boîte courrier 187, 75252 Paris Cedex 05, France. christophe_chalons@ljll.univ-paris-diderot.fr

We are interested in the numerical approximation of the entropy solutions of the gas dynamics equations in Lagrangian and Eulerian coordinates with general pressure closure laws. The celebrated Godunov's method is an example of conservative and entropy satisfying numerical strategy that provides good approximations. A key ingredient of this method is the resolution at each mesh interface of the so-called Riemann problem, which consists in the resolution of the governing equations with an initial condition made of two constant states separated by a discontinuity. In the classical Godunov's method, the Riemann problems are solved exactly, which may be expensive for general pressure laws.

In order to reduce the computational cost of the Godunov's method, Approximate Riemann Solutions (ARS) are introduced and used in place of the exact Riemann solutions. Among the well-known ARS (see for instance [7]), and without any attempt to be exhaustive, are the Harten-Lax-Van Leer's ARS, the Jin-Xin's ARS [8] and the ARS based on a Sulicu's pressure relaxation [9] (see also [5], [4], [3], [1]...) These so-called Approximate Godunov's methods are shown to be positivity preserving and entropy satisfying.

It turns out however that none of these ARS is able to provide exact Riemann solutions in the simple case of an isolated entropy shock wave. More precisely, if the exact Riemann solution simply consists of an isolated shock wave separating two constant states and propagating with a velocity given by the Rankine-Hugoniot relations, then the Approximate Riemann Solution is actually an approximation in the sense that it does not coincide with the exact solution. In this work, our objective is to propose an ARS that is able to exactly calculate such simple solutions.

More precisely, we present a new class of ARS for the gas dynamics equations in Lagrangian and Eulerian coordinates with general pressure laws. The design of these new ARS relies on a generalized Suliciu's pressure relaxation approach. They give by construction the exact solutions for isolated shock discontinuities. They are proved to be positivity preserving and entropy satisfying under a classical CFL restriction. Motivated by a former work [2], all these approximate solutions are used to develop new Godunov-type methods generating infinitely sharp discrete shock profiles. The results extend to the gas dynamics setting a recent work [6] devoted to the Jin and Xin's relaxation method in the scalar framework.

References

- Bouchut F., A reduced stability condition for nonlinear relaxation to conservation laws, J. Hyp. Diff. Eq., vol 1(1), pp 149-170, (2004).
- [2] Chalons C. and Coquel F., Capturing infinitely sharp discrete shock profiles with the Godunov scheme, Proceedings of the Eleventh International Conference on Hyperbolic Problems. S. Benzoni-Gavage and D. Serre (Eds), Springer, pp 363-370 (2008).
- [3] Chalons C. and Coquel F., Navier-stokes equations with several independant pressure laws and explicit predictor-corrector schemes. Numerisch Math, 101(3), pp. 451-478, (2005).
- [4] Chalons C. and Coulombel J.F., Relaxation approximation of the Euler equations. Journal of Mathematical Analysis and Applications, 348(2), pp. 872-893, (2008).
- [5] Coquel F., Godlewski E., In A., Perthame B. and Rascle P., Some new Godunov and relaxation methods for two phase flows, Proceedings of an International conference on Godunov methods : Theory and Applications, Kluwer Academic/Plenum Publishers (2001).
- [6] Coquel F., Jin S., Liu J.-G. and Wang L., Jin-Xin-Glimm scheme for scalar conservation laws, Preprint (2012).
- [7] Godlewsky E. and Raviart P.A., Numerical approximation of hyperbolic systems of conservation laws, Springer (1995).
- [8] Jin S. and Xin Z., The Relaxation Schemes for Systems of Conservation Laws in Arbitrary Space Dimension, Comm. Pure. Appl. Math., vol 48, pp 235-276 (1995).
- [9] Suliciu, I., On the thermodynamics of fluids with relaxation and phase transitions. Fluids with relaxation, Int. J. Engag. Sci., vol 36, pp 921-947 (1998).

Joint work with: Frédéric Coquel (CNRS & Centre de Mathématiques Appliquées, U.M.R. 7641, Ecole Polytechnique, Route de Saclay, 91128 Palaiseau Cedex, France. E-mail: frederic.coquel@cmap.polytechnique.fr)

8.6 Session 33 — Room I — Multi Physics Models IV

S33 – Multi Physics Models IV – Room I, 17.20–17.50

The Riemann problem for three-phase flow in virgin reservoirs for general permeabilities

Pablo Castañeda Instituto Nacional de Matemática Pura e Aplicada (IMPA), Brazil castaneda@impa.br

We focus on a system of two conservation laws representing a large class of models of immiscible flow in porous media relevant for petroleum engineering. The Riemann solutions are found for a range of initial conditions important in applications, involving the injection of two fluids (water, gas) into a horizontal reservoir containing a third fluid (oil) to be displaced.

Despite loss of hyperbolicity, the solution for each data exists and is unique. Also, it depends L_1 continuously on the Riemann data. Such solutions always present a lead shock involving one of the injected fluids and the fluid already present. There is a threshold solution separating solutions according to which of the injected fluids is present in the lead shock.

This class of solution was discovered for a particular model with quadratic permeabilities in [1]. However, general models considered here possess non-local shock curves [2], a novel feature. Another distinction from the previous work is the nature of the threshold solution. In all models the threshold solution consists of a 1-rarefaction starting at the left Riemann datum and finishing at a left 1-characteristic generalized 2-Lax shock that jumps to the right Riemann datum. The particular case presented in [1] was easier, because this threshold lied naturally on a straight line, which is not the case for the general solution presented here. We perform this analysis utilizing results in [3] on the umbilic point.

References

- [1] A.V. Azevedo, A. de Souza, F. Furtado, D. Marchesin and B. Plohr, The solution by the wave curve method of three-phase flow in virgin reservoirs, Transp. Porous Media 83 (2010), pp. 99–125.
- [2] E. Isaacson, D. Marchesin, B. Plohr and B. Temple, Multiphase flow models with singular Riemann problems, Comp. Appl. Math. 11 (1992), pp. 147–166.
- [3] H. Medeiros, Stable hyperbolic singularities for three-phase flow models in oil reservoir simulation, Acta Appl. Math. 28 (1992), pp. 135–159

Joint work with: Frederico Furtado (University of Wyoming, USA) and Dan Marchesin (IMPA, Brazil).

S33 - Multi Physics Models IV - Room I, 17.50-18.20

Application of discontinuous flux for polymer flooding in Multi-dimensional oil reservoir simulation

Sudarshan Kumar Kenettinkara TIFR Centre for Applicable Mathematics Bangalore-560065, India sudarshan@math.tifrbng.res.in

Session 33 — Room I — Multi Physics Models IV

We propose a finite volume method to study the Buckley leverett equation with polymer flooding in the presence of gravity, The system of equations is given by

$$s_t + \nabla \cdot F(s, c, x) = 0$$

(sc)_t + \nabla \cdot (cF(s, c, x)) = 0

where s = s(x,t), c = c(x,t), $(x,t) \in \mathbb{R}^2 \times (0,\infty)$ are saturation of water and concentration of the polymer respectively and the flux function, $F(s,c,x) \in \mathbb{R}^2$ is given by

$$F(s, c, x) = [v(x) - (\rho_w - \rho_o)g\lambda_o(s)K(x)\hat{y}]f(s, c)$$
$$f(s, c) = \frac{\lambda_w(s, c)}{\lambda_w(s, c) + \lambda_o(s)}$$

where ρ_w , ρ_o are the densities of water and oil, g is the acceleration due to gravity and $\hat{y} = [0, 1]$ is the unit vector pointing in the positive y-direction (opposite to gravity), K(x) is the absolute permeability of the rock, $\lambda_w(s, c)$ and $\lambda_o(s, c)$ are mobilities of water and oil respectively. The velocity v which is assumed to be incompressible, is given by

$$v = -(\lambda_w + \lambda_o)K\nabla p - (\lambda_w\rho_w + \lambda_o\rho_o)gK\hat{y}.$$

This problem was studied in [4] numerically in the abscence of gravity. In the presence of gravity, the exact Riemann solver for this problem is too complicated due to the fact that we have to deal with a system of two variables and the permeability K could be discontinuous in the space variable. The presence of gravity also complicates the solution since the flux is no longer monotone and can also change sign. Here by using the idea of discontinuous flux in the space variable [1, 2], a numerical scheme is proposed and implemented. In the equation for s the dependence of the c variables is taken to lead to a discontinuous flux in the space variable. Then a Godunov flux called the DFLU flux can be written down for the discontinuous flux using previous ideas from [1, 2]. The discontinuity in permeability K and the effect of gravity is taken account of in a natural way and does not lead to any complications, with the DFLU flux retaining its very simple form in all cases, and hence being computationally very efficient. Higher order accurate scheme is constructed by introducing slope limiter in space variable and a strong stability preserving Runge-Kutta scheme [3] in the time variable. The resulting schemes are shown to respect a maximum principle. Our numerical results are compared with other standard schemes on some canonical examples like the quarter 4-spot problem. The results from the exact Godunov flux for the system case is shown to be close to the results using the DFLU flux on some 1-D problems. The difficulties of handling the problem in a highly heterogenous media in the presence of gravity attracts the importance of the proposed work.

References

- Adimurthi and J. Jaffré and G. D. Veerappa Gowda, Godunov-type methods for conservation laws with a flux function discontinuous in space, SIAM Journal in Numerical Analysi, 42 (2004), pp. 179-208
- [2] Adimurthi and J. Jaffré and G. D. Veerappa Gowda, Application of the DFLU flux to systems of conservation laws, preprint (2011)
- [3] Sigal Gottlieb And Chi-Wang Shu, Total variation diminishing Runge-Kutta schemes, Mathematics of Computation, (1998), pp.73-85.
- [4] Prabir Daripa, James Glimm, Brent Lindquist and Oliver McBryan, Polymer floods: a case study of nonlinear wave analysis and of instability control in tertiary olil recovery, Siam J.Appl. Math 48.(1988), pp.353-373.
- [5] Thormod Johanson and Ragnar Winther, The solution of Reimann problem for a hyperbolic system of conservation laws modeling polymer flooding, *Siam J.Math. Anal* 19. (1988), pp. 541-566

Joint work with: G D Veerappa Gowda (TIFR Centre for Applicable Mathematics), Praveen. C (TIFR Centre for Applicable Mathematics).

* * * ------

S33 – Multi Physics Models IV – Room I, 18.20–18.50

Spectral WENO schemes with Adaptive Mesh Refinement for multi-species kinematic flow models

Raimund Bürger Universidad de Concepción, Concepción, Chile rburger@ing-mat.udec.cl

The sedimentation of a polydisperse suspension with particles belonging to N size classes (species) can be described by a system of N nonlinear, strongly coupled scalar first-order conservation laws [1]. Its solutions usually exhibit kinematic shocks separating areas of different composition. A similar system of conservation laws is given by the multi-class Lighthill-Whitham-Richards (MCLWR) traffic flow model [2, 3]. Based on the so-called secular equation [4], which provides access to the spectral decomposition of the Jacobian of the flux vector for this class of models, Bürger et al. [5] proposed a spectral weighted essentially non-oscillatory (WENO) scheme for the numerical solution of the model. In [6] it is demonstrated that the efficiency of this scheme can be improved by the technique of Adaptive Mesh Refinement (AMR), which concentrates computational effort on zones of strong variation. Numerical experiments for the cases N = 4 and N = 7 for the polydisperse sedimentation model [6], and some examples for the MCLWR model are presented.

References

- [1] R. Bürger, R. Donat, P. Mulet and C.A. Vega, Hyperbolicity analysis of polydisperse sedimentation models via a secular equation for the flux Jacobian, *SIAM J. Appl. Math.*, **70** (2010), pp. 2186–2213.
- [2] S. Benzoni-Gavage and R.M. Colombo, An n-populations model for traffic flow, Eur. J. Appl. Math., 14 (2003), pp. 587–612.
- [3] G.C.K. Wong and S.C. Wong, A multi-class traffic flow model—an extension of LWR model with heterogeneous drivers, *Transp. Res. A*, 36 (2002), pp. 827–841.
- [4] J. Anderson, A secular equation for the eigenvalues of a diagonal matrix perturbation, *Lin. Alg. Appl.*, 246 (1996), pp. 49–70.
- [5] R. Bürger, R. Donat, P. Mulet and C.A. Vega, On the implementation of WENO schemes for a class of polydisperse sedimentation models, J. Comput. Phys., 230 (2011), pp. 2322–2344.
- [6] R. Bürger, P. Mulet and L.M. Villada, Spectral WENO schemes with Adaptive Mesh Refinement for models of polydisperse sedimentation. Preprint 2011-30, Centro de Investigación en Ingeniería Matemática, Universidad de Concepción; submitted.

Joint work with: Pep Mulet (Universitat de València, Valencia, Spain) and Luis Miguel Villada (Universidad de Concepción, Concepción, Chile).

S33 – Multi Physics Models IV – Room I, 18.50–19.20

Stability, instability and symmetry-breaking bifurcations for the stationary states of the one-dimensional NLS with a defect

Riccardo Adami Department of Mathematics and Applications, University of Milan Bicocca riccardo.adami@unimib.it

One-dimensional nonlinear Schrödinger equation is currently used in order to describe the dynamics of the so-called cigar-shaped Bose-Einstein condensates. These are ultracold boson gases confined in elongated optical or magnetic traps, in which all particles lie in the same quantum state. Their effective dynamics han been proved by Lieb end Seiringer to be one-dimensional. The action of possible inhomogeneities or impurities can be modeled by adding a point interaction, namely a pointwise condition on the wave function and on its derivative. Such a condition must fulfil some consistency requirements, in order to guarantee the conservation of the number of particles and of a suitable notion of energy. Such requirements are satisfied by the whole class of linear one-dimensional point interactions, defined in the linear case as the self-adjoint extensions of the laplacian restricted to functions that vanish in some neighbourhood of the origin, and naturally generalized to the NLS.

The resulting system is usually referred to as the NLS with a defect and shares with the corresponding linear problem the useful feature of being exactly solvable: stationary states and their energies can be explicitly computed.

Depending on the particular choice of the point interaction, the system can exhibit the so-called defect modes, namely, nonlinear stationary states that have no counterpart in the ordinary NLS. Some of such modes can be interpreted as nonlinear deformations of a corresponding bound state already present in the linear Schrödinger equation with the same defect.

Once established the existence of the defect modes, it is natural to investigate whether they are stable or not. If we restrict our scope to power focusning nonlinearities and nonlinear ground states, then the analysis becomes simpler. Beyond the well-known case of the Dirac's delta defect, treated in a series of papers by Fukuizumi, Ohta, Ozawa, Jeanjean, Fibich, Ksherim Sivan, Le Coz, we explicitly studied the cases of a resonant point interaction, recently highlighted in the literature by Golovaty and Hryniv, and the case of a delta prime-defect.

The latter revealed a particularly rich structure in the family of stationary states, provided with a pitchfork bifircation with spontaneous symmetry breaking in the ground state. This analysis has been partially extended to the case of graphs. In order to establish the existence and the stability properties of ground states we used both Grillakis-Shatah-Strauss' theory on linear stability and Lions' concentration-compactness method.

References

- Adami R., Cacciapuoti C., Finco D., Noja D., Stationary states of NLS on star graphs, preprint, arXiv:1104.3839 (submitted) (2011).
- [2] Adami R., Noja D.: Existence of dynamics for a 1-d NLS equation perturbed with a generalized point defect, J. Phys. A Math. Theor. 42, 49, (2009) 495302, 19pp.
- [3] Adami R., Noja D.: Stability and symmetry breaking bifurcation for the ground states of a NLS equation with a δ' interaction, arXiv:1112.1318, submitted (2011).
- [4] Adami R., Noja D., Sacchetti A., On the mathematical description of the effective behaviour of onedimensional Bose-Einstein condensates with defects, Chapter in the book Bose-Einstein Condensates: Theory, Characteristics, and Current Research, Nova Publisher (2010).

Joint work with: Claudio Cacciapuoti, (Hausdorff Institut, Universität Bonn), Domenico Finco, (Università Telematica Nettuno), Diego Noja (Università di Milano Bicocca), Nicola Visciglia (Università di Pisa).

8.7 Session 34 — Room A — Theory of Conservation Laws IV

S34 - Theory of Conservation Laws IV - Room A, 17.20-17.50

Selective relaxation model for general fluid systems

Edwige Godlewski

UPMC Univ Paris 06, UMR 7598, Laboratoire Jacques-Louis Lions, F-75005, Paris, France CNRS, UMR 7598, Laboratoire Jacques-Louis Lions, F-75005, Paris, France edwige.godlewski@upmc.fr

Following a previous work [1] on general *fluid models* as described by Després in [4], we propose a relaxation framework for these fluid models, among which we find Euler system and the ideal MHD. Our approach may be seen as a natural extension of the Suliciu approach [5]. In particular the relaxation is selective. Indeed, the relaxation approximation procedure of the original system may be performed field by field, in order to preserve some exact waves of interest. Then, the (non linear) relaxation system may involve only degenerate fields. A situation we shall refer to as "totally linearly degenerate".

Following Yong's results [6][7], several stability properties are proved in order to justify the relaxation procedure and its efficiency in the numerical approximation of the entropy weak solutions of the *equilibrium* system, i.e., of the original nonlinear system of PDEs.

The Godunov scheme is efficient in the case of a totally linearly degenerate system for which the solution of the Riemann problem is explicit and the resulting scheme for the fluid system is particularly simple. Indeed, the Godunov solver for the homogeneous relaxation system results in an HLLC-type solver for the equilibrium system which satisfies discrete entropy inequalities under a natural Gibbs principle, valid under natural subcharacteristic conditions.

Moreover, the equivalence between the Eulerian and Lagrangian frames permits to derive a numerical method for the system in Eulerian coordinates with the same nice properties.

References

- Ambroso, A., Chalons, C., Coquel, F., Godlewski, E., Lagoutière, F., Raviart, P.-A. and Seguin, N. Coupling of general Lagrangian systems, *Math. Comp.* Volume no. 77 (2008), 909–941
- [2] Chalons, C. and Coulombel, J.-F. Relaxation approximation of the Euler equations, J. Math. Anal. Appl. Volume no. 348 (2008), pp. 872-893
- [3] Coquel, F. Godlewski, E. and Seguin, N. Relaxation of fluid systems, to appear in Mathematical Models & Methods In Applied Sciences, DOI No: 10.1142/S0218202512500145 Accepted 2011-11-06
- [4] Després, B. Lagrangian systems of conservation laws. Invariance properties of Lagrangian systems of conservation laws, approximate Riemann solvers and the entropy condition, *Numer. Math.* Volume no. 89 (2001), pp. 99–134
- [5] Suliciu, I. On the thermodynamics of fluids with relaxation and phase transitions. Fluids with relaxation, in *Internat. J. Engrg. Sci.*, Volume no. 36 (1998), pp.921-947
- [6] Yong, W.-A. Singular perturbations of first-order hyperbolic systems with stiff source terms, Journal of Differential Equations Volume no.155 (1999) pp. 89-132
- [7] Yong, W.-A. Entropy and global existence for hyperbolic balance laws, Arch. Rational. Mech. Anal. 172 (2004) pp. 247–266

Joint work with: Frédéric Coquel (CNRS, UMR 7641, CMAP Ecole Polytechnique, Palaiseau), frederic.coquel@cmap.polytechnique.fr, Nicolas Seguin (UPMC Univ Paris 06, UMR 7598, Laboratoire Jacques-Louis Lions, F-75005, Paris, France CNRS, UMR 7598, Laboratoire Jacques-Louis Lions, F-75005, Paris, France, nicolas.seguin@upmc.fr S34 - Theory of Conservation Laws IV - Room A, 17.50-18.20

Blow up at the hyperbolic boundary for a system arising from chemical engineering

Marguerite Gisclon LAMA, CNRS UMR 5127, Université de SAVOIE, FRANCE gisclon@univ-savoie.fr

We investigate a model arising in chemical engineering and related to gas chromatography. This model describes an isothermal adsorption process of separation of a gaseous mixture. The common velocity of the various species is not constant because the sorption effect is taken into account. We give first some results in the case of two active components for the Cauchy problem in one dimension. Exchanging the roles of the space and the time variables we obtain a strictly hyperbolic system with a zero eigenvalue. Using a Godunov type scheme and the Front Tracking Algorithm (cf [4]) we prove existence and uniqueness in the class of piecewise smooth functions and we find all the entropies (cf [1]-[2]). Our aim (cf [3]) is to construct a solution with a velocity which blows up at the corresponding characteristic "hyperbolic boundary" (t = 0). It is already known that systems of two hyperbolic conservation laws may blow up in the L^{∞} norm and, in literature, there are examples of blow up for one dimensional strictly hyperbolic systems of PDE's with at least 3 equations. There is no example for a 2×2 strictly hyperbolic system, except the example built by Robin Young (cf [5]), involving two Burgers equations linearly coupled at the two boundaries. All examples occur in cases where strict hyperbolicity is lost as the solution explodes. In our example, we also loose the strict hyperbolicity but the blow up takes place only at the characteristic boundary which becomes twice characteristic and only the velocity blows up. Our example, although artificial, comes from a realistic chemical model. It illustrates what may occur when BV regularity is not ensured for the velocity at the physical boundary.

References

- C. Bourdarias and M. Gisclon and S. Junca, Existence of weak entropy solutions for gas chromatography system with one or two actives species and non convex isotherms, *Commun. Math. Sci.*, Volume no. 5 (2007), pp 67-84
- [2] C. Bourdarias and M. Gisclon and S. Junca, Strong stability with respect to weak limits for a hyperbolic system arising from gas chromatography, *Methods Appl. Anal.*, Volume no. 17 (2010), pp 301-330
- [3] C. Bourdarias and M. Gisclon and S. Junca, Blow up at the hyperbolic boundary for a 2x2 system arising from chemical engineering, J. Hyperbolic Differ. Equ., Volume no. 7 (2010), pp 295-316
- [4] A. Bressan and P. Goatin. Stability of L[∞] solutions of Temple class systems, Differential Integral Equation, Volume no. 13 (2000), pp 1503-1528
- [5] R.Young. Blow up of solutions and boundary instabilities in nonlinear hyperbolic equations, Commun. Math. Sci., Volume no. 1 (2003), pp 269-292

Joint work with: Christian Bourdarias (LAMA, Université de Savoie), Stéphane Junca (Université de Nice).

S34 - Theory of Conservation Laws IV - Room A, 18.20-18.50

Using Geometric Singular Perturbation Theory to Understand Singular Shocks

Barbara Lee Keyfitz

Department of Mathematics, The Ohio State University bkeyfitz@math.ohio-state.edu

There are classes of conservation laws which do not possess Riemann solutions of the standard type (composed of shocks, rarefactions and linear waves), even in regions where the equations are strictly hyperbolic and genuinely nonlinear. This is a "large data" phenomenon. For some systems, candidates for solutions of lower regularity, now called singular shocks, have been postulated [3,4,8]. By means of singular shocks, Riemann problems can be resolved. However, it is unclear in what sense singular shocks satisfy the conservation law. In this talk, I expand on work by Stephen Schecter [7] which uses Geometric Singular Perturbation Theory (GSPT) [2] to prove that approximations to singular shocks satisfy the self-similar Dafermos-DiPerna regularization [1] of the conservation law system [5]. In addition to demonstrating a mechanism for the approximation, GSPT also demonstrates the detailed structure of singular shock profiles. Some examples include the classic model, which gave rise to the discovery of singular shocks, of gas dynamics with the wrong variables conserved. In addition, singular shocks occur in a recent model in chromatography [6], and GSPT solves this model.

References

- C. M. Dafermos and R. J. DiPerna, The Riemann problem for certain classes of hyperbolic systems of conservation laws. J. Differential Equations 20 (1976), 90–114.
- [2] C. K. R. T. Jones, Geometric singular perturbation theory. Dynamical systems (Montecatini Terme, 1994), Lecture Notes in Mathematics, Vol. 1609, Springer, Berlin, 1995, pp. 44–118.
- [3] B. L. Keyfitz and H. C. Kranzer, A viscosity approximation to a system of conservation laws with no classical Riemann solution. In *Nonlinear Hyperbolic Problems (Bordeaux, 1998)*, (eds. C. Carasso *et al.*), Lecture Notes in Mathematics, Vol. **1402**, Springer, Berlin, 1989, pp. 185–197.
- [4] B. L. Keyfitz and H. C. Kranzer, Spaces of weighted measures for conservation laws with singular shock solutions, J. Differential Equations, 118 (1995), 420–451.
- [5] B. L. Keyfitz and C. Tsikkou, Conserving the Wrong Variables in Gas Dynamics: A Riemann Solution with Singular Shocks, Q. Applied Mathematics, to appear.
- [6] M. Mazzotti, Non-classical composition fronts in nonlinear chromatography Delta-shock, Indust. & Eng. Chem. Res., 48 (2009), 7733–7752.
- [7] S. Schecter, Existence of Dafermos profiles for singular shocks, J. Differential Equations 205 (2004), 185-210.
- [8] M. Sever, Distribution solutions of nonlinear systems of conservation laws, Memoirs of the AMS, 889 (2007), 1–163.

Ting-Hao Hsu (The Ohio State University), and Charis Tsikkou (The Ohio State University)

* * *

S34 - Theory of Conservation Laws IV - Room A, 18.50-19.20

Global existence of strong solution for shallow water system with large initial data on the irrotational part

Boris Haspot

Ceremade UMR CNRS 7534 Université de Paris Dauphine, Place du Marchal DeLattre De Tassigny 75775 PARIS CEDEX 16 haspot@ceremade.dauphine.fr

The motion of a general barotropic compressible fluid is described by the following system:

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho u) = 0, \\ \partial_t(\rho u) + \operatorname{div}(\rho u \otimes u) - \operatorname{div}(\mu(\rho)D(u)) - \nabla(\lambda(\rho)\operatorname{div} u) + \nabla P(\rho) = 0, \\ (\rho, u)_{t=0} = (\rho_0, u_0). \end{cases}$$
(1)

Here $u = u(t, x) \in \mathbb{R}^N$ stands for the velocity field, $\rho = \rho(t, x) \in \mathbb{R}^+$ is the density and $D(u) = \frac{1}{2}(\nabla u + t\nabla u)$. We denote by λ and μ the two viscosity coefficients of the fluid, which are assumed to satisfy $\mu > 0$ and $\lambda + 2\mu > 0$. Such a condition ensures ellipticity for the momentum equation and is satisfied in the physical cases where $\lambda + \frac{2\mu}{N} > 0$. In the sequel we shall only consider the shallow-water system which corresponds to:

 $\mu(\rho) = \mu \rho \text{ with } \mu > 0 \text{ and } \lambda(\rho) = 0.$

We supplement the problem with initial condition (ρ_0, u_0) . Throughout the paper, we assume that the space variable $x \in \mathbb{R}^N$ or to the periodic box \mathbb{T}_a^N with period a_i , in the i-th direction. We restrict ourselves to the case $N \geq 2$.

We are interested in showing the existence of global strong solutions with large initial data on the irrotational part for the shallow-water system. We introduce a new notion of *quasi-solutions* (see [BH,BH1]) when the initial velocity is assumed to be irrotational, these last one exhibit regularizing effects both on the velocity and in a very surprising way also on the density (indeed the density is a priori governed by an hyperbolic equation). We would like to point out that this smoothing effect is purely non linear and is absolutely crucial in order to deal with the pressure term as it provides new damping effects in high frequencies on the density. More precisely we can verifies that $(\rho_1, -\mu\nabla \ln \rho_1)$ is a *quasi solution* of the system (1) when P = 0 and with ρ_1 which checks an heat equation. The rest of the proof consists in working around, more precisely we shall construct solution of the form $(\rho, u) = (\rho_1 + h_2, -\mu\nabla \ln \rho_1 + u_2)$. The main difficulty consists in proving the existence of global strong solution for the system verified by (h_2, u_2) (we are going to follow some of the ideas of [CD,arma]), in particular we are going to use the notion of *effective velocity* in order to obtain suitable damping effects on the density h_2 in high frequencies (we refer to [arma]).

Our result gives a first kind of answer to the problem of the existence of global weak solution for the shallowwater system in dimension $N \ge 2$ (which is actually open). We conclude by giving new pointwise decay estimates on the solution and some new blow-up results for the shallow water system (1) depending only on the behavior of the density.

References

- [CD] F. Charve and R. Danchin, A global existence result for the compressible Navier-Stokes equations in the critical L^p framework, Archive for Rational Mechanics and Analysis, 198(1), 2010, 233-271.
- [arma] B. Haspot, Existence of global strong solutions in critical spaces for barotropic viscous fuids, Archive for Rational Mechanics and Analysis, Volume 202, Issue 2 (2011), Page 427-460.
- [BH] B. Haspot, Existence of strong global solutions for the shallow-water equations with large initial data, preprint arXiv:1110.6100.
- [BH1] B. Haspot, Global existence of strong solution for shallow water system with large initial data on the irrotational part, preprint arXiv:1201.5456.

8.8 Session 35 — Room D — Kinetic Models II

S35 – Kinetic Models II – Room D, 17.20-17.50

The Limit of the Boltzmann Equation to the Euler Equations for Riemann Problems

Yi Wang

Institute of Applied Mathematics, AMSS, Chinese Academy of Sciences, Beijing, China wangyi@amss.ac.cn

The convergence of the Boltzmann equaiton to the compressible Euler equations when the Knudsen number tends to zero has been a long standing open problem in the kinetic theory. In the setting of Riemann solution that contains the generic superposition of shock, rarefaction wave and contact discontinuity to the Euler equations, we succeed in justifying this limit by introducing hyperbolic waves with different solution backgrounds to capture the extra masses carried by the hyperbolic approximation of the rarefaction wave and the diffusion approximation of contact discontinuity.

References

- [1] Feimin Huang, Yi Wang, Yong Wang and Tong Yang, The Limit of the Boltzmann Equation to the Euler Equations for Riemann Problems, http://arxiv.org/abs/1109.6751.
- [2] Feimin Huang, Yi Wang, Tong Yang, Vanishing Viscosity Limit of the Compressible Navier-Stokes Equations for Solutions to a Riemann Problem, Archive for Rational Mechanics and Analysis, 203 (2012), 379-413.
- [3] Feimin Huang, Yi Wang, Tong Yang, Fluid Dynamic Limit to the Riemann Solutions of Euler Equations: I. Superposition of rarefaction waves and contact discontinuity, Kinet. Relat. Models, 3 (2010), 685C728.
- [4] Feimin Huang, Yi Wang, Tong Yang, Hydrodynamic limit of the Boltzmann equation with contact discontinuities, Comm. Math. Phy., 295 (2010), 293C326.

Joint work with: Feimin Huang(Institute of Applied Mathematics, AMSS, Chinese Academy of Sciences, Beijing, China), Yong Wang (Institute of Applied Mathematics, AMSS, Chinese Academy of Sciences, Beijing, China), Tong Yang (Department of Mathematics, City University of Hong Kong, Hong Kong, China)

* * * ------

S35 – Kinetic Models II – Room D, 17.50–18.20

A non singular Vlasov equation for magnetic plasmas

Frédérique Charles LJLL, Université Pierre et Marie Curie (Paris 6), UMR 7598, 4 place Jussieu, 75252 Paris Cedex 05. frederique.charles@ann.jussieu.fr The mathematical description of laboratory fusion plasmas produced in Tokamaks is challenging. Therefore it is useful to understand simplified models. Here we consider one of those which keeps both the complexity of the Vlasov equation for ions and the Hall effect in Maxwell's equation. Based on energy dissipation, a fundamental physical property, we can show that the model is nonlinear stable.

The model is

$$(-\lambda^2 \Delta \ln n_e = n_I - n_e, \tag{a}$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \wedge \left(\frac{1}{n_e} n_I \ \mathbf{u}_I \wedge \mathbf{B}\right) + \nabla \wedge \left(\frac{1}{n_e} \mathbf{J} \wedge \mathbf{B}\right) + \nabla \wedge \left(\eta \nabla \wedge \mathbf{B}\right) = 0, \qquad (b)$$

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{\partial}{\partial \mathbf{v}} \left[\left(\left(-\frac{T_{\mathbf{e}}}{n_e} \nabla n_e + \frac{\mathbf{J} - n_I \mathbf{u}_I}{n_e} \wedge \mathbf{B} \right) + \mathbf{v} \wedge \mathbf{B} \right) f \right] = 0, \quad (c)$$
$$\nabla \cdot \mathbf{B} = 0 \quad (d).$$

where the following notations are used

$$\mathbf{J} = \nabla \wedge \mathbf{B} \qquad \text{(the electric current)},$$

$$n_I(t, x) = \int_{\mathbb{R}^3} f(t, x, \mathbf{v}) d\mathbf{v} \qquad \text{(the number density in ions)}, \qquad (2)$$

$$f(t, x) \mathbf{u}_I(t, x) = \int_{\mathbb{R}^3} f(t, x, \mathbf{v}) \mathbf{v} d\mathbf{v} \qquad \text{(the macroscopic velocity of ions)} \qquad (3)$$

$$n_I(t,x)\mathbf{u}_I(t,x) = \int_{\mathbb{R}^3} f(t,x,\mathbf{v})\mathbf{v}d\mathbf{v} \qquad \text{(the macroscopic velocity of ions)}.$$
 (3)

Here η is a positive bounded function which here describes the plasma resistivity, the constant $T_{\rm e}$ is the mean electron temperature: that is η and $T_{\rm e}$ are parameters of the model. The (small) Debye length is λ .

Mathematical analysis shows that this model is weakly continuous. It is used to build up an approximate solution which converges to a weak solution since some sharp a priori estimates are satisfied. The mathematical analysis shows the major asset of keeping the inverse of the electronic density $\frac{1}{n_e}$ instead of using the trivial but dangerous simplification $\frac{n_I}{n_e} \approx 1$ in some parts of the model: in particular n_e is bounded away from zero and is more regular than n_I ; this is the explanation why this model is non singular. It is key properties to obtain the theoretical results.

References

[1] F. Charles, B. D., B. Perthame, R. Sentis Nonlinear stability of a Vlasov equation for magnetic plasmas in preparation.

Joint work with: Bruno Després (LJLL-UPMC), Benoit Perthame (LJLL-UPMC) and Rémi Sentis (CEA-DIF)

_____ * * * _____

S35 – Kinetic Models II – Room D, 18.20-18.50

Boundary singularity for Boltzmann equation

I-Kun Chen Academia Sinica ikunchen@math.sinica.edu.tw

In Kinetic Theory, it is observed that there is a logarithmic singularity for the fluid velocity around the solid boundary. This is understood in WKB model also supported by computations for linearized Boltzmann equation. The goal of this talk is to confirm this basic phenomenon, under the setting of thermal transpiration problem,

(1)

for linearized Boltzmann equation for sufficiently large Knudsen number. In addition, we improve the solution to Gaussian-Like. We use an iterated scheme, with the "gain" part of the collision operator as a source. The scheme yields an explicit leading term. The remaining converging terms are estimated through a refined pointwise estimate and Maxwellian upper bound for the gain part. Our analysis is motivated by the previous studies of asymptotic and computational analysis.

Joint work with: Tai-Ping Liu (Academia Sinica), Shigeru Takata (Kyoto University)

S35 – Kinetic Models II – Room D, 18.50–19.20

Hydrodynamic limit of the Gross-Pitaevskii equation

Kung-Chien Wu University of Cambridge kcw28@dpmms.cam.ac.uk

In this talk, we consider the hydrodynamic limit of the Gross-Pitaevskii equation with general initial data and nonconstant density, the limit equation is the anelastic system (generalized incompressible Euler equation) plus a fast singular oscillating term.

References

- H.L. Li and C.K. Lin, Zero Debye length asymptotic of the quantum hydrodynamic model of semiconductors, *Commun. Math. Phys.*, 256 (2005), pp. 195–212.
- [2] C. K. Lin and K. C. Wu, Hydrodynamic limits of the nonlinear Klein-Gordon equation, J. Math Pures Appl.(2012), doi:10.1016/j.matpur.2012.02.002.
- [3] Chi-Kun Lin and K.C. Wu, Semiclassical limit of the NLS for general initial data and nonconstant density, in preparation.
- [4] N. Masmoudi, Incompressible, inviscid limit of the compressible Navier-Stokes system., Ann. Inst. Henri Poincare, Anal. non lineaire 18 (2001), pp. 199–224.
- [5] K.C. Wu, Asymptotic limit of the QHD model for general initial data and nonconstant density, in preparation.

Joint work with: Chi-Kun Lin (National Chiao-Tung University)
8.9 Session 36 — Room B — Control and Geometric Problems for Hyperbolic Equations

S36 – Control and Geometric Problems for Hyperbolic Equations – Room B, 17.20–17.50

Optimal control of re-entrant manufacturing systems

Matthias Kawski Arizona State University kawski@asu.edu

Highly re-entrant semiconductor manufacturing systems may be modeled controlled hyperbolic conservation laws

$$\partial_t \rho(t, x) + \partial_x \left(\lambda(W(t)) \,\rho(t, x) \right) = 0 \quad \text{with} \quad W(t) = \int_0^1 \rho(t, x) \, dx, \tag{1}$$

on the semi-infinite strip $[0, \infty) \times [0, 1]$ with velocity $\lambda(\cdot) \in C^1([0, +\infty); (0, +\infty))$. For fixed initial data $\rho(0, x) = \rho_0(x)$, $0 \le x \le 1$ the control input is the influx $u(t) = \lambda(W(t))\rho(t, 0)$, $t \ge 0$. The natural control objective is to minimize the error signal between a given demand forecast y_d and the out-flux $y(t) = \lambda(W(t))\rho(t, 1)$, or in the case of perishable demand, to minimize the alternate error signal

$$\beta(t) = \int_0^t y_d(s) \, ds - \int_0^t \lambda(W(s)) \rho(s, 1) \, ds, \tag{2}$$

while keeping the state $\rho(\cdot, x)$ bounded.

Extending results presented in [1], we analyze the optimal controls for minimizing the error signal in the L^1 -norm, and investigate using a *push-pull-point* as an additional control for systems which may be modeled as coupled two-stage systems of the above form.

References

 Jean-Michel Coron, Matthias Kawski, and Zhiqiang Wang, Analysis of a conservation law modeling a highly re-entrant manufacturing system, *Discrete and Continuous Dynamical Systems - Series B*, vol. 14 no. 4 (2010) pp. 1337 - 1359.

S36 - Control and Geometric Problems for Hyperbolic Equations - Room B, 17.50-18.20

Asymptotic stabilization of the hyperelastic-rod wave equation

Giuseppe Maria Coclite University of Bari coclitegm@dm.uniba.it We investigate the problem of asymptotic stabilization of the hyperelastic-rod wave equation on the real line

$$\partial_t u - \partial_{txx}^3 u + 3u \partial_x u = \gamma \left(2 \partial_x u \, \partial_{xx}^2 u + u \, \partial_{xxx}^3 u \right) \quad t > 0, \ x \in \mathbb{R},$$
(1)

where u(t, x) represents the radial deformation in a cylindrical compressible hyperelastic rod, and $\gamma \in \mathbb{R}$ is some given constant depending on the material and on the prestress of the rod (see Dai [3, 4, 5]). Observe that if $\gamma = 1$, then (1) is the classical Camassa–Holm equation [2, 6] modelling the propagation of unidirectional shallow water waves on a flat bottom.

The asymptotic stabilizability of the Camassa–Holm equation through a stationary feedback law was recently established, within the space of H^2 solutions, in [7] by means of a forcing term acting as a control, and in [8] by means of a boundary feedback. Here, we assume $\gamma > 0$, and consider the equation (1) with an additional force term of the form

$$f: H^1(\mathbb{R}) \to H^{-1}(\mathbb{R}), \qquad f[u] = -\lambda(u - \partial_{xx}^2 u),$$

for some $\lambda > 0$. With the same approach of [1], we show the existence of a semigroup of global weak dissipative solutions of the corresponding closed-loop system

$$\partial_t u - \partial_{txx}^3 u + 3u \partial_x u = \gamma \left(2 \partial_x u \, \partial_{xx}^2 u + u \, \partial_{xxx}^3 u \right) + f[u] \qquad t > 0, \ x \in \mathbb{R},$$

$$\tag{2}$$

defined for every initial data $u_0 \in H^1(\mathbb{R})$, and we prove that any such solution decays esponentially to 0 as $t \to \infty$.

References

- A. Bressan and A. Constantin, Global dissipative solutions of the Camassa-Holm equation, Anal. Appl. (Singap.), 5 (2007), no. 1, pp. 1–27.
- [2] R. Camassa and D. D. Holm, An integrable shallow water equation with peaked solitons, *Phys. Rev. Lett.*, **71** (1993), pp. 1661-1664.
- [3] H.-H. Dai, Exact travelling-wave solutions of an integrable equation arising in hyperelastic rods, Wave Motion, 28 (1998), pp. 367-381.
- [4] H.-H. Dai, Model equations for nonlinear dispersive waves in a compressible Mooney-Rivlin rod, Acta Mech., 127 (1998), pp. 193-207.
- [5] H.-H. Dai and Y. Huo, Solitary shock waves and other travelling waves in a general compressible hyperelastic rod, R. Soc. Lond. Proc. Ser. A, 456 (2000), pp. 331-363.
- [6] R. S. Johnson, Camassa-Holm, Korteweg-de Vries and related models for water waves. J. Fluid Mech., 455 (2002), pp. 63–82.
- [7] O. Glass, Controllability and asymptotic stabilization of the Camassa-Holm equation. J. Differential Equations, 245 (2008), no. 6, pp. 1584–1615.
- [8] V. Perrollaz, Initial boundary value problem and asymptotic stabilization of the Camassa-Holm equation on an interval. J. Funct. Anal., 259 (2010), no. 9, pp. 2333–2365.

Joint work with: Fabio Ancona (University of Padova)

- * * * -

S36 - Control and Geometric Problems for Hyperbolic Equations - Room B, 18.20-18.50

Extensions for systems of conservation laws

Irina A. Kogan North Carolina State University iakogan@ncsu.edu

A frame of eigenvectors for the Jacobian Df of the flux f, called eigenfarme, plays an important role in the analysis of hyperbolic conservation laws $u_t + f(u)_x = 0$ in one space variable. We start by reviewing our earlier results on constructing systems with a prescribed eigenframe [1]. Those systems of conservation laws are determined by solving a certain algebraic-differential " λ -system" for the associated eigenvlaues. We next consider the question of how many extensions the resulting systems have. We show how these can be determined by solving a related algebraic-differential " β -system." The unknowns in the latter system are the lengths of the given eigenvectors as measured with the metric determined by an extension. From these lengths the extension itself can be found by quadrature. Our analysis goes one step further than determining whether extensions exist or not. By analyzing the β -systems we obtain information about how many extensions there are. More precisely, we determine on how many arbitrary functions and/or constants extensions of any conservative system with a prescribed eigenframe depend.

- Helge Kristian Jenssen and Irina A. Kogan, Conservation Laws with Prescribed Eigencurves, J. of Hyperbolic Differential Equations, Volume 7, no.2 (2010), pp. 211-254
- [2] Helge Kristian Jenssen and Irina A. Kogan, Extensions for systems of conservation laws, *Communications in PDEs*, to appear.

Joint work with: Helge Kristian Jenssen (*Pennsylvania State University*)

S36 - Control and Geometric Problems for Hyperbolic Equations - Room B, 18.50-19.20

On a nonlocal hyperbolic conservation law arising from a gradient constraint problem

Paulo Amorim CMAF - University of Lisbon pamorim@ptmat.fc.ul.pt

In some models involving nonlinear conservation laws, physical mechanisms exist which prevent the formation of shocks. This gives rise to conservation laws with a constraint on the gradient of the solution. We approach this problem by studying a related conservation law with a spatial nonlocal term. We prove existence, uniqueness and stability of solution of the Cauchy problem for this nonlocal conservation law. In turn, this allows us to provide a notion of solution to the conservation law with a gradient constraint. The proof of existence is based on a time-stepping technique, and an L^1 -contraction estimate follows from stability results of Karlsen and Risebro.

9 Abstracts of contributed lectures — Thursday 9.15–9.45

9.1 Session 37 — Room F — Numerical Methods XIII

S37 – Numerical Methods XIII – Room F, 9.15–9.45

Analysis of Asymptotic Preserving schemes with the modified equation

Bruno Després *LJLL-UPMC* despres@ann.jussieu.fr

Introduction

Consider the hyperbolic heat equation in dimension one $x \in \mathbb{R}$

$$\partial_t u_{\varepsilon} + \frac{1}{\varepsilon} \partial_x v_{\varepsilon} = 0, \quad \partial_t v_{\varepsilon} + \frac{1}{\varepsilon} \partial_x u_{\varepsilon} = -\frac{\sigma}{\varepsilon^2} v_{\varepsilon}. \tag{1}$$

To fix the notation with will consider that $0 < \varepsilon \le 1$ is the scaling parameter which can take value arbitrarily in [0,1]. The other coefficient is $0 < \sigma \le 1$. The formal asymptotic limit of (1) writes $\partial_t u - \frac{1}{\sigma} \partial_{xx} u = 0$.

Asymptotic Preserving techniques [1], [2] and [4] are very useful to control the accuracy of discretization methods in transitional regimes where ε covers the whole range [0, 1]. This family of methods and schemes are particularly appealing for the numerical discretization of physical problems with very different scales, from microscopic scale to macroscopic scale. Essentially it amounts to design numerical methods such that the numerical error goes to zero with the mesh size Δx uniformly with respect to the small parameter ε . However the a priori understanding of the structures of these numerical methods is not so easy.

This presentation will be devoted to show that the modified system

$$\begin{cases}
\partial_t \widehat{u}_{\alpha,\varepsilon} + \frac{M}{\varepsilon} \left(\partial_x \widehat{v}_{\alpha,\varepsilon} - \alpha \partial_{xx} \widehat{u}_{\alpha,\varepsilon} \right) = 0, \\
\partial_t \widehat{v}_{\alpha,\varepsilon} + \frac{M}{\varepsilon} \left(\partial_x \widehat{u}_{\alpha,\varepsilon} - \alpha \partial_{xx} \widehat{v}_{\alpha,\varepsilon} \right) = -\frac{\sigma M}{\varepsilon^2} \widehat{v}_{\alpha,\varepsilon},
\end{cases}$$
(2)

where the "Magic" coefficient is defined by $M = \frac{\varepsilon}{\varepsilon + \sigma \alpha}$ displays interesting theoretical properties that can be used to reach a better understanding of A.P. schemes for (1). The coefficient α stands for the underlying numerical diffusion, that is $\alpha \approx \frac{\Delta x}{2}$ in the context of the modified equation.

The main theoretical result is the following: for well prepared data, solutions of (2) are uniformly close to solutions of the initial system (1) in the sense that

$$\|u_{\varepsilon}(t) - \widehat{u}_{\alpha,\varepsilon}(t)\|_{L^{2}(\mathbb{R})} + \|v_{\varepsilon}(t) - \widehat{v}_{\alpha,\varepsilon}(t)\|_{L^{2}(\mathbb{R})} \le C\alpha, \qquad t \le T.$$
(3)

This estimate is uniform with respect to α and show no dependency with respect to any negative powers of ε : that is C is bounded from above independently of ε . We will show that the Gosse-Toscani [1] is naturally compatible with (2). We refer to the recent work [3] for a fully discrete proof of (3). Numerical examples will illustrate the theoretical properties.

References

- L. Gosse, G. Toscani An asymptotic-preserving well-balanced scheme for the hyperbolic heat equations C. R. Acad. Sci Paris, Ser. I 334 (2002) 337-342.
- [2] S. Jin, D. Levermore Numerical schemes for hyperbolic conservation laws with stiff relaxation terms. JCP 126,449-467, 1996.
- [3] C. Buet, E. Franck and B. Després Design of asymptotic preserving Finite Volume schemes for the hyperbolic heat equation on unstructured meshes, to appear in Numer. Math.

[4] M. Lemou, L. Mieussens A new asymptotic preserving scheme based on micro-macro formulation for linear kinetic equations in the diffusion limit. SIAM J. Sci. COMPUT. Vol. 31, 1, pp 334-368

Joint work with: Christophe Buet (CEA-DIF), Emmanuel Franck (CEA-DIF and LJLL-UPMC)

S37 – Numerical Methods XIII – Room F, 9.45–10.15

* * * *

Two-waves PVM-WAF type method for non-conservative systems

Manuel J. Castro Díaz Universidad de Málaga. Dpto. Análisis Matemático castro@anamat.cie.uma.es

In this work a new two-waves WAF (Weighted Average Flux) type method for non-conservative systems is presented. WAF method was introduced by Prof. E. F. Toro in [6]. The extension to the multidimensional case was performed by Billet and Toro in [1], and more recently to unstructured meshes [5]. There are also many works related to the applications of the WAF method to conservative systems and balance laws: see for example [7-8] and [4].

The two-waves PVM-WAF scheme that we introduce here is defined in terms of a suitable non-linear combination of two different PVM (Polynomial Viscosity Matrix) and it has the property that it is second order accurate for smooth solutions for general 1D hyperbolic systems, while the original two-waves WAF method only guarantees this accuracy for 2×2 1D systems. Let us recall that PVM schemes have been introduced by the authors in the framework of balance laws and non-conservative hyperbolic system in [2]. They are defined in terms of viscosity matrices computed by a suitable polynomial evaluation of a Roe matrix. These methods have the advantage that they only need some information about the eigenvalues of the system to be defined, and no spectral decomposition of Roe Matrix is needed. These methods can be seen as a generalization of the schemes introduced by Degond et al. in [3].

An efficient implementation on GPUs is also discussed and some numerical tests will be presented to check the good properties of the new scheme.

References

- S.J. Billet, E.F. Toro. On WAF-type schemes for multidimensional hyperbolic conservation laws. J. Comput. Phys. 130 (1997), pp. 1-24.
- [2] M.J. Castro, E.D. Fernández-Nieto. A class of computationally fast first order finite volume solvers: PVM methods. Submitted to SIAM Journal of Scientific Computing.
- [3] P. Degond, P.F. Peyrard, G. Russo, Ph. Villedieu. Polynomial upwind schemes for hyperbolic systems. C. R. Acad. Sci. Paris 328, (1999), pp. 479-483.
- [4] E.D. Fernández-Nieto, G. Narbona-Reina. Extension of WAF type methods to non-homogeneous Shallow Water Equations with pollutant. *Journal of Scientic Computing*, 36, (2008), pp. 193-217.
- [5] L. Müller, E.F. Toro WAF schemes on unstructured meshes. Proceedings of Wonapde 2010 congress, (2010).
- [6] E.F. Toro. A Weighted Average Flux Method for Hyperbolic Conservation Laws. Proceedings of the Royal Society of London, Series A, Mathematical and Physical Sciences, 423 (1989), pp. 401-418.
- [7] E.F. Toro. Riemann problems and the WAF method for solving two-dimensional shallow water equations. *Philos. Trans. R. Soc. Lond. A*, **338** (1992), pp. 4368.

[8] E.F. Toro. The weighted average flux method applied to the time dependent Euler equations. *Philos. Trans. R. Soc. Lond. A* 341 (1992) pp. 499530.

Joint work with: Enrique Fernández-Nieto (University of Sevilla. Dpto. de Matemática Aplicada.), Gladys Narbona Reina (University of Sevilla. Dpto. de Matemática Aplicada), Marc de la Asunción (Universidad de Málada. Dpto. Aníalisis Matemático).

9.2 Session 38 - Room B - Navier-Stokes and Euler Equations V

S38 – Navier-Stokes and Euler Equations V – Room B, 9.15–9.45

Boundary layer problem: Navier-Stokes equations and Euler equations

Nikolai, Vasilievich Chemetov CMAF/University of Lisbon, Av. Prof. Gama Pinto, 2, 1649-003 Lisbon, Portugal chemetov@ptmat.fc.ul.pt

This talk is concerned with the boundary layer turbulence. We consider an incompressible viscous fluid in domains with permeable walls. The permeability is described by the Navier slip boundary conditions.

The goal is to study the fluid behavior at vanishing viscosity. We show that the vanishing viscous limit is a solution of the Euler equations with the Navier slip boundary conditions on the inflow region of the boundary.

References

- [1] Chemetov N.V., Cipriano F., The inviscid limit for the Navier-Stokes equations with slip condition on permeable walls, *submitted*.
- [2] Chemetov N. V., Antontsev S. N., Euler equations with non-homogeneous Navier slip boundary conditions, *Physica D: Nonlinear Phenomena*, Vol. 237, Issue 1, 92–105 (2008).
- [3] Chemetov N.V., Cipriano F., Shallow water model for lakes with friction and penetration, Math. Methods in the Applied Sciences, Vol. 33, Issue 6, 687-703 (2010).

Joint work with: Fernanda Cipriano (GFM / New University of Lisbon)

S38 - Navier-Stokes and Euler Equations V - Room B, 9.45-10.15

Low Mach Number Singular Limits of the Compressible Navier-Stokes-Smoluchowski System

Joshua Ballew University of Maryland jballew@amsc.umd.edu The Navier-Stokes-Smoluchowski system models the behavior of fluid-particle interaction in physical situations in which particles interact with fluids under forces such as gravity and buoyancy. Such models arise in applications to medicine, biotechnology, and atmospheric science, among other fields. The system under investigation in this work describes the evolution of particles dispersed in a viscous compressible fluid and is expressed through the conservation of fluid mass, the balance of momentum and the balance of particle density often referred as the Smoluchowski equation. The coupling between the dispersed and dense phases is obtained through the drag forces that the fluid and the particles exert mutually by the action-reaction principle. The governing equations form the so-called Navier-Stokes-Smoluchowski system (NSS). In this presentation, I briefly cover the existence of suitable weak solutions as outlined in [1] as an extension of the existence result in [2]. In addition, approximations to the NSS model in the form of singular limits are considered. In particular, I look at conditions for which the speed of the fluid flow is small compared to the speed of sound in the fluid, also known as the low Mach number case. Under a low stratification condition of the scaling of the system, the solutions converge to a solution of the mathematically simpler incompressible fluid model as the Mach number approaches zero. In the strong stratification case, it is expected, at least formally, that the solutions will converge to functions obeying the anelastic condition. Both of these problems involve using bounds from the energy inequality for the systems to provide estimates that allow us to show the convergence of the solutions. These techniques are motivated by the work in [3], [4], and [5]. In the case for strong stratification, we assume that the external forces acting on the fluid and particles depend only on the vertical component of position, which is physically realizable in gravitation and buoyancy forces.

References

- [1] J. Ballew and K. Trivisa, Suitable weak solutions and low stratification singular limit for a fluid particle interaction model, preprint (2012) to appear on *Quarterly of Applied Mathematics*
- [2] J. A. Carrillo, T. Karper, and K. Trivisa, On the dynamics of a fluid-particle interaction model: The bubbling regime, *Nonlinear Analysis*, **74** (2011), pp. 2778-2801
- [3] E. Feireisl, Flows of viscous compressible fluids under strong stratification: incompressible limits for longrange potential forces, *Mathematical Models and Methods in Applied Sciences*, **21** (2011), pp. 7-27
- [4] E. Feireisl and A. Novotný, Singular Limits in Thermodynamics of Viscous Fluids, Birkhäuser, (2009)
- [5] E. Feireisl and A. Novotný, On the low Mach number limit for the full Navier-Stokes-Fourier system, Arch. Rational Mech. Anal., 186 (2007), pp. 77-107

Joint work with: Konstantina Trivisa (University of Maryland)

9.3 Session 39 — Room G — Numerical Methods XIV

S39 – Numerical Methods XIV – Room G, 9.15–9.45

Simulation of Poroelastic Wave Propagation using CLAWPACK

Grady Lemoine University of Washington gl@uw.edu

We use the CLAWPACK (Conservation LAWs PACKage) finite volume method code [1] to solve Biot's equations [2] for dynamics of a porous, fluid-saturated elastic medium. These equations were developed to model fluid-saturated rock formations, but are also applicable to other porous solids, such as *in vivo* bone. At low frequency Biot's equations are a system of hyperbolic PDEs with a relaxation source term, which may be stiff depending on the time scales associated with wave propagation. We discuss the development of a Riemann solver for orthotropic poroelasticity on arbitrary mapped grids, as well as issues associated with incorporating the stiff relaxation term. We also show numerical results on Cartesian grids, comparing against recent discontinuous Galerkin results [3], and on logically rectangular mapped grids capable of modeling moderately complex geometry.

References

- [1] R. J. LeVeque, M. J. Berger, et al., CLAWPACK software, www.clawpack.org.
- [2] M. A. Biot, Mechanics of deformation and acoustic propagation in porous media, Journal of Applied Physics, 33 (1962), pp. 1482-1498.
- [3] J. de la Puente, M. Dumbser, M. Käser, and H. Igel, Discontinuous Galerkin methods for wave propagation in poroelastic media, Geophysics, 73 (2008), pp. T77-T97.

Joint work with: M.-J. Yvonne Ou (University of Delaware), Randall J. LeVeque (University of Washington)

S39 – Numerical Methods XIV – Room G, 9.45–10.15

* * * -

Hyperbolic explicit-Parabolic linearly implicit finite difference methods for degenerate convection diffusion equation

Fausto Cavalli University of Brescia fausto.cavalli@ing.unibs.it

Convection diffusion equations arise in the modelling of several physical phenomena, ranging from fluid mechanics to astrophysics, from semiconductors to reactive flows, in two-phase flow modeling and in phenomena involving front propagation.

There are several approaches based on finite difference to solve such kind of equations, mostly based on explicit time integration (see for example [1,2,3] and references therein). One of the most unpleasant feature of explicit methods is that, to grant stability, the time integration step Δt has to be proportional to h^2 , where h is the spatial discretization parameter. This can be computationally burdensome when a fine spatial discretization is required or the partial differential equation has to be integrated for a long time. I propose a strategy to overcome these issues, based on the integration of the convection diffusion equation through suitable Implicit parabolic-Explicit hyperbolic (IMEX) methods, which are used, for example, in the case of non linear convection coupled with linear diffusion ([5]). This enables the use of classical non linear reconstruction techniques for the convective term and to avoid a parabolic constraint on the time step thanks to the implicit handling of the diffusive term. However, since the implicit diffusion term would require the use of non linear solvers and since this could be both expensive (due to the high number of iteration needed to grant accuracy) and difficult to apply (due to the possibly of strong degeneracy of the parabolic term), according to what was accomplished in the case of parabolic equations in [4], I propose a linearization technique that allows to solve simply a linear implicit problem. The global scheme is high order accurate and its stability is subjected only to the CFL stability condition imposed by the hyperbolic term. I present some preliminary theoretical results joint with numerical simulations that describe the behaviour of the approach.

References

- A. Kurganov and E. Tadmor, New High-Resolution Central Schemes for Nonlinear Conservation Laws and ConvectionDiffusion Equations, *Journal of Computational Physics*, Volume no. 160 (2000), pp. 241–282
- [2] S. Evje and K.H. Karlsen, Monotone Difference Approximations of BV Solutions to Degenerate Convection-Diffusion Equations, SIAM Journal on Numerical Analysis, Volume no. 37 (2000), pp. 1838–1860
- [3] F. Cavalli, G. Naldi, G. Puppo, M. Semplice, A Family of Relaxation Schemes for Nonlinear Convection Diffusion Problems, *Communications in computational physics*, Volume no. 5 (2009), pp. 532–545
- [4] F. Cavalli, Linearly implicit approximation of relaxation systems in diffusive regime, preprint (2012)
- [4] U. M Ascher, S. J. Ruuth and R. J. Spiteri, Implicit-explicit Runge-Kutta methods for time-dependent partial differential equations, *Journal Applied Numerical Mathematics* Volume 25 (1997)

9.4 Session 40 — Room A — Convective Flows

Decay rate of convection equations with degenerate diffusion

Christian F. Klingenberg Mathematics Dept., Würzburg Univ., Germany klingen@mathematik.uni-wuerzburg.de

We study large-time behaviour of solutions for convection equations with degenerate diffusion. An example of what we consider is the initial value problem to

$$u_t + (u^q)_x + cu^n = (u^m)_{xx}$$
(1)

where (for non-negative initial data in L^1) we get time decay estimates (under certain assumptions) of the type

$$||u||_{\infty} \le K(1+t)^{-1/q}$$

with q > 1.

We shall give conditions on the coefficients of more general equations

$$u_t + F(u, x, t)_x + H(u, x, t) = G(u, t)_{xx},$$
(2)

where G(u) may have a finite number of degenerate points, such that for non-negative L^1 initial data we show that the L^{∞} decay rate of the solutions is given by

$$||u||_{\infty} \le K(1+t)^{-1/2}.$$
(3)

Our approach is as follows: we first first prove the decay rate of derivative of solutions of such equations. Using that we obtain the decay rate of solutions of degenerate convection diffusion equations. The analysis depends on a Lax-Oleinik type estimate.

The second part of the presentation deals with degenerate convection diffusion equation in several space dimensions, which are of the form

$$u_t = \Delta u^m + \sum_{i=1}^{N} f_i(u)_{x_i},$$
(4)

with the initial data $u(x,0) = u_0(x_1, x_2, \cdots, x_N) \ge 0$. N denotes the space dimension.

We prove the decay rate of derivatives of the solution. We give conditions such that we obtain

$$(u^q)_{x_i}(x,t) \le \frac{M}{(1+t)^{\alpha/2}}$$

for every i and any t > 0, for $0 < \alpha < 1$ and $q \ge a$ given positive function of m and N.

The first part is a generalization of [1] using a different and simpler technique. Details of our work are available in [2].

References

- Laurencot, Long-Time Behaviour for Diffusion Equations with Fast Convection, Annali di Matematica pura ed applicata, (IV), Vol. CLXXV. (1998), pp. 233-25
- [2] C. Klingenberg, U. Koley, Y. Lu, Decay rate of degenerate convection diffusion equations in both one and several Space dimensions, Würzburg Univ. Math. dept. preprint (2012)

Joint work with: Ujjwal Koley (Mathematics Dept., Würzburg Univ., Germany), Yunguang Lu (Hangzhou Normal University, China)

S40 - Convective Flows - Room A, 9.45-10.15

* * * -

Entropy solutions via JKO scheme for a class of degenerate convection-diffusion equations

Marco Di Francesco Universitat Autònoma de Barcelona difrancesco@mat.uab.cat

We consider the scalar convection diffusion equation

$$u_t = (a(y)u^m)_y + (u^m)_{xx}, \quad m > 1,$$
(1)

with $a \in W^{2,\infty}$ and $a \ge c > 0$, posed on $y \in \mathbb{R}$. By a simple mass preserving change of variables, the equation (1) is transformed into the equation

$$\rho_t = (\rho(a(x)\rho^{m-1})_x)_x, \tag{2}$$

which is the formal Wasserstein gradient flow of the functional

$$\mathcal{F}[\rho] = \frac{1}{m} \int a(x)\rho^m dx.$$
(3)

We shall prove that the JKO scheme (cf. [JKO98,AGS08]) for \mathcal{F} produces in the limit a unique solution for (2) which is an entropy solution in the sense of Kruzkov [Kru70,Car99,KR03] for the original equation (1). The strategy relies on a slight modification of a flow interchange lemma contained in [MMS09].

References

- [AGS08] L. Ambrosio, N. Gigli, and G. Savaré, Gradient flows in metric spaces and in the space of probability measures. 2nd ed., Lectures in Mathematics, ETH Zürich. Basel: Birkhäuser., (2008).
- [Car99] J. Carrillo, Entropy solutions for nonlinear degenerate problems, Arch. Ration. Mech. Anal., 147 (4) (1999), pp. 269–361.

- [JKO98] R. Jordan, D. Kinderlehrer, and F. Otto, The variational formulation of the Fokker-Planck equation, SIAM J. Math. Anal., 29 (1) (1998), pp. 1–17.
- [KR03] K. H. Karlsen and N. H. Risebro, On the uniqueness and stability of entropy solutions of nonlinear degenerate parabolic equations with rough coefficients, *Discrete Contin. Dyn. Syst.*, 9 (5) (2003), pp. 1081–1104.
- [Kru70] S. N. Kružkov, First order quasilinear equations in serveral independent variables, Math. USSR Sb, 10 (1970), pp. 217–243.
- [MMS09] D. Matthes, R. J. McCann, and G. Savaré, A family of nonlinear fourth order equations of gradient flow type, Comm. Partial Differential Equations, 34 (10-12) (2009), pp. 1352–1397.

Joint work with: Daniel Matthes (TU Munich)

9.5 Session 41 — Room E — Electromagnetic Flows I

S41 – Electromagnetic Flows I – Room E, 9.15–9.45

A Discontious Galerkin Method for the Magnetic Induction Equation with Hall Effect

Paolo Corti ETH, Zürich paolo.corti@sam.math.ethz.ch

Fast magnetic reconnection is important in many applications like Solar Physics and the design of plasma devices. A common modeling framework utilizes the equations of Magnetohydrodynamics (MHD) coupled with a generalized Ohm's law that incorporates electron inertia and the Hall effect. The resulting equations include third-order mixed spatial and temporal derivatives along with the dispersive Hall term. The design of numerical schemes is challenging on account of the non-linearities and high-order derivatives.

As a preliminary step we consider a model based on the magnetic induction equations together with the Hall effect and electron inertia. The non-linearity is present in the form of the Hall term. We derives estimates in H(Curl) which together with the divergence constraint lead to apriori energy estimates. The presence of resistivity and electron inertia suggests the use of implicit-explicit time integration schemes. The implicitness of the schemes results in a series of large linear systems to solve at every time step. A discontinuous Galerikn (DG) discretisation allows to satisfy at the same time an energy estimate and to efficiently precondition the resulting linear systems using affine spaces. Numerical experiments illustrating the scheme are presented.

Joint work with: Ralf Hiptmair (ETH, Zürich), Siddhartha Mishra (ETH, Zürich).

S41 – Electromagnetic Flows I – Room E, 9.45–10.15

- * * * -

Semi-implicit solutions to Radiation-Magnetohydrodynamics

Andrew David McMurry University of Oslo a.d.mcmurry@cma.uio.no 153

Solutions to the equations of Radiation-Magnetohydrodynamics are of vital importance to many astrophysical problems, but very few codes for solution of the system exist. We present a finite volume code which solves the M1 moment model of radiative transfer in conjunction with MHD. The M1 model reduces the directional dimensionality of the transfer equation. The RMHD system includes wave speeds of the order of the speed of light, which can be more than 10^4 times the fastest MHD wave speed in typical problems. In order to be able to use a timestep size of the same order as that required for MHD, we use a semi-implicit method, where MHD is solved explicitly as usual and radiative transfer is solved implicitly, using an iterative parallel non-linear system solver. We present results using grey radiation (averaged over the frequency dimension of the transfer equation).

Joint work with: Siddartha Mishra (University of Oslo), Franz Fuchs

9.6 Session 42 — Room D — PDEs in Mathematical Physics

S42 – PDEs in Mathematical Physics – Room D, 9.15–9.45

On Uniquenes Properties of Solutions to the Benjamin-Ono Equation

Felipe Linares IMPA, Rio de Janeiro, Brazil linares@impa.br

This talk is concerned with some special uniqueness properties of solutions to the IVP associated to the Benjamin-Ono equation. These will be deduced as a consequence of some persistent properties in weighted Sobolev spaces. In particular, we shall show that the uniqueness results established in [1] do not extend to any pair of non-vanishing solutions of the BO equation. Also, we shall prove that the uniqueness result established in [1] under a hypothesis involving information of the solution at three different times can not be relaxed to two different times.

References

 G. Fonseca and G. Ponce, The IVP for the Benjamin-Ono equation in weighted Sobolev spaces, J. Funct. Anal. 260 (2011), pp. 436–459.

Joint work with: German Fonseca (UNAL), Gustavo Ponce (UCSB)

S42 – PDEs in Mathematical Physics – Room D, 9.45–10.15

Smoothing effect and Fredholm property for first-order hyperbolic PDEs

Irina Kmit Institute of Mathematics, Humboldt University of Berlin, Rudower Chaussee 25, D-12489 Berlin kmit@informatik.hu-berlin.de We present recent results on regularity and Fredholm properties for first-order one-dimensional hyperbolic PDEs [1-4]. We show that large classes of boundary operators (appearing in traveling-wave models of laser and population dynamics and chemical kinetics) cause an effect that smoothness increases with time. This means that solutions improve smoothness dynamically, more precisely, they eventually become k-times continuously differentiable for each particular k. This phenomenon allows us to work out a regularization procedure via construction of a parametrix. We construct parametrices for periodic problems for dissipative first-order linear hyperbolic PDEs and show that these problems are modeled by Fredholm operators of index zero. Our Fredholm results cover non-strictly hyperbolic systems with discontinuous coefficients, but they are new even in the case of strict hyperbolicity and smooth coefficients.

References

- I. Kmit, Smoothing effect and Fredholm property for first-order hyperbolic PDEs (2012), E-print available at http://arxiv.org/abs/1202.6282
- [2] I. Kmit, L. Recke, Fredholmness and smooth dependence for linear time-periodic hyperbolic problems, Journal of Differential Equations 252 (2012), No. 2, pp. 1962–1986.
- [3] I. Kmit, Smoothing solutions to initial-boundary problems for first-order hyperbolic systems, Applicable Analysis 90 (2011), No. 11, pp. 1609–1634.
- [4] I. Kmit, L. Recke, Fredholm alternative for periodic-Dirichlet problems for linear hyperbolic systems, Journal of Mathematical Analysis and Applications 335 (2007), No. 1, pp. 355–370.

9.7 Session 43 — Room C — Theory of Conservation Laws V

S43 - Theory of Conservation Laws V - Room C, 9.15-9.45

Remarks on the Theory of the Divergence-Measure Fields

Hermano Frid

Institute for Pure and Applied Mathematics-IMPA, Rio de Janeiro, Brazil hermano@impa.br

We review the theory of the (extended) divergence-measure fields providing an up to date account of its basic results established by Chen and Frid (1999, 2002), as well as the more recent important contributions by Silhavý (2008, 2009). We include a discussion on some pairings that are important in connection with the definition of normal trace for divergence-measure fields. We also review its application to the uniqueness of Riemann solutions to the Euler equations in gas dynamics, as given by Chen and Frid (2002). While reviewing the theory, we simplify a number of proofs allowing an almost self-contained exposition.

* * *

S43 - Theory of Conservation Laws V - Room C, 9.45-10.15

Coupling techniques for nonlinear hyperbolic equations.

Benjamin Boutin IRMAR - Université de Rennes 1, France benjamin.boutin@univ-rennes1.fr

We analyze the coupling between different nonlinear hyperbolic problems across possibly resonant interfaces, say at x = 0 considering the one space variable problem:

$$\partial_t w + \partial_x f^{\pm}(w) = 0, \quad t > 0, \ \pm x > 0.$$

A supplemented coupling condition, modeling the transient exchange of informations at the interface, reads as the continuity of the unknown w or of a nonlinear transformation u of it, say

$$u(t, 0^{-}) = u(t, 0^{+}), \quad t > 0.$$

Such a coupling condition is formulated in a weak form, following [5] and [6]. A difficulty arising with thin interfaces lies in the fact that the initial value problem, even with apparently well-defined interfaces conditions, is often ill-posed, so that the thin interface model does not fully determine the dynamics of the relevant solution.

In the present work, we view the coupling interface as a standing wave v for an augmented system of partial differential equations:

$$\partial_t u + A(u, v)\partial_x u = 0, \quad \partial_t v = 0, \tag{1}$$

The definition of weak solutions for the nonconservative system (1) in the resonant regime has been tackled via the self-similar vanishing viscosity analysis (see [1] and [4]). In [2], we extend this analysis to systems under fairly general assumptions and obtain existence of self-similar weak solutions to the Riemann problem for (1). The internal structure of the coupling interface is also analyzed, and distinct solutions for Riemann data leading to the resonance phenomena are constructed. Multiplicity of self-similar solutions thus do persist for this augmented model with thin interface, even with this regularization mechanism.

We propose then another regularization strategy based on thick interfaces for the same augmented system of partial differential equations (1). The Kružkov's theory applies and ensures the well-posedness of this thick interface model. A new well-balanced finite volume scheme approximates its entropy solution, and preserves the equilibria satisfying the thick coupling condition (see [3]).

References

- [1] B. Boutin, F. Coquel, and E. Godlewski, Dafermos' regularization for interface coupling of conservation laws, in *Hyperbolic problems: Theory, Numerics, Applications*, Springer Verlag, Berlin (2008), pp. 567-575
- [2] B. Boutin, F. Coquel, and P.G. LeFloch, Coupling techniques for nonlinear hyperbolic equations. I. Self-similar diffusion for thin interfaces, Proc. Roy. Soc. Edinburgh Sect. A, 141 (2011), pp. 921-956.
- [3] B. Boutin, F. Coquel, and P.G. LeFloch, Coupling techniques for nonlinear hyperbolic equations. III. The well-balanced approximation of thick interfaces. Submitted.
- [4] C. M. Dafermos, Solution of the Riemann problem for a class of hyperbolic systems of conservation laws by the viscosity method, Arch. Rational Mach. Anal., 52 (1973), pp. 1-9.
- [5] F. Dubois, and P.G. LeFloch, Boundary layers in weak solutions of hyperbolic conservation laws, J. Differential Equations, 71 (1988), pp. 93-122
- [6] E. Godlewski, and P.A. Raviart, The numerical interface coupling of nonlinear hyperbolic systems of conservation laws. I. The scalar case, *Numer. Math.*, 97 (2004), pp. 81-130

Joint work with: Frédéric Coquel (Centre de Mathématiques Appliquées & Centre National de la Recherche Scientifique, École Polytechnique, 91128 Palaiseau, France), Philippe G. LeFloch (Laboratoire Jacques-Louis Lions & Centre National de la Recherche Scientifique, Université Pierre et Marie Curie, Paris 6, 75252 Paris, France)

9.8 Session 44 — Room H — BioFluids Models I

S44 – BIOFLUIDS MODELS I – ROOM H, 9.15–9.45

Schemes with well-controlled dissipation (WCD)

Jan Ernest Seminar for Applied Mathematics, ETH Zürich, Switzerland. jernest@ethz.ch

We study the approximation of entropy solutions to nonlinear hyperbolic conservation laws depending on underlying small scale effects. Such small scale dependent shock waves arise e.g. in non-strictly hyperbolic models or in nonconservative models. Standard finite difference or finite volume schemes for hyperbolic systems may fail to converge to the non-classical solutions of above problems. We claim that the equivalent equation can be used to ensure that non-classical shock waves are approximated correctly. We design a new class of numerical schemes which we call schemes with well-controlled dissipation (WCD), which yields approximations to the physically relevant solutions and allows for resolving shocks of arbitrary strength. As an introductionary example we consider the diffusive-dispersive regularization of the cubic conservation law

$$U_t + \left(U^3\right)_x = \epsilon U_{xx} + \delta \epsilon^2 U_{xxx}$$

where δ is fixed. It is well established, that as ϵ vanishes, the solution converges towards a non-classical solution. We show by means of numerical examples that the WCD scheme converges to the correct non-classical solution even for shocks of high amplitude. Further applications involving non-classical shocks for elasticity systems and reduced MHD models will be presented.

Joint work with: Siddhartha Mishra (Seminar for Applied Mathematics, ETH Zürich) and Philippe G. LeFloch (Laboratoire Jacques-Louis Lions, Centre National de la Recherche Scientifique, Université Pierre et Marie Curie)

S44 – BioFluids Models I – Room H, 9.45–10.15

Hydrodynamical behaviour for chemotaxis

François James Mathématiques – Analyse, Probabilités, Modélisation – Orléans (MAPMO), Université d'Orléans & CNRS UMR 6628, Fédération Denis Poisson, Université d'Orléans & CNRS FR 2964, 45067 Orléans Cedex 2, France francois.james@univ-orleans.fr Chemotaxis is the phenomenom in which a population of cells rearranges its structure according to some chemical, called *the chemoattractant*, present in the environmement. At the kinetic level of description, the Othmer-Dunbar-Alt model describes the dynamics of bacteria like E. Coli taking into account the run and tumble process during the motion. Denoting f(t, x, v) the distribution function at time t depending on the position x and the velocity $v \in \{-c, c\}$, and S the chemoattractant concentration, the system of equations writes in one space dimension (see e.g. [2])

$$\begin{cases} \partial_t f_{\epsilon} + v \partial_x f_{\epsilon} = \frac{1}{\epsilon} (\Phi(-v \partial_x S_{\epsilon}) f_{\epsilon}(-v) - \Phi(v \partial_x S_{\epsilon}) f_{\epsilon}(v)), \\ -\partial_{xx} S_{\epsilon} + S_{\epsilon} = \rho_{\epsilon} = f_{\epsilon}(v) + f_{\epsilon}(-v). \end{cases}$$

The constant c represents the constant velocity of cells. The parameter ϵ is a scaling factor and in applications it is considered as very small ($\epsilon \ll 1$). The hydrodynamical limit $\epsilon \to 0$ of this system leads to the following macroscopic model (see [3])

$$\begin{aligned} \left(\begin{array}{l} \partial_t \rho + \partial_x (a(\partial_x S)\rho) = 0, \\ a(\partial_x S) = c \, \frac{\Phi(-c\partial_x S) - \Phi(c\partial_x S)}{\Phi(-c\partial_x S) + \Phi(c\partial_x S)}, \\ -\partial_{xx}S + S = \rho. \end{aligned} \right) \end{aligned}$$

Introducing the elementary potential K solving $-\partial_{xx}K + K = \delta_0$, the latter system reduces to the scalar conservation law

$$\partial_t \rho + \partial_x (a(\partial_x K * \rho)\rho) = 0. \tag{1}$$

This equation is known as the aggregation equation and it is now classical that regular solutions blow up in finite time when K is not smooth. Thus measure solutions have to be considered, together with a suitable definition of the product $a(\partial_x K * \rho)\rho$, when ρ is a measure. Using the framework of duality solutions developed in [1], as well as a careful strategy to manage the uniqueness of such solutions we are able to give a complete study of the problem (see [3]), as well as convenient numerical schemes that recover the dynamics of aggregates. One of these schemes can be recovered from a suitable asymptotic-preserving schemes (with respect to ϵ) on the kinetic model.

References

- F. Bouchut, F. James, One-dimensional transport equations with discontinuous coefficients, Nonlinear Analysis TMA 32 (1998), nº 7, 891–933.
- [2] Y. Dolak, C. Schmeiser, Kinetic models for chemotaxis: Hydrodynamic limits and spatio-temporal mechanisms, J. Math. Biol. 51, 595–615 (2005).
- [3] F. James, N. Vauchelet, Chemotaxis : from kinetic equations to aggregate dynamics, to appear in Nonlinear Differential Equations and Applications (NoDEA).

Joint work with: Nicolas Vauchelet (UPMC Univ Paris 06, UMR 7598, Laboratoire Jacques-Louis Lions & CNRS, UMR 7598, Laboratoire Jacques-Louis Lions & INRIA Paris-Rocquencourt, Equipe BANG. F-75005, Paris, France).

9.9 Session 45 — Room I — Hydrodynamics and Simulations

S45 – Hydrodynamics and Simulations – Room I, 9.15–9.45

Asymptotical Solutions in a Reactive non-ideal Hydrodynamic medium

Rajan Arora Indian Institute of Technology Roorkee, Saharanpur Campus, Saharanpur-247001, U.P., India rajan_a100@yahoo.com, rajanfpt@iitr.ac.in Using the weakly non-linear geometrical acoustics theory, we obtain the small amplitude high frequency asymptotic solution to the basic equations governing one dimensional unsteady planar, spherically and cylindrically symmetric flow in a reactive non-ideal hydrodynamic medium. The transport equations for the amplitudes of resonantly interacting waves are derived. The evolutionary behavior of non-resonant wave modes culminating into shock waves is also studied.

References

- Y. B. Zel'Dovich, On the theory of the propagation of detonation in gaseous systems, J. of Experimental and Theoretical Physics of the U.S.S.R., 10 (1940), pp. 542-568.
- [2] V. Choquet-Bruhat, Ondes asymptotique et approchees pour systemes d'equations aux derivees partielles nonlineaires, J. Math. Pures Appl. 48 (1969), pp. 119-158.
- [3] J. K. Hunter and J. Keller, Weakly nonlinear high frequency waves, Comm. Pure Appl. Math., 36 (1983), pp. 547-569.
- [4] V. D. Sharma and Gopala Krishna Srinivasan, Wave interaction in a non-equilibrium gas flow, Int. J. Non-linear Mechanics 40, (2005), pp. 1031-1040.
- [5] Rajan Arora, Asymptotical Solutions for vibrationally relaxing gas, Journal of Mathematical Modelling and Analysis, 14(4), (2009), pp. 423-434.

S45 – Hydrodynamics and Simulations – Room I, 9.45–10.15

A high-order unstaggered constrained transport method for the 3D ideal magnetohydrodynamic equations based on the method of lines

Bertram Taetz

Department of Mathematics, Ruhr-University Bochum, Universitätsstr. 150, 44780 Bochum, Germany Bertram.Taetz@rub.de

We study finite volume methods for the 3D ideal magnetohydrodynamic (MHD) equations. Numerical methods for solving the MHD equations in more than one space dimension must confront the challenge of controlling errors in the discrete divergence of the magnetic field. One approach that has been shown successful in stabilizing MHD calculations are constrained transport (CT) schemes. CT schemes can be viewed as predictor-corrector methods for updating the magnetic field, where a magnetic field value is first predicted by a method that does not exactly preserve the divergence-free condition on the magnetic field, followed by a correction step that aims to control these divergence errors. One way to use a CT method without introducing a second (staggered grid) is by solving an evolution equation for the magnetic potential during each time step and computing a divergence-free update of the magnetic field by taking the curl of the magnetic potential as first introduced by Rossmanith [SIAM J. Sci. Comput. 28, 5 (2006)] for 2D Cartesian grids. The evolution equation for the vector potential in 3D is only weakly hyperbolic, which makes the direct use of Riemann solvers complicated, since the system matrix is defective in some directions and due to this, fails to have a full set of linearly independent eigenvectors in those directions. This requires special numerical treatment, as mentioned in Helzel et al. [J]. Comp. Phys. 227, 9527 (2011)]. To deal with the weakly hyperbolic evolution equation for the magnetic vector potential, a key step in this work is to use the method of lines approach with a third order non-conservative finite volume method based on Castro et al. [Math. Comput. 79, 1427 (2010)] for the spatial discretisation. We couple the evolution of the magnetic potential with the evolution of the MHD equations by using a third order strong stability preserving Runge-Kutta time stepping method in time. This gives a third order accurate method for the whole system. We can summarize the properties of the method to be

- (1) third order accurate in space and time on smooth solutions while giving high-resolution on problems with shocks,
- (2) able to solve the weakly hyperbolic evolution equation of the magnetic potential for an CT type update to control errors in the divergence of the magnetic field,
- (3) applicable on both Cartesian and logically rectangular (2D) and hexhedral (3D) mapped grids.

Special artificial resistivity limiters are used to control unphysical oscillations in the magnetic potential and magnetic field components, computed by the CT, across shocks. Several test computations confirm the desired properties mentioned above.

References

- C. Helzel, J.A. Rossmanith, B. Taetz, An unstaggered constrained transport method for the 3d ideal magnetohydrodynamic equations, J. Comput. Phys., 230 (2011), pp. 3803-3829
- [2] M.J. Castro and A. Pardo and C. Parés and E.F. Toro On some fast well-balanced first order solvers for nonconservative systems *Mathematics of Computation*, **79** (2010)
- [3] J.A. Rossmanith An unstaggered, high-resolution constrained transport method for magnetohydrodynamic flows SIAM J. Sci. Comp., 28 (2006), pp.1766-1797

Joint work with: Christiane Helzel (Department of Mathematics, Ruhr-University Bochum, Universitätsstr. 150, 44780 Bochum, Germany), and James A. Rossmanith (Department of Mathematics, University of Wisconsin, 480 Lincoln Drive, Madison, WI 53706-1388, USA)

10 Abstracts of contributed lectures — Thursday 11.20–12.50

10.1 Session 46 — Room F — Numerical Methods XV

S46 – Numerical Methods XV – Room F, 11.20-11.50

ENO interpolation is stable: high resolution, the sign property and entropy stability

Eitan Tadmor University of Maryland tadmor@cscamm.umd.edu

ENO is an adaptive procedure to recover piecewise-smooth data with high resolution. The ENO procedure was introduced in 1987 by Harten et. al. [3] in the context of accurate simulations for piecewise smooth solutions of nonlinear conservation laws, and since then, it have been used with a considerable success in Computational Fluid Dynamics; we refer to the review articles of Harten and Shu [4,5] and the references therein. Despite the extensive literature on the construction and implementation of ENO method and its variants for the last 25 years, we were not aware of any global, mesh independent, stability results. It is in this context that we proved in [1] the following stability of the ENO procedure: the jump of the ENO pointvalues at each cell interface has the same *sign* and in fact, the same *size* as the jump of the underlying data across that interface.

This sign property, which is shown to hold for ENO interpolation of arbitrary order of accuracy and on non-uniform meshes, manifests a remarkable rigidity of the piecewise-polynomial ENO procedure. Similar sign properties hold for the ENO reconstruction procedure from cell averages, which is used, in [2], for the construction of a new class of arbitrarily high-order, entropy stable — so-called TeCNO schemes, for nonlinear conservation laws.

References

- U. Fjordholm, S. Mishra & E. Tadmor, ENO reconstruction and ENO interpolation are stable, J. FoCM, to appear, 2012. http://arxiv.org/abs/1112.1131.
- [2] U. Fjordholm, S. Mishra & E. Tadmor, Arbitrarily high order accurate entropy stable essentially nonoscillatory schemes for systems of conservation laws, SIAM Journal on Numerical Analysis, to appear, 2012.
- [3] A. Harten, B. Engquist, S. Osher and S. R. Chakravarty, Uniformly high order accurate essentially nonoscillatory schemes J. Comput. Phys., 71 (2), 1987, 231-303.
- [4] A. Harten, Recent developments in shock-capturing schemes. Proc. International Congress of Mathematicians, Vol. I, II (Kyoto, 1990), 1549-1559, Math. Soc. Japan, Tokyo, 1991.
- [5] C.W. Shu, Essentially non-oscillatory and weighted essentially non-oscillatory schemes for hyperbolic conservation laws, in "Advanced Numerical Approximation of Nonlinear Hyperbolic Equations", Lecture notes in Mathematics 1697, 1997 C.I.M.E. course (A. Quarteroni ed.), Springer Verlag 1998, pp. 325-432.

Joint work with: Ulrik S. Fjordholm and Siddhartha Mishra (Seminar for Applied mathematics, ETH Zurich)

Session 46 — Room F — Numerical Methods XV

__ * * * -

S46 – Numerical Methods XV – Room F, 11.50–12.20

Central-Upwind Schemes for the System of Shallow Water Equations with Horizontal Temperature Gradients

Alina Chertock North Carolina State University chertock@math.ncsu.edu

We consider a modification of the Saint-Venant system of shallow water equations, in which the water temperature fluctuations are taken into account. In two space dimensions the system takes the form:

$$\begin{cases} h_t + (hu)_x + (hv)_y = 0, \\ (hu)_t + \left(hu^2 + \frac{g}{2}h^2\theta\right)_x + (huv)_y = -gh\theta B_x, \\ (hv)_t + (huv)_x + \left(hv^2 + \frac{g}{2}h^2\theta\right)_y = -gh\theta B_y, \\ (h\theta)_t + (uh\theta)_x + (vh\theta)_y = 0, \end{cases}$$
(1)

where h(x, y, t) denotes the water depth, u(x, y, t) and v(x, y, t) denote the fluid velocity in x- and y-direction respectively, B(x, y) represents the bottom topography, and g is the gravitational constant. The variable θ denotes the potential temperature field. Specifically, θ is the reduced gravity $\Delta \Theta / \Theta_{\text{ref}}$ computed as the potential temperature difference $\Delta \Theta$ from some reference value Θ_{ref} .

The studied model was introduced in [3,4,5] for modeling ocean currents and is reffered to as the Ripa system. System (1) takes into account temperature variations, which effect the pressure term in the Saint-Venant system. In the Ripa system, the temperature is transported by the fluid, which makes the model substantially more complicated than the classical Saint-Venant system: If one of the velocity components vanishes, the characteristic speed becomes zero and the system exhibits a "nonlinear resonance" in the sense that wave speeds from different families of waves coincide. Moreover, there are no Riemann invariants for this system and therefore it is very hard to design upwind schemes for the Ripa system, since they are based on (approximate) Riemann problem solvers.

In general, designing a well-balanced scheme for the Ripa system is a highly nontrivial task since steady states at rest,

$$\theta \equiv \text{constant}, \quad w = h + B \equiv \text{constant}, \quad u = v \equiv 0$$
 (2)

and

$$B \equiv \text{constant}, \quad p = \frac{g}{2}h^2\theta \equiv \text{constant}, \quad u = v \equiv 0,$$
 (3)

which are characterized by zero velocity and the differential (not integrable!) form, no longer correspond to the flat water surface. In [2], we developed a well-balanced central-upwind scheme for the Ripa system. Our scheme is capable of exactly preserving these two special types of steady states at rest. To preserve steady states (2), we have implemented the same technique as for the original Saint-Venant system. However, steady states (3) are of different nature since they correspond to steady contact waves with constant pressure. They are similar to the steady-state solutions appearing in compressible multi-fluids. As in the multi-fluid case, a good scheme must be able to preserve constant pressure and velocity across contact waves to avoid appearance of spurious pressure and velocity oscillations. To achieve this goal, we have extended the interface tracking method, which we previously developed for compressible multi-fluids [1], to the Ripa system.

References

- A. Chertock, S. Karni, and A. Kurganov, Interface tracking method for compressible multifluids, M2AN Math. Model. Numer. Anal., 42 (2008), pp. 991–1019.
- [2] A. Chertock, A. Kurganov and Y. Liu, Central-upwind schemes for the system of shallow water equations with horizontal temperature gradients, submitted.

- [3] P. Dellar, Common hamiltonian structure of the shallow water equations with horizontal temperature gradients and magnetic fields, *Phys. Fluids*, **303** (2003), pp. 292–297.
- [4] P. Ripa, Conservation laws for primitive equations models with inhomogeneous layers, *Geophys. Astrophys. Fluid Dynam.*, 70 (1993), pp. 85–111.
- [5] P. Ripa, On improving a one-layer ocean model with thermodynamics, J. Fluid Mech., 303 (1995), pp. 169–201.

Joint work with: Alexander Kurganov (Tulane University), Yu Liu (Tulane University).

S46 – Numerical Methods XV – Room F, 12.20–12.50

A positive, entropic, full-well-balanced scheme for the shallow-water model

Christophe Berthon Laboratoire de Mathématiques Jean Leray christophe.berthon@univ-nantes.fr

The present work concerns the derivation of an approximate Riemann solver to discretize the well-known shallowwater model. During the two last decades, numerous methods have been introduced with a special attention on the preservation of the stationary states given by

$$hu = \mathrm{cste}, \quad \frac{\mathrm{u}^2}{2} + \mathrm{g}(\mathrm{h} + \mathrm{Z}) = \mathrm{cste},$$

where h > 0 stand for the water height and $u \in \mathbb{R}$ the water velocity. The function Z is an imposed smooth topography. In general the so-called well-balanced schemes are able to restore the lack at rest solution, i.e. the stationary solution with u = 0. Several attempts were recently proposed to derive schemes able to preserve a large class of steady states. In [1], a positive entropic scheme which preserves the subsonic stationary states is given. In [2], a full well-balanced technique is suggested to capture all the stationary states. Unfortunately, this scheme may involve negative water height.

Here, by involving a suitable approximate Riemann solver (see [3] for related techniques), we obtain a scheme which is positive, entropy preserving and full-well-balanced since it preserves all the stationary states.

References

- F. Bouchut, T. Morales, A subsonic-well-balanced reconstruction scheme for shallow water flows, SIAM J. Numer. Anal., 48 (2010), pp. 1733-1758.
- [2] M. J. Castro, A. Pardo Milanés, C. Parés, Well-balanced numerical schemes based on a generalized hydrostatic reconstruction technique, *Math. Models Methods Appl. Sci.*, **17** (2007), pp. 2055-2113.
- [3] C. Chalons, F. Coquel, E. Godlewski, P.-A. Raviart, N. Seguin, Godunov-type schemes for hyperbolic systems with parameter dependent source. The case of Euler system with friction, *Math. Models Methods Appl. Sci.*, **20** (2010), pp. 2109-2166.

Joint work with: Christophe Chalons (Université Paris Diderot-Paris 7 & Laboratoire J.-L. Lions)

10.2 Session 47 — Room B — Navier-Stokes and Euler Equations VI

S47 – Navier-Stokes and Euler Equations VI – Room B, 11.20–11.50

Nonlinear stability of a boundary layer solution to the Euler-Poisson equations in plasma physics

Masashi Ohnawa Waseda University ohnawa@aoni.waseda.jp

We study the initial boundary value problem to the Euler-Poisson equations (1) over $\mathbb{R}^N_+ := \{(x_1, x') \in \mathbb{R}^N | x_1 > 0, x' \in \mathbb{R}^{N-1}\}$ for N = 1, 2, 3:

$$\rho_t + \operatorname{div}(\rho u) = 0, \tag{1a}$$

$$(\rho u)_t + \operatorname{div}\left(\rho u \otimes u\right) + K\nabla\rho + \rho\nabla\phi = 0,\tag{1b}$$

$$-\Delta\phi = \rho - e^{\phi}.\tag{1c}$$

This system of equations describes the isothermal flow of positive ions, where unknown functions ρ , u and ϕ stand for the density and the velocity of positive ions and the electrostatic potential. The positive constant K corresponds to the temperature of ions. The third equations is obtained by assuming the Boltzmann relation: $\rho_e = e^{\phi}$. In plasma physics, the Bohm criterion is known as a necessary condition for the formation of a boundary layer called sheath.

In this presentation, first we define the sheath by a monotone stationary solution to the system of Euler-Poisson equations (1) over one-dimensional half space and then show that the Bohm criterion together with the physically natural boundary condition on the electric potential is sufficient for the unique existence of a monotone stationary solution. We also prove the asymptotic stability of the stationary solution under the degenerate or nondegenerate Bohm criterion and justifies the Bohm criterion from the mathematical point of view. Details are seen in [1,2].

We prescribe the initial and the boundary data as

x

$$(\rho, u)(0, x) = (\rho_0, u_0)(x), \quad \inf_{x \in \mathbb{R}^N_+} \rho_0(x) > 0,$$

$$\lim_{x \to \infty} (\rho_0, u_0)(x) = \rho_0(x) = \rho_0(x) = 0, \quad (2)$$

$$\lim_{k \to \infty} (\rho_0, u_0)(x_1, x) = (\rho_+, u_+, 0, \dots, 0) \in \mathbb{R}^{d-1},$$
(2)

$$\phi(t,0,x') = \phi_b \tag{3}$$

for an arbitrary $x' \in \mathbb{R}^{N-1}$, where $\rho_+ > 0$, u_+ and ϕ_b are constants. The reference point of the value of the potential ϕ is taken as $x_1 = \infty$:

$$\lim_{x_1 \to \infty} \phi(t, x_1, x') = 0 \quad \text{for an arbitrary } x' \in \mathbb{R}^{N-1}.$$
 (4)

It is easily seen that constructing a classical solution to (1c) requires $\rho_{+} = 1$.

The planar stationary solution $(\tilde{\rho}, \tilde{u}, 0, \dots, 0, \tilde{\phi})(x_1)$ is a solution to (1) independent of the time variable t or tangential variable x':

$$\left(\tilde{\rho}\tilde{u}\right)_{r_1} = 0,\tag{5a}$$

$$\left(\tilde{\rho}\tilde{u}^2 + K\tilde{\rho}\right)_{x_1} + \tilde{\rho}\tilde{\phi}_{x_1} = 0,\tag{5b}$$

$$-\tilde{\phi}_{x_1x_1} = \tilde{\rho} - e^{\tilde{\phi}}.$$
(5c)

Assumptions corresponding to (2)-(4) are also made, that is,

$$\inf_{x_1 \in \mathbb{R}_+} \tilde{\rho}(x_1) > 0, \quad \lim_{x_1 \to \infty} (\tilde{\rho}, \tilde{u}, \tilde{\phi})(x_1) = (\rho_+, u_+, 0), \quad \tilde{\phi}(0) = \phi_b.$$
(6)

The conditions on the unique existence of the monotone solution to (5) and (6) are obtained in [2]. By those results, we know that the condition

$$u_{+}^{2} > K + 1, \quad u_{+} < 0$$
(7)

together with $|\phi_b| \ll 1$ or

$$u_{+}^{2} = K + 1, \quad u_{+} < 0 \tag{8}$$

together with $\phi_b \leq 0$ and $|\phi_b| \ll 1$ is sufficient for the unique existence and the stability of the monotone stationary solution.

To study the asymptotic stability of the stationary solution, we introduce unknown functions $v := \log \rho$, $\tilde{v} := \log \tilde{\rho}$ and the perturbation

$$(\psi, \eta, \sigma)(t, x_1, x') := (v, u, \phi)(t, x_1, x') - (\tilde{v}, \tilde{U}, \phi)(x_1),$$

where $\tilde{U} = (\tilde{u}, 0, ..., 0)$. The equations for (ψ, η, σ) are obtained from (1) and (5) while the initial and the boundary data are obtained from (2), (3) and (6).

Theorem. (nondegenerate case) For N = 1, 2, 3, let m = [N/2] + 2. Assume K > 0 and (7) hold. Suppose also $(e^{\lambda x_1/2}\psi_0, e^{\lambda x_1/2}\eta_0) \in (H^m(\mathbf{R}^N_+))^{N+1}$ for a certain positive constant λ . If $\lambda + (|\phi_b| + ||(e^{\lambda x_1/2}\psi_0, e^{\lambda x_1/2}\eta_0)||_{H^m}) / \lambda$ is small enough, (1)–(3) has a unique solution (ρ, u, ϕ) such that $(e^{\lambda x_1/2}\psi, e^{\lambda x_1/2}\eta, e^{\lambda x_1/2}\eta) \in (\mathfrak{X}^0_m([0,\infty)))^{N+1} \times \mathfrak{X}^2_m([0,\infty))$. Moreover, the solution verifies

$$\|(e^{\lambda x_1/2}\psi, e^{\lambda x_1/2}\eta)(t)\|_m^2 + \|e^{\lambda x_1/2}\sigma(t)\|_{m+2}^2 \le C\|(e^{\lambda x_1/2}\psi_0, e^{\lambda x_1/2}\eta_0)\|_m^2 e^{-ct}.$$

We also show the algebraic convergence rate in case the initial perturbation decays algebraically. Under the degenerate condition (8), similar results are obtained with algebraic weight. Readers are referred to [1] for their proofs.

Notation. For a real number x, [x] denotes a maximum integer which does not exceed x. For a nonnegative integer $l \ge 0$, $H^l(\mathbb{R}^N_+)$ denotes the *l*-th order Sobolev space in the L^2 sense, equipped with the norm $\|\cdot\|_l = \|\cdot\|_{H^l}$. The function space \mathfrak{X}_i^j (i = 0, 1, 2, 3, j = 0, 1, 2) is defined by

$$\mathfrak{X}_{i}^{j}([0,T]) := \bigcap_{k=0}^{i} C^{k}([0,T]; H^{j+i-k}(\mathbb{R}^{N}_{+})).$$

References

- [1] S. Nishibata, M. Ohnawa and M. Suzuki, Asymptotic stability of boundary layers to the Euler-Poisson equations arising in plasma physics, to appear in *SIAM J. Math. Anal.*
- M. Suzuki, Asymptotic stability of stationary solutions to the Euler-Poisson equations arising in plasma physics, *Kinetic and Related Models*, 4 (2011), pp. 569-588.

Joint work with: Shinya Nishibata (Tokyo Institute of Technology), Masahiro Suzuki (Tokyo Institute of Technology).

S47 – Navier-Stokes and Euler Equations VI – Room B, 11.50–12.20

- * * * -

Steady self-similar inviscid flow

Joseph Roberts University of Michigan, Ann Arbor, USA joeprob@umich.edu In multi-dimensional flow, there are several examples in which there exists a distinguished point around which the flow is (to first order) constant along rays starting at this point. These include regular reflection (four shock waves meeting at a point) or Mach reflection (three shocks meeting with a contact or another type of wave). However, other configurations such as triple points (three shocks with no other waves in between) are not possible in most reasonable models. Beyond these special cases, the possible combinations of such waves meeting at a point have not been classified.

From the point of view of an observer moving with this distinguished point, the flow is steady. This leads to systems of the form

$$U_t + f^x(U)_x + f^y(U)_y = 0,$$

where $U(t, x, y) = U(\phi)$, with $\phi = \measuredangle(x, y) \in [0, 2\pi)$. Here U and the flux functions f^x and f^y take values in \mathbb{R}^m . The fluxes are assumed to be smooth and possess an entropy-entropy flux pair with uniformly convex entropy on some open nonempty set in state space. We consider admissible weak solutions that are L^{∞} -close to some constant supersonic background state \overline{U} ; that is $||U - \overline{U}||_{L^{\infty}} < \epsilon$. This background state is supersonic in the sense that the polynomial

$$P(x:y) = \det\left(xf_U^y(\overline{U}) - yf_U^x(\overline{U})\right)$$

(where (x : y) are homogeneous coordinates) has exactly m distinct real roots (this is equivalent to requiring the steady form of the system of conservation laws to be strictly hyperbolic). We carry out all our analysis with the appropriate weak form, assuming only that $U \in L^{\infty}$.

Without loss of generality, we can rotate coordinates so that none of the roots of the above polynomial lie on the y-axis. Then the solutions to that polynomial correspond to the generalized eigenvalues for the system – that is, the values of ξ that solve

$$\det\left(f_U^y(\overline{U}) - \xi f_U^x(\overline{U})\right) = 0,$$

where $\xi = y/x$. We assume that these eigenvalues are either genuinely nonlinear or linearly degenerate. We prove, for $\epsilon > 0$ sufficiently small, that U must be constant outside of 2m thin sectors associated to those ξ solving the above equation, which we can group as m forward (x > 0) and m backward (x < 0) sectors. Moreover, we show that linearly degenerate sectors each contain at most one contact discontinuity and that genuinely nonlinear forward sector can contain infinitely many shocks and compression waves, but there cannot be consecutive compression waves.

Interestingly, we also prove that U must have bounded variation. As a corollary, we can interpret the *y*-axis as being Riemann data and use our results regarding forward sectors to prove that self-similar solutions to the Riemann problem for strictly hyperbolic 1d-conservation laws (with degenerate or genuinely nonlinear characteristic fields) are unique in the class of L^{∞} functions sufficiently close to a specified background state. Although we cannot have uniqueness in backward sectors, the fact that such L^{∞} solutions must be BV is interesting because BV is sharp in the sense that any commonly used function space more restrictive than BV does not contain all possible Riemann problem solutions.

Finally, we verify that the compressible isentropic Euler equations satisfy all the required conditions for our results. We give a required entropy-entropy flux pair that is uniformly convex under a standard assumption on the pressure law. For this system, there are two genuinely nonlinear fields and one degenerate field. We consider our background state to have horizontal supersonic velocity. The following figure summarizes our results for this case.



References

[1] V. Elling and J. Roberts, Steady self-similar inviscid flow, arxiv:1104.0331 (submitted), 2011.

Joint work with: Volker Elling (University of Michigan, Ann Arbor, USA)

S47 – Navier-Stokes and Euler Equations VI – Room B, 12.20-12.50

A dispersive property of the Euler-Korteweg model

Corentin Audiard

1- UPMC Univ Paris 06, UMR 7598, Laboratoire Jacques-Louis Lions, F-75005, Paris, France 2- CNRS, UMR 7598, Laboratoire Jacques-Louis Lions, F-75005, Paris, France corentin.audiard@ljll.math.upmc.fr

The Euler-Korteweg system consists in a quasi-linear dispersive perturbation of the Euler equations by the so-called Korteweg tensor which is intended to take into account capillary effects. The system reads

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho u) = 0, \\ \partial_t u + (u \cdot \nabla)u + \nabla g_0(\rho) = \nabla \left(K(\rho) \Delta \rho + \frac{1}{2} K'(\rho) |\nabla \rho|^2 \right), \end{cases}$$
(EK)

The Cauchy problem has been studied in any dimension $d \ge 1$ by Benzoni-Danchin-Descombes [1], who obtained local well-posedness results when the velocity is in $H^s(\mathbb{R}^d)$ for s > d/2+1. They noticed that one may expect to find some smoothing effect due to the dispersion (more precisely the local gain of 1/2 derivative). Our aim here is to give such results in any dimension under their local existence assumptions. Though dispersive smoothing is well known for the -possibly quasi-linear- Schrödinger [3] or Korteweg de Vries equation [4], (EK) exhibits several singular features. Besides technical difficulties arising from its quasi-linear nature, special attention is devoted to two points

• Not all Cauchy data produce a solution satisfying dispersive estimates. Namely we will describe why the irrotionality of u(t = 0) is essential,

• The system admits traveling waves solutions [2] whose profile do not satisfy the decay assumptions usually required in dispersive smoothing results. We give a sufficient condition ensuring that smoothing occurs for solutions of (EK) linearized near a traveling profile.

The main technics involved are the construction of a symbol in Doi's spirit [5] that leads formally to dispersive estimates, and the use of para-differential calculus to tackle the non-linearities.

References

- S. Benzoni-Gavage and R. Danchin and S. Descombes, On the well-posedness for the Euler-Korteweg model in several space dimensions, *Indiana Univ. Math. J.*, Volume no. 56 (2007), issue 4 pp. 1499-1579
- [2] Benzoni-Gavage, Sylvie and Danchin, Raphaël and Descombes, Stéphane and Jamet, Stability issues in the Euler-Korteweg model, in *Control methods in PDE-dynamical systems*, Contemp. Math, vol.426, A.M.S. (2007), pp. 103-127
- [3] Kenig, Carlos E. and Ponce, Gustavo and Vega, Luis, The Cauchy problem for quasi-linear Schrödinger equation, *Invent. Math.*, Volume no. 158 (2004), pp. 343-388
- [4] Kenig, Carlos E. and Ponce, Gustavo and Vega, Luis, Well-posedness and scattering results for the generalized Korteweg-de Vries equation via the contraction principle, *Comm. Pure Appl. Math.*, Volume no. 46 (1993), pp. 527-620
- [5] Doi, Shin-ichi, Remarks on the Cauchy problem for Schrödinger-type equations, Comm. Partial Differential Equations Volume no. 21 (1996), pp. 163-178

10.3 Session 48 — Room G — Numerical Methods XVI

S48 – Numerical Methods XVI – Room G, 11.20–11.50

Error estimates for monotone finite difference approximations to degenerate convection-diffusion equations

Erlend Briseid Storrøsten CMA, University of Oslo erlenbs@math.uio.no

We consider semi-discrete monotone finite difference schemes for the nonlinear, possibly strongly degenerate convection-diffusion equation

$$\begin{cases} \partial_t u + \partial_x f(u) = \partial_x^2 A(u), & (x,t) \in \Pi_T = \mathbb{R} \times (0,T), \\ u(x,0) = u^0(x), & x \in \mathbb{R}. \end{cases}$$
(1)

Using a doubling of variables type of argument we show that the L^1_{loc} difference between the approximate solution and the unique entropy solution converges at a rate $\mathcal{O}(\Delta x^{1/3})$ where Δx is the spatial mesh size. We also consider the adaptation of the proof to the multidimensional case.

References

[1] K. H. Karlsen, U. Koley and N. H. Risebro, An error estimate for the finite difference approximation to degenerate convection-diffusion equations, *Numer. Math.*, to appear.

[2] E. B. Storrøsten, K. H. Karlsen and N. H. Risebro, Error estimates for monotone finite difference approximations to degenerate convection-diffusion equations, in preparation.

Joint work with: Kenneth H. Karlsen (CMA, University of Oslo), Nils Henrik, Risebro (CMA, University of Oslo)

S48 – Numerical Methods XVI – Room G, 11.50–12.20

- * * * -

Implicit-Explicit Runge-Kutta schemes for the Boltzmann-Poisson equation for semiconductors

Vittorio Rispoli Department of Mathematics, University of Ferrara rspvtr@unife.it

In this talk, we consider a new class of Implicit-Explicit (IMEX) Runge-Kutta schemes for the linear semiconductor Boltzmann equation that works in both the kinetic and diffusive regimes. In the latter case, the system is governed by a parabolic convection-diffusion equation. For such problems, it is suitable to use a method that is able to capture the asymptotic behavior of the equation with an implicit treatment of the limiting diffusive term. To this aim, we reformulate the problem by properly combining the limiting diffusion term with the convective flux, in order to compute the correct limit. Then we discretize the resulting system using high order IMEX Runge-Kutta schemes. Our approach originates in the zero relaxation limit an IMEX method for the corresponding convection-diffusion system where, as desired, the diffusion term is discretized implicitly.

References

- S. Boscarino, L. Pareschi, G. Russo, Implicit-Explicit Runge-Kutta schemes for hyperbolic systems and kinetic equations in the diffusion limit, arXiv:1110.4375 (2011).
- [2] G. Dimarco, L. Pareschi, V. Rispoli, Implicit-Explicit Runge-Kutta schemes for the Boltzmann-Poisson equation for semiconductors, work in progress
- [3] S. Jin and L. Pareschi, Discretization of the multiscale semiconductor Boltzmann equation by diffusive relaxation schemes, J. Comp. Phys. 161 (2000) pp.312-330

Joint work with:

Lorenzo Pareschi, (Department of Mathematics, University of Ferrara), Giacomo Dimarco, (Institut de Mathématiques de Toulouse, University of Toulouse)

* * * -

S48 – Numerical Methods XVI – Room G, 12.20-12.50

Boundary treatment in cut cell finite difference methods for compressible gas dynamics in domain with moving boundaries

Armando Coco

Dipartimento di Matematica ed Informatica, Università di Catania, v.le A. Doria, 6, 95125 Catania, Italy coco@dmi.unict.it

A new method is presented for the solution of compressible Euler equations in time dependent domains. The computational domain $\Omega(t) \subseteq \mathcal{R}$ is identified by a region in which a time dependent level-set function $\phi(\mathbf{x}, t)$ is negative. The rectangular region \mathcal{R} in which the $\Omega(t)$ is immersed is discretized by a regular square grid. In all our talk we assume that Ω is the region in \mathcal{R} external to a moving obstacle D(t).

Two sets of nodes are identified in \mathcal{R} at each time t: internal nodes $\mathbf{x} \in \Omega(t)$, and *ghost* nodes, i.e. nodes in \mathcal{R} , which are external to Ω , but are *close* to the boundary (i.e. within one or few grid points from an internal node). For fixed domains, i.e. if Ω does not depend on time, the sets of internal and ghost points do not change, otherwise it has to be updated at every time step. Conservative finite difference will be used as space discretization. In one time step, from t^n to t^{n+1} , the evolution of the system is performed as follows: for points that will be internal at time t^{n+1} the field variables are evolved by integrating the semidiscrete system in time, while the values of all the ghost points which are required to close the system of equations are computed by making use of boundary conditions.

During the talk, particular care will be given to the imposition of boundary conditions. For Euler equation, each node contains four quantities in two space dimensions, say density, pressure and two velocities, therefore four equations are needed for each ghost point. Of course, because of the hyperbolic nature of the problem, the conditions cannot be applied independently, and have to be compatible with the equations.

We assume that the boundary conditions on the obstacle D(t) are the classical no slip conditions of inviscid Euler equation on a wall, so one boundary condition states that the normal velocity of the gas on ∂D is equal to the normal velocity in the points of ∂D . The second condition is obtained balancing centrifugal force on the gas with pressure gradient. The third condition is obtained from adiabaticity, and relates variations in pressure and density, and the last condition, imposed on the transversal velocity, is a condition on the enthalpy, commonly adopted in gas dynamics.

Because the conditions on one ghost point are related to the conditions on neighbor ghost points, they are not independent, rather they constitute a system that has to be solved quickly in order to proceed with the integration of the equations on internal points. High order extrapolation will be able to define the equations for the ghost points to high order accuracy in space.

A recently developed relaxation procedure, successfully applied in the numerical solution of elliptic problems, is applied to the solution of the system of equations to compute the field at ghost nodes.

Several examples are performed, which illustrate the flexibility and robustness of the methods.

References

- Chertock, A., Kurganov, A., A simple Eulerian finite-volume method for compressible fluids in domains with moving boundaries, *Commun. Math. Sci.*, Volume no. 6 (2008), 531-556.
- [2] Chertock, A., Coco, A., Kurganov, A., Russo, G., A Second-Order Finite-Difference Method for Compressible Fluids in Domains with Moving Boundaries, *in preparation*.
- [3] Coco, A., Russo, G., A fictitious time method for the solution of Poisson equation in an arbitrary domain embedded in a square grid, *Journal of Computation Physics*, under revision.

Joint work with: Alina Chertock (Department of Mathematics, North Carolina State University State University, Raleigh, NC, 27695, USA), Alexander Kurganov (Mathematics Department, Tulane University, New Orleans, LA 70118, USA), Giovanni Russo (Dipartimento di Matematica ed Informatica, Università di Catania, v.le A. Doria, 6, 95125 Catania, Italy).

10.4 Session 49 - Room A - Wave Analysis I

S49 – Wave Analysis I – Room A, 11.20-11.50

Asymptotic behavior of solutions for Damped wave equations with non-convex convection term on the half line

Itsuko Hashimoto Division of Mathematics and Physics Kanazawa university and Osaka City university advanced mathematical institute itsuko@staff.kanazawa-u.ac.jp

We consider the initial-boundary value problem on the half line for damped wave equations with a nonlinear convection term:

$$\begin{cases} u_{tt} - u_{xx} + u_t + f(u)_x = 0, & x > 0, \ t > 0, \\ u(0,t) = u_{-}, & t > 0, \\ \lim_{x \to \infty} u(x,t) = 0, & t > 0, \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), & x > 0, \end{cases}$$
(1)

where the function f = f(u) is a given smooth function satisfying f(0) = 0 and u_{-} is a given constant with $u_{-} < 0$. In this problem, we assume that the initial data $u_{0}(x)$ satisfies $u_{0}(0) = u_{-}$ and $\lim_{x \to \infty} u_{0}(x) = 0$ as the compatibility conditions.

We study the asymptotic stability of nonlinear waves for damped wave equations with non-convex convection term satisfying

$$0 < |f'(0)| < 1, \quad 0 = f(0) < f(u), \quad \text{for} \quad u \in [u_{-}, 0).$$
(2)

We note that the first condition in (2) is so-called sub-characteristic condition. Ueda-Kawashima [2] and Ueda [1] dealt with the damped wave equation (1) as a derivation of relaxation system and by applying the Chapman-Enskog expansion to the relaxation system, they suggested that the dissipative structure of (1) is similar to one of viscous conservation laws. Then Ueda [1] and Ueda-Nakamura-Kawashima [3] actually showed that the solution of (1) tends toward the stationary solution ϕ , provided that the initial perturbation is suitably small. Here, the stationary solution $\phi = \phi(x)$ is defined by the solution of the stationary problem corresponding to (1):

$$\begin{cases} f(\phi) = \phi_x, & x > 0, \\ \phi(0) = u_-, & \lim_{x \to \infty} \phi(x) = 0. \end{cases}$$
(3)

In this talk, we prove that even for a quite wide class of the convection term, such a stationary solution is asymptotically stable. To investigate the stability of the stationary wave ϕ , we assume that $u_0 - \phi$ and u_1 are integrable. Then we can define the following functions:

$$z_0(x) = -\int_x^\infty (u_0(y) - \phi(y)) \, dy \in L^2, \qquad z_1(x) = -\int_x^\infty u_1(y) \, dy \in L^2.$$

By using these functions, we obtain the stability results as follows.

Theorem 1. Let $\phi(x)$ be the stationary solution satisfying the problem (3). Assume that $z_0 \in H^2$ and $z_1 \in H^1$. Then there is a positive constant ε_0 such that if $||z_0||_{H^2} + ||z_1||_{H^1} \leq \varepsilon_0$, then the initial-boundary value problem (1) has a unique global solution u(x,t) satisfying $u - \phi \in C^0([0,\infty); H^1) \cap C^1([0,\infty); L^2)$ and the asymptotic behavior:

$$\lim_{t \to \infty} \sup_{x > 0} |u(x, t) - \phi(x)| = 0.$$

$$\tag{4}$$

Theorem 2. Let $\phi(x)$ be the stationary solution satisfying the problem (3). (i) Assume that $z_0 \in H^2_{\alpha}$ and $z_1 \in H^1_{\alpha}$ for $\alpha > 0$. Let u(x,t) be the global solution to the problem (1), which is constructed in Theorem 1. Then it holds that

$$||u(t) - \phi||_{H^1} \le CE_{\alpha}(1+t)^{-\alpha/2}$$

for $t \ge 0$, where C is a positive constant and $E_{\alpha} = ||z_0||_{H^2_{\alpha}} + ||z_1||_{H^1_{\alpha}}$. (ii) Assume that $z_0 \in H^2_{\alpha,exp}$ and $z_1 \in H^1_{\alpha,exp}$ for $\alpha > 0$. Let u(x,t) be the global solution to the problem (1), which is constructed in Theorem 1. Then it holds that

$$\|u(t) - \phi\|_{H^1} \le C E_{\alpha, exp} e^{-\beta t},$$

for $t \ge 0$, where β is a positive constant depending on α , C is a positive constant and $E_{\alpha,exp} = ||z_0||_{H^2_{\alpha,exp}} + ||z_1||_{H^1_{\alpha,exp}}$.

Notation. For $\alpha > 0$, $L^2_{\alpha} = L^2_{\alpha}(\mathbb{R}_+)$ and $L^2_{\alpha,exp} = L^2_{\alpha,exp}(\mathbb{R}_+)$ denotes the polynomially and exponentially weighted L^2 space with the norm defined by

$$\|u\|_{L^{2}_{\alpha}} = \left(\int_{0}^{\infty} (1+x)^{\alpha} |u(x)|^{2} dx\right)^{1/2}, \|u\|_{L^{2}_{\alpha,exp}} = \left(\int_{0}^{\infty} e^{\alpha x} |u(x)|^{2} dx\right)^{1/2},$$

respectively. **References**

- Y. Ueda: Asymptotic stability of stationary waves for damped wave equations with a nonlinear convection term, Adv. Math. Sci. Appl. 18 (2008), 329-343.
- Y. Ueda, S. Kawashima: Large time behavior of solutions to a semilinear hyperbolic system with relaxation, J. Hyperbolic Differ Equ. 4 (2007), 147-179.
- [3] Y. Ueda, T. Nakamura, S. Kawashima: Stability of degenerate stationary waves for viscous gases, Arch. Rational Mech. Anal. 198 (2010), 735-762.

Joint work with: Yoshihiro Ueda (Graduate School of Maritime Sciences, Kobe University).

S49 – Wave Analysis I – Room A, 11.50-12.20

Wave-wave interactions of a gasdynamic type

Liviu Florin Dinu Institute of Mathematics of the Romanian Academy Liviu.Dinu@imar.ro

For a quasilinear system of a gasdynamic type [ex. Euler isentropic/anisentropic; possibly multidimensional] we constructively consider, in presence of certain integrability restrictions, some highly nontrivial and significant classes of solutions.

Two *analytic* approaches are considered: a Burnat type approach [structured by a duality connection between the hodograph character and the physical character], essentially *restricted* here to a genuinely nonlinear version; and a Martin type approach [associated with a Monge–Ampère type representation], which shows *unconditionally* a genuinely nonlinear character. • Each of these two approaches results in two *significant classes* of solutions. In the *isentropic* case a genuinely nonlinear Burnat type approach constructively structure: \bullet some [possibly multidimensional] simple waves solutions – here called *waves* [a first significant class], and \bullet some [possibly multidimensional] wave-wave regular interaction solutions [a second significant class].

The present paper includes two selfsimilar isentropic exemples - significant and highly nontrivial - of twodimensional wave-wave regular interaction solutions. • The two examples are concurrently associated to some cases of a quantifiable "amount" of genuine nonlinearity. • The isentropic types of wave-wave regular interactions constructed appear to parallel, from an analytic, local and *regular* prospect, some details [interactions of simple waves solutions] of the Zhang and Zheng two-dimensional qualitative, global and *irregular* construction. • The two examples mentioned above suggest that a *regular* character of the wave-wave interaction described essentially reflects facts of a *multidimensional* and *skew* construction.

In the *anisentropic* case – and in two independent variables – a Martin type approach is associated with a *particular* gasdynamic example, to constructively structure an anisentropic *analogue* of the isentropic pair of classes mentioned above: the anisentropic pair which puts together \bullet some *pseudo* simple waves solutions [as a first significant class] and \bullet some *pseudo* wave-wave regular interaction solutions [as a second significant class]. Details concerning the nature of the mentioned *analogous* character are also presented.

A classifying parallel is concurrently presented between the two analogous pairs of classes [isentropic, anisentropic] – making evidence of some consonances and, respectively, of some nontrivial contrasts of the two mentioned constructions [genuinely nonlinear Burnat type, Martin type].

The regular passage [which uses the two analogous pairs of classes] from an isentropic description to an anisentropic description appears to be *fragile*. Some essential details of this fragility are presented. **References**

- [1] Liviu Florin Dinu and Marina Ileana Dinu, Gasdynamic regularity: some classifying geometrical remarks, Balkan Journal of Geometry and Its Applications, Volume no.15 (2010), pp. 41-52
- [2] Liviu Florin Dinu and Marina Ileana Dinu, Gasdynamic interactions: two significant examples, in Proceedings of the 33rd National Conference on Fluid Mechanics, under the Aegis of the Romanian Academy, Publishing editor Richard Selescu (2011), pp. 51-60
- [3] Liviu Florin Dinu, Wave-wave interactions of a gasdynamic type, preprint (2012), to appear on *Commu*nications in Mathematical Analysis

S49 – Wave Analysis I – Room A, 12.20-12.50

On cavitation in elastodynamics

Jan Giesselmann ACMAC, University of Crete jan.giesselmann@acmac.uoc.gr

In this contribution we study the equations of elastodynamics

$$\mathbf{y}_{tt} = \operatorname{div}(\sigma(\nabla \mathbf{y})),\tag{1}$$

with unknown deformation $\mathbf{y} : B_1(0) \times [0,T) \to \mathbb{R}^d$ for some $T > 0, d \in \mathbb{N}, B_1(0) := {\mathbf{x} \in \mathbb{R}^d : \|\mathbf{x}\| < 1}$ and given stress response $\sigma : \mathbb{R}^{d \times d}_+ \to \mathbb{R}^{d \times d}$, where $\mathbb{R}^{d \times d}_+ := {M \in \mathbb{R}^{d \times d} : \det M > 0}$. The system given by (1) is equivalent to the system of conservation laws

$$\mathbf{u}_t - \nabla \mathbf{v} = 0$$

$$\mathbf{v}_t - \operatorname{div}(\sigma(\mathbf{u})) = 0,$$

(2)

where $\mathbf{u} = \nabla \mathbf{y}$ is the strain and $\mathbf{v} = \mathbf{y}_t$ is the velocity. In particular, we are interested in radially symmetric solutions having the form

$$\mathbf{y}(\mathbf{x},t) = t\varphi\left(\frac{\|\mathbf{x}\|}{t}\right)\frac{\mathbf{x}}{\|\mathbf{x}\|}$$
(3)

for some $\varphi: [0,\infty) \to [0,\infty)$, to the initial boundary value problem

$$\mathbf{y}(\mathbf{x}, 0) = \lambda \mathbf{x}, \ \mathbf{y}_t(\mathbf{x}, 0) = 0 \quad \forall \mathbf{x} \in B_1(0),$$
$$\mathbf{y}(\mathbf{x}, t) = \lambda \mathbf{x} \quad \forall \mathbf{x} \in \partial B_1(0) \text{ and } 0 \le t < T$$

for some $\lambda > 0$. Such problems were investigated in [1] in the static and in [2] in the dynamic case. In those works solutions containing a cavity at the origin, i.e. $\varphi(0) > 0$, are constructed.

For d = 1 a solution of (1) which has a cavity corresponds to a solution of (2) where u is a L^1 -function plus a δ -distribution. We give a meaning to $\sigma(u)$ for convex σ (for d = 1), using sequences of functions which approximate u. We will see that there can only be a solution with cavity in case $\lim_{u\to\infty} \frac{\sigma(u)}{u} = 0$.

For d = 3 we define energies of solutions containing cavities in a way which takes into account the contribution of the cavity, i.e. a region of infinite strain. This differs from the definition of energy in [1,2]. For this new definition of energy we show that the solution with a cavity has a higher energy at every time than the trivial solution $\mathbf{y}(\mathbf{x}, t) = \lambda \mathbf{x}$.

References

- J. M. Ball, Discontinuous Equilibrium Solutions and Cavitation in Nonlinear Elasticity, *Philos. Trans.* Roy. Soc. London Ser. A, **306**, (1982) no. 1496, pp. 557–611
- [2] K. A. Pericak-Spector, S. J. Spector, Nonuniqueness for a hyperbolic system: cavitation in nonlinear elastodynamics, Arch. Rational Mech. Anal. 101, (1988), no. 4, pp. 293–317

Joint work with: Athanasios E. Tzavaras (University of Crete).

10.5 Session 50 — Room E — Electro-Magnetic Flows & High Frequency Phenomena

S50 – Electro-Magnetic Flows & High Frequency Phenomena – Room E, 11.20–11.50

The Riemann problem for a full-wave Maxwell model modeling electromagnetic propagation in a nonlinear Kerr medium

Denise Aregba-Driollet Univ. Bordeaux, IMB aregba@math.u-bordeaux1.fr

In some contexts optical beams propagating in a nonlinear Kerr medium can be modelled by Maxwell's equations with the constitutive relations

$$\begin{bmatrix}
B &= \mu_0 H \\
D &= \epsilon_0 E + P
\end{bmatrix}$$

where P is the nonlinear polarization:

$$P = P_K = \epsilon_0 \epsilon_r |E|^2 E.$$

 μ_0 , ϵ_0 are the free space permeability and permittivity, ϵ_r is the relative permittivity. Choosing as unknowns the electric displacement D and the magnetic field H this model reads as a 6×6 three-dimensionnal nonlinear system of conservation laws:

$$\begin{cases} \partial_t D - \operatorname{curl} H = 0, \\ \partial_t H + \mu_0^{-1} \operatorname{curl}(\mathbf{P}(D)) = 0 \end{cases}$$

where \mathbf{P} is the reciprocal function of \mathbf{D} :

$$\mathbf{D}(E) = \epsilon_0 (1 + \epsilon_r |E|^2) E.$$

This system is called Kerr system in the following. It is endowed with a strictly convex entropy, namely the energy density, so that it is hyperbolic symmetrizable. It is supplemented with the divergence free conditions on D and H.

For smooth solutions with small data, global existence is proved in [4]. As far as one is concerned with weak solutions, there is no general existence result. As a first step into the comprehension of such solutions, Kerr shocks and related relaxation Kerr-Debye shock profiles have been studied in [1], the Kerr-Debye system being obtained by approximating the instantaneous polarization P by a model with a finite response time.

Solving the Riemann problem enlightens the theoritical properties of weak solutions as well as it is an essential tool for the design of numerical schemes. For Kerr system, partial results on this problem can be found in [3]. In the present work we solve the Riemann problem for the 6×6 full wave system. For each direction $\omega \in \mathbb{R}^3$, the Kerr system owns six eigenvalues with variable multiplicity. The characteristic fields 1, 3, 4, 6 are linearly degenerate, and the others are neither linearly degenerate nor genuinely nonlinear. More precisely let us denote u = (D, H), $\lambda_i(\omega, u)$ the ith eigenvalue and $r_i(\omega, u)$ the related eigenvector.

$$\lambda_1(\omega, u) = \lambda_2(\omega, u)$$
 and $\lambda_5(\omega, u) = \lambda_6(\omega, u)$ if and only if $D \times \omega = 0$,

and for $i \in \{2, 5\}$:

$$\lambda'_i(\omega, u) \cdot r_i(\omega, u) \neq 0$$
 if and only if $D \times \omega \neq 0$.

For all Riemann data, we construct a Lax entropy solution as a combination of simple waves: 1-contact discontinuity, 2-shock or rarefaction, 3-4-contact discontinuity, 5-shock or rarefaction, 6-contact discontinuity. We prove that entropy solutions structured like this are unique. When the solution contains shocks, those shocks are admissible in the sense of Liu [5] and in the sense of Lax, and they satisfy an energy inequality.

Generally speaking, the 1 and 6 contact discontinuities are rotating modes which put the intermediate electric displacements and ω in a same plane where shocks and rarefactions occur. Special physical configurations such as TE or TM propagation give rise to reduced 3×3 or even 2×2 models. We point out the fact that even if a rotating contact discontinuity of the full wave system may be a weak solution of a reduced model, it is not an admissible solution for it. The entropy solution of the reduced model differs from the one of the full wave model, and this induces numerical problems like the ones observed in [3].

References

- D. Aregba-Driollet and B. Hanouzet, Kerr-Debye relaxation shock profiles for Kerr equations, Commun. Math. Sci., 9 (2011), pp. 1-31.
- [2] G. Carbou and B. Hanouzet, Relaxation approximation of Kerr Model for the three dimensional initialboundary value problem, J. Hyperbolic Differ. Equ., 6 (2009), pp. 577-614.
- [3] A. de La Bourdonnaye, High-order scheme for a nonlinear Maxwell system modelling Kerr effect, J. Comput. Phys., 160 (2000), pp. 500–521.
- [4] M. Kanso, PhD thesis, 2012.
- [5] T.-P. Liu, The entropy condition and the admissibility of shocks, J. Math. Anal. Appl., 53 (1976), pp. 78–88.
- [6] R.-W. Ziolkowski, The incorporation of microscopic material models into FDTD approach for ultrafast optical pulses simulations, *IEEE Transactions on Antennas and Propagation*, 45 (1997), pp. 375-391.

* * *

S50 – Electro-Magnetic Flows & High Frequency Phenomena – Room E, 11.50–12.20

High-frequency limit of the Maxwell-Landau-Lifshitz system in the diffractive optics regime

Yong Lu Université Paris-Diderot (Paris 7), Institut de Mathématiques de Jussieu, UMR CNRS 7586 luyong@math.jussieu.fr

We study semilinear Maxwell-Landau-Lifshitz systems in one space dimension of the form:

$$\partial_t v + A(e_1)\partial_y v + \frac{L_0 v}{\varepsilon} = B(v, v), \tag{1}$$

where the unknown $v \in \mathbb{R}^9$, the space variable is $y \in \mathbb{R}^1$. The symmetric matrix $A(e_1)$ and skew-symmetric matrix L_0 are defined as

$$A(e_1) = \begin{pmatrix} 0 & -e_1 \times & 0\\ e_1 \times & 0 & 0\\ 0 & 0 & 0 \end{pmatrix}, \quad L_0 = \begin{pmatrix} 0 & 0 & 0\\ 0 & -e_1 \times & e_1 \times\\ 0 & e_1 \times & -e_1 \times \end{pmatrix},$$
(2)

where the vector $e_1 = (1, 0, 0) \in \mathbb{R}^3$. We consider highly oscillatory initial data of the form

$$v(0,y) = a(y)e^{iky/\varepsilon} + \overline{a(y)}e^{-iky/\varepsilon} + \varepsilon a_1(y,ky/\varepsilon) + \varepsilon^2 a_2(y,ky/\varepsilon),$$
(3)

where $a_1(y,\theta)$ and $a_2(y,\theta)$ are real-valued and 2π -periodic in θ .

We first construct WKB approximate solutions v^a over long times $O(1/\varepsilon)$. The leading terms of the WKB solutions solve cubic Schrödinger equations. Then we show that the Schrödinger approximation stays close to the exact solution of Maxwell-Landau-Lifshitz over its existence time $O(1/\varepsilon)$ with an error estimate that is comparable to the initial error. Precisely, for prepared and regular initial data (3), we obtain the following result

$$\sup_{t \in [0, T/\varepsilon]} \| (v - v^a)(t) \|_{L^{\infty}(\mathbb{R}^1_y)} \le C \| (v - v^a)(0) \|_{L^{\infty}(\mathbb{R}^1_y)} \le C\varepsilon,$$

where T > 0 independent of ε , v and v^a are respectively the exact solution and the approximate solution. In the context of Maxwell-Landau-Lifshitz, this extends the analysis of Colin and Lannes [2] from times $O(|\ln \varepsilon|)$ up to $O(1/\varepsilon)$.

References

- J.-L. Joly, G. Métivier, J. Rauch, Transparent nonlinear geometric optics and Maxwell-Bloch equations, J. Diff. Eq., vol. 166 (2000), 175-250.
- [2] T. Colin, D. Lannes, Justification of and long-wave correction to Davey-Stewartson systems from quadratic hyperbolic systems, Discrete and Continuous Dynamical Systems, vol. 11, n. 1 (2004), 83-100.

* * *

S50 – Electro-Magnetic Flows & High Frequency Phenomena – Room E, 12.20–12.50

Mather measures in semiclassical analysis

Lorenzo Zanelli Università di Bologna lorenzo.zanelli@unibo.it

We discuss the Mather's minimization problem, i.e. to find measures which are Action minimizing and invariant under the Lagrangian flow, in the framework of semiclassical analysis. We show that the Legendre transform of a relevant class of Mather measures is the semiclassical limit of the Wigner transform of energy quasimodes.

References

- N. Anantharaman, Gibbs measures and semiclassical approximation to action-minimizing measures, arXiv:math/0204190.
- [2] O. Bernardi, A. Parmeggiani, L. Zanelli, Mather measures associated with a class of Bloch wave functions, preprint (2012), to appear on Annales Henri Poincaré.
- [2] L-C. Evans: Towards a quantum analog of weak KAM theory. Comm. Math. Phys. 244 (2004), no. 2, 311-334.
- [3] L-C. Evans, Further PDE methods for weak KAM theory. Calc. Var. Partial Differential Equations 35 no. 4, 435-462 (2009).
- [4] D-A. Gomes, A-O. Lopes, J. Mohr: Wigner measures and the semi-classical limit to the Aubry-Mather measure. arxiv: 1111.3187
- [5] D-A. Gomes; C. Valls: Wigner measures and quantum Aubry-Mather theory. Asymptot. Anal. 51 (2007), no. 1, 47-61.
- [6] J.N. Mather, Action minimizing invariant measures for positive def- inite Lagrangian systems. Math. Z. 207, 169-207 (1991).
- [7] M. Ruzhansky, V. Turunen: Quantization of pseudo-differential operators on the torus. J. Fourier Anal. Appl. 16 no. 6, 943-982 (2010).

Joint work with: Olga Bernardi (Università di Padova), Alberto Parmeggiani (Università di Bologna)

10.6 Session 51 — Room D — Traffic Flow and Population Dynamics

S51 - Traffic Flow and Population Dynamics - Room D, 11.20-11.50

Scalar conservation laws with moving density constraints arising in traffic flow modeling

Maria Laura Delle Monache INRIA Sophia Antipolis - Méditerranée, EPI OPALE, 2004, route des Lucioles - BP 93, 06902 Sophia Antipolis Cedex (France) maria-laura.delle_monache@inria.fr The aim of this work is to study the well-posedness of a PDE-ODE coupled model with application to traffic flow.

We refer to a model proposed in [2]: a slow moving large vehicle along a road reduces its capacity and thus generates a moving bottleneck for the cars flow. From the macroscopic point of view this can be modeled by a PDE-ODE coupled model consisting in a scalar conservation law with moving density constraint and an ODE describing the slower vehicle motion, i.e.,

$$\begin{cases} \partial_t \rho + \partial_x f(\rho) = 0, & (t, x) \in \mathbb{R}^+ \times \mathbb{R}, \\ \rho(0, x) = \rho_0(x), & x \in \mathbb{R}, \\ \rho(t, y(t)) \le \alpha R, & t \in \mathbb{R}^+, \\ \dot{y}(t) = \omega(\rho(t, y(t)+)), & t \in \mathbb{R}^+, \\ y(0) = y_0. \end{cases}$$
(1)

Above, $\rho = \rho(t, x) \in [0, R]$ is the scalar conserved quantity representing the mean traffic density, R is the maximal density allowed on the road and the flux function $f : [0, R] \to \mathbb{R}^+$ is a strictly concave function such that f(0) = f(R) = 0. It is given by the formula

$$f(\rho) = \rho v(\rho),$$

where v is a smooth decreasing function denoting the mean traffic speed and here set to be $v(\rho) = V(1 - \rho/R)$, V being the maximal velocity allowed on the road.

The time-dependent variable y denotes the bus position, that moves with a traffic density dependent speed of the form

$$\omega(\rho) = \begin{cases} V_b & \text{if } \rho \le \rho^* \doteq R(1 - V_b/V), \\ v(\rho) & \text{otherwise,} \end{cases}$$
(2)

that is, the slow vehicle moves with constant speed $V_b < V$ as long as it is not slowed down by downstream traffic conditions. When this happens, it moves with the mean traffic speed.

Finally, the constant coefficient $\alpha \in [0, 1]$ gives the reduction rate of the road capacity due to the presence of the bus.

The above model can be viewed as a generalization to moving constraints of the problem consisting in a scalar conservation law with a (fixed in space) constraint on the flux, introduced in [1]. Here, the constraint location moves due to the surrounding traffic conditions, which in turn is modified by the presence of the slower vehicle, thus resulting in a strong non-trivial coupling. Compared to the model recently proposed by [3], problem (1)-(2) offers a more realistic definition of the bus velocity and a description of its impact on traffic flow which is simpler to handle both from the analytical and the numerical point of view.

References

- R. M. Colombo and P. Goatin, A well posed conservation law with a variable unilateral constraint, J. Differential Equations, 234(2) (2007), pp. 654-675
- [2] F. Giorgi, Prise en compte des transports en commune de surface dans la modisation macroscopique de l'coulement du trafic, Institut National des Sciences Appliques de Lyon, (2002)
- [3] C. Lattanzio, A. Maurizi and B. Piccoli, Moving bottlenecks in car traffic flow: a PDE-ODE coupled model, SIAM J. Math. Anal., 43(1) (2011), pp. 50-67

Joint work with: Paola Goatin (INRIA Sophia Antipolis - Méditerranée, EPI OPALE, 2004, route des Lucioles - BP 93, 06902 Sophia Antipolis Cedex (France))
S51 - Traffic Flow and Population Dynamics - Room D, 11.50-12.20

On the Management of Vehicular Traffic

Massimiliano Daniele Rosini ICM, Warsaw University mrosini@icm.edu.pl

Several realistic situations in vehicular traffic that give rise to queues can be modeled through conservation laws with initial-boundary data and unilateral constraints on the flux. We provide a rigorous analytical framework for these descriptions, comprising stability with respect to the initial data, to the boundary inflow and to the constraint. We present a framework to rigorously state optimal management problems and prove the existence of the corresponding optimal controls. Specific cases are dealt with in detail through *ad hoc* numerical integrations. These are here obtained implementing the wave front tracking algorithm, which appears to be very precise in computing, for instance, the exit times.

- D. Amadori, Initial-boundary value problems for nonlinear systems of conservation laws, NoDEA Nonlinear Differential Equations Appl., 4 (1997), pp. 1-42
- [2] D. Amadori and R. M. Colombo, Continuous dependence for 2 × 2 conservation laws with boundary, J. Differential Equations, 138 (1997), pp. 229-266
- [3] F. Ancona and A. Marson, Scalar non-linear conservation laws with integrable boundary data, Nonlinear Anal., 35 (1999), pp. 687-710
- [4] B. Andreianov, P. Goatin, and N. Seguin, Finite volume schemes for locally constrained conservation laws, *Numer. Math.*, **115** (2010), pp. 609-645
- [5] C. Bardos, A. Y. le Roux, and J.-C. Nédélec, First order quasilinear equations with boundary conditions, Comm. Partial Differential Equations, 4 (1979), pp. 1017-1034
- [6] A. Bressan, Hyperbolic systems of conservation laws, Oxford University Press, (2000)
- [7] W. Chen, S. C. Wong, C. W. Shu, and P. Zhang, Front tracking algorithm for the Lighthill-Whitham-Richards traffic flow model with a piecewise quadratic, continuous, non-smooth and non-concave fundamental diagram, Int. J. Numer. Anal. Model., 6 (2009), pp. 562-585
- [8] R. M. Colombo and P. Goatin, A well posed conservation law with a variable unilateral constraint, J. Differential Equations, 234 (2007), pp. 654-675
- [9] R. M. Colombo, P. Goatin, and M. D. Rosini, On the modeling and management of traffic, ESAIM: Mathematical Modelling and Numerical Analysis, 45 (2011), pp. 853-872
- [10] R. M. Colombo and A. Groli, Minimising stop and go waves to optimise traffic flow, Appl. Math. Lett., 17 (2004), pp. 697-701
- [11] C. M. Dafermos, Polygonal approximations of solutions of the initial value problem for a conservation law, J. Math. Anal. Appl., 38 (1972), pp. 33-41
- [12] C. Daganzo, The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory, *Transpn. Res.-B*, **28B** (1994), pp. 269-287
- [13] F. Dubois and P. LeFloch, Boundary conditions for nonlinear hyperbolic systems of conservation laws, J. Differential Equations, 71 (1988), pp. 93-122
- [14] M. Garavello and B. Piccoli, *Traffic flow on networks*, American Institute of Mathematical Sciences (AIMS), (2006)

- [15] J. Goodman, Initial Boundary Value Problems for Hyperbolic Systems of Conservation Laws, California University, (1982)
- [16] H. Greenberg, An analysis of traffic flow, Operations Res., 7 (1959), pp. 79-85
- [17] B. Greenshields, A study of traffic capacity, Proceedings of the Highway Research Board, 14 (1935), pp. 448-477
- [18] H. Holden and N. H. Risebro, Front tracking for hyperbolic conservation laws, Springer-Verlag, New York, (2002)
- [19] W. L. Jin and H. M. Zhang, The formation and structure of vehicle clusters in the Payne-Whitham traffic flow model, *Transp. Res. B*, **37** (2003), pp. 207-223
- [20] B. S. Kerner and P. Konhäuser, Cluster effect in initially homogeneous traffic flow, *Physical Review E*, 48(1993), pp. R2335-R2338
- [21] B. S. Kerner and P. Konhäuser, Structure and parameters of clusters in traffic flow, *Physical Review E*, 50 (1994), pp. 54-83
- [22] A. Klar, Kinetic and Macroscopic Traffic Flow Models, School of Computational Mathematics: Computational aspects in kinetic models, (2002)
- [23] S. N. Kružhkov, First order quasilinear equations with several independent variables, Mat. Sb. (N.S.), 81 (1970), pp. 228-255
- [24] R. J. LeVeque, Finite volume methods for hyperbolic problems, Cambridge University Press, (2002)
- [25] M. J. Lighthill and G. B. Whitham, On kinematic waves. II. A theory of traffic flow on long crowded roads, Proc. Roy. Soc. London. Ser. A., 229 (1955), pp. 317-345
- [26] G. Newell, A simplified theory of kinematic waves in highway traffic, part II, Transp. Res. B, 27B (1993), pp. 289-303
- [27] E. Y. Panov, Existence of strong traces for quasi-solutions of multidimensional conservation laws, J. Hyperbolic Differ. Equ., 4 (2007), pp. 729-770
- [28] P. I. Richards, Shock waves on the highway, Operations Res., 4 (1956), pp. 42-51
- [29] D. Serre, Systems of conservation laws. 1 & 2, Cambridge University Press, Cambridge, (1999)
- [30] B. Temple, Global solution of the Cauchy problem for a class of 2 × 2 nonstrictly hyperbolic conservation laws, Adv. in Appl. Math., 3 (1982), pp. 335-375
- [31] E. Tomer, L. Safonov, N. Madar, and S. Havlin, Optimization of congested traffic by controlling stop-andgo waves, *Phys. Rev. E (3)*, 65 (2002), pp. 065101

Joint work with: Paola Goatin (INRIA Sophia Antipolis - Méditerranée), Rinaldo Mario Colombo (Department of Mathematics, Brescia University).

* * * -

S51 – Traffic Flow and Population Dynamics – Room D, 12.20–12.50

An adaptive finite-volume method for a model of two-phase pedestrian flow

Stefan Berres Universidad Católica de Temuco sberres@uct.cl

A flow composed of two populations of pedestrians moving in different directions is modeled by a two-dimensional system of convection-diffusion equations. An efficient simulation of the two-dimensional model is obtained by a finite-volume scheme combined with a fully adaptive multiresolution strategy. Numerical tests show the flow behavior in various settings of initial and boundary conditions, where different species move in countercurrent or perpendicular directions. The equations are characterized as hyperbolic-elliptic degenerate, with an elliptic region in the phase space, which in one space dimension is known to produce oscillation waves. When the initial data are chosen inside the elliptic region, a spatial segregation of the populations leads to pattern formation. The entries of the diffusion-matrix determine the stability of the model and the shape of the patterns.

References

- S. Berres, R. Bürger and A. Kozakevicius, Numerical approximation of oscillatory solutions of hyperbolicelliptic systems of conservation laws by multiresolution schemes, Adv. Appl. Math. Mech., 1 (2009), 581–614.
- [2] S. Berres, R. Ruiz-Baier, H. Schwandt, E.M. Tory, An adaptive finite-volume method for a two-phase model of pedestrian flow, *Networks and Heterogeneous Media*, 6 (2011), 401–423.
- [3] J.H. Bick, G.F. Newell, A Continuum Model for Two-Directional Traffic Flow, Quart. Appl. Math., 18 (1960), 191–204.
- [4] H. Frid, I-S. Liu, Oscillation waves in Riemann problems inside elliptic regions for conservation laws of mixed type, Z. Angew. Math. Phys., 46 (1995), 913–931.

Joint work with: Ricardo Ruiz-Baier (*Ecole Polytechnique Fédérale de Lausanne*), Hartmut Schwandt (*Technische Universität Berlin*), Elmer M. Tory (*Mount Allison University*).

10.7 Session 52 — Room C — Theory of Conservation Laws VI

S52 - Theory of Conservation Laws VI - Room C, 11.20-11.50

Generalizing the Bardos-LeRoux-Nédélec boundary condition for scalar conservation laws

Boris P. Andreianov Laboratoire de Mathématiques CNRS UMR 6623, Université de Franche-Comté, Besançon, France boris.andreianov@univ-fcomte.fr In the study of boundary-value problems for scalar conservation laws, the Bardos, LeRoux and Nédélec paper [1] is the crucial reference. In [1], the interpretation of the Dirichlet boundary condition for problem

$$\partial_t u + \operatorname{div} \varphi(u) = 0 \text{ in } \Omega \subset \mathbb{R}^N, \ u(0, \cdot) = u_0$$

was established. Indeed, if the Dirichlet boundary condition is imposed at the level of the viscosity approximation, the approximate solutions may develop a boundary layer, so that the limit solution verifies a different boundary condition. The BV assumption on the data, used in [1] for existence of boundary traces of u, has now been dropped thanks to "strong trace" regularity results of Vasseur [4] and Panov [5]. The zero-flux boundary condition, at least as important in practice as the Dirichlet one, has received much less attention in the literature. Bürger, Frid and Karlsen in [3] treat the important case of a compactly supported φ : under this assumption, there is no boundary layer.

Our interest goes to the zero-flux condition beyond the setting of [3], as well as to the Robin, obstacle, mixed boundary conditions. All these examples are particular cases of general "dissipative" boundary conditions for conservation laws, i.e., the conditions that are compatible with the L^1 -dissipative structure of the conservation law. In general, they are stated under the form $\varphi(u) \cdot \nu(x) \in \beta_{(t,x)}(u)$, where $\nu(x)$ is the exterior unit normal at a point $x \in \partial\Omega$, and $\beta := (\beta_{(t,x)})_{t \in (0,T), x \in \partial\Omega}$ is a measurable family of maximal monotone graphs.

Our interpretation of the boundary condition involves a projection $\beta_{(t,x)}$ of $\beta_{(t,x)}$ which is the closest to $\beta_{(t,x)}$ maximal monotone subgraph of $\varphi(.) \cdot \nu(x)$ that contains all the points of crossing between the graphs $\beta_{(t,x)}$ and $\varphi(.) \cdot \nu(x)$. This can be seen as a generalization of the Dubois and LeFloch's [2] graphical interpretation of the Bardos-LeRoux-Nédélec condition. The first results that we presented in [6] and [7] are now extended so that to yield a full well-posedness theory. This includes several equivalent definitions of entropy solutions for conservation laws with dissipative boundary condition; the associated uniqueness and comparison theorems; results of convergence of vanishing viscosity approximation; and different stability results with respect to perturbations of β .

References

- C. Bardos, A.Y. Le Roux, J.-C. Nédélec. First order quasilinear equations with boundary conditions. Comm. Partial Diff. Equ. 4 (1979), pp. 1017–1034.
- F. Dubois and Ph. LeFloch. Boundary conditions for nonlinear hyperbolic systems of conservation laws. J. Diff. Equ. 71 (1988), pp. 93–122.
- [3] R. Bürger, H. Frid and K.H. Karlsen. On the well-posedness of entropy solutions to conservation laws with a zero-flux boundary condition. J. Math. Anal. Appl. 326 (2007), pp. 108–120.
- [4] A. Vasseur. Strong traces for weak solutions to multidimensional conservation laws. Arch. Rat. Mech. Anal. 160(3):181–193, 2001.
- [5] E. Yu. Panov. Existence of strong traces for quasi-solutions of multidimensional conservation laws. J. Hyp. Diff. Equ., 4 (2007), pp. 729–770.
- [6] B. Andreianov and K. Sbihi. Strong boundary traces and well-posedness for scalar conservation laws with dissipative boundary conditions, in *Hyperbolic problems: theory, numerics, applications*, S. Benzoni-Gavage and D. Serre, eds, Springer, Berlin (2008), pp. 937–945
- [7] B. Andreianov and K. Sbihi. Scalar conservation laws with nonlinear boundary conditions. C. R. Acad. Sci. Paris, Ser. I, 345 (2007), pp. 431–434.

Joint work with: Karima Sbihi (Toulouse)

Session 52 — Room C — Theory of Conservation Laws VI

S52 - Theory of Conservation Laws VI - Room C, 11.50-12.20

Well-posedness of continuity equations with low regularity coefficients defined in domains with boundary

Laura V. Spinolo IMATI-CNR, Pavia, Italy spinolo@imati.cnr.it

The talk will focus on continuity equations with weakly differentiable coefficients. Existence and uniqueness results for the Cauchy problem have been established for coefficients with Sobolev and BV (bounded total variation) regularity by DiPerna-Lions [4] and by Ambrosio [1], respectively. These results have been applied to the analysis of several nonlinear partial differential equations, including multidimensional systems of conservation laws [2].

During the talk I will discuss existence and uniqueness results for continuity equations with BV coefficients defined in domains with boundary. Data are assigned at the initial time and on the portions of the boundary where the coefficients are inward-pointing. Under suitable hypotheses on the orientation of the coefficients at the boundary, the above-mentioned well-posedness results can be extended to the case when the total variation is only locally bounded. However, I will exhibit an example showing that, in the general case, uniqueness is violated when the total variation of the coefficients blows up at the boundary of the domain.

References

- Ambrosio L., Transport equation and Cauchy problem for BV vector fields, *Invent. Math.*, 158 (2004), pp. 227-260.
- [2] Ambrosio L., Bouchut F. and De Lellis C., Well-posedness for a class of hyperbolic systems of conservation laws in several space dimensions, *Comm. Partial Differential Equations*, **29** (2004), pp. 1635-1651.
- [3] Crippa G., Donadello C. and Spinolo L.V., Well-posedness of continuity equations with low regularity coefficients defined in domains with boundary. *In preparation*.
- [4] DiPerna, R. J. and Lions, P.-L., Ordinary differential equations, transport theory and Sobolev spaces, *Invent. Math.*, 98 (1989), pp. 511-547.

Joint work with: Gianluca Crippa (Universität Basel, Switzerland), Carlotta Donadello (Université de Franche-Comté, Besançon, France)

* * * _____

S52 – Theory of Conservation Laws VI – Room C, 12.20–12.50

Lower compactness estimates for scalar balance laws

Khai T. Nguyen Dipartimento di Matematica, Università degli Studi di Padova, Italy khai@math.unipd.it We study the compactness in L^1_{loc} of the semigroup $(S_t)_{t\geq 0}$ of entropy weak solutions to strictly convex scalar conservation laws in one space dimension. The compactness of S_t for each t > 0 was established by P. D. Lax [1]. Upper estimates for the Kolmogorov's ε -entropy of the image through S_t of bounded sets C in $L^1 \cap L^{\infty}$ which is denoted by

$$H_{\varepsilon}(S_t(C) \mid L^1(\mathbb{R})) := \log_2 N_{\varepsilon}(S_t(C)).$$

where $N_{\varepsilon}(S_t(C))$ is the minimal number of sets in a cover of $S_t(C)$ by subsets of $L^1(\mathbb{R})$ having diameter no larger than 2ε , were given by C. De Lellis and F. Golse [3]. Here, we provide lower estimates on this ε -entropy of the same order as the one established in [3], thus showing that such an ε -entropy is of size $\approx (1/\varepsilon)$. Moreover, we extend these estimates of compactness to the case of convex balance laws.

References

- Lax P. D., Weak solutions of nonlinear hyperbolic equations and their numerical computation, Comm. Pure Appl. Math., Volume no. 7 (1954), pp. 159-193.
- [2] Lax P. D., Accuracy and resolution in the computation of solutions of linear and nonlinear equations. Recent advances in numerical analysis, *Publ. Math. Res. Center Univ. Wisconsin.*, Academic Press, New York (1978), pp. 107-117.
- [3] De Lellis C., Golse F., A Quantitative Compactness Estimate for Scalar Conservation Laws, Comm. Pure Appl. Math., Volume no. 7 (2005), pp. 989-998.
- [4] Fabio Ancona, Olivier Glass and Khai T. Nguyen, Lower compactness estimates for scalar balance laws, preprint (2011), to appear on Comm. Pure Appl. Math.

Joint work with: Fabio Ancona (Dipartimento di Matematica, Università degli Studi di Padova, Italy), Olivier Glass (Ceremade, Université Paris-Dauphine, France).

10.8 Session 53 — Room H — BioFluids Models II

S53 – BioFluids Models III – Room H, 11.20–11.50

Fractional Conservation Laws and Keller-Segel Type System in Higher Dimensions

Suleyman Ulusoy Zirve University suleyman.ulusoy@zirve.edu.tr

In this talk, we will introduce our results on Levy mixed hyperbolic-parabolic equations and also on a Keller-Segel type system in higher dimensions. If time permits we will also talk about the Keller-Segel type system with a nonlinear, nonlocal diffusion operator.

- [1] Eric A. Carlen and Suleyman Ulusoy, On a higher dimensional analog of Keller-Segel type system, preprint (2012).
- [2] Kenneth H. Karlsen and Suleyman Ulusoy, Hyperbolic Keller-Segel system with fractional degenerate diffusion, preprint (2012).
- [3] Kenneth H. Karlsen and Suleyman Ulusoy, Stability of entropy solutions for Levy mixed hyperbolicparabolic equations, *Electronic Journal of Differential Equation*, Volume no. 2011 (2011), pp. 1-23.

Joint work with: Kenneth H. Karlsen (University of Oslo), Eric A. Carlen (Rutgers University).

S53 – BioFluids Models III – Room H, 11.50-12.20

Multi-dimensional Degenerate Keller-Segel system with new diffusion exponent 2n/(n+2)

Jinhuan Wang

Department of Mathematics, Liaoning University, Shenyang 110036, P. R. China AND Department of Mathematical Sciences, Tsinghua University, Beijing, 100084, People's Republic of China wangjinhuan@math.tsinghua.edu.cn

Joint work with Li Chen and Jian-Guo Liu

This talk will deal with a degenerate diffusion Patlak-Keller-Segel system in $n \ge 3$ dimension with homogeneous degenerate diffusion:

$$\begin{cases}
\rho_t = \Delta \rho^m - \operatorname{div}(\rho \nabla c), & x \in \mathbb{R}^n, \ t \ge 0, \\
-\Delta c = \rho, & x \in \mathbb{R}^n, \ t \ge 0, \\
\rho(x,0) = \rho_0(x), & x \in \mathbb{R}^n,
\end{cases}$$
(1)

where diffusion exponent is taken to be $m = \frac{2n}{n+2} \in (1,2)$. This model is widely used to describe the collective motion of cells. Here $\rho(x,t)$ represents the bacteria density and c(x,t) represents the chemical substance concentration.

The main difference between the current work [1] and many recent works on the same model is that we study the diffusion exponent m = 2n/(n+2) which is smaller than the usually used exponent $m^* = 2 - 2/n$ in many recent works [2]. With the exponent m = 2n/(n+2), the associated free energy is conformal invariant. Moreover, there is a family of stationary solution $U_{\lambda,x_0}(x) = C(n)(\frac{\lambda}{\lambda^2 + |x-x_0|^2})^{\frac{n+2}{2}}, \forall \lambda > 0, x_0 \in \mathbb{R}^n$, and the L^m norm of the stationary solution is fixed constant independ ed of λ and x_0 .

Our main results: At first, for radially symmetric solutions, we prove that if the initial data is strictly below $U_{\lambda,0}(x)$ for some λ then the solution vanishes in L^1_{loc} as $t \to \infty$; if the initial data is strictly above $U_{\lambda,0}(x)$ for some λ then the solution either blows up at a finite time or has a mass concentration at r = 0 as time goes to infinity. Next, for general initial data, we prove that there is a global weak solution provided that the L^m norm of initial density is less than a universal constant, and the weak solution vanishes as time goes to infinity. Finally, We give a finite time blow up of the solution if the L^m norm for initial data is larger than the L^m norm of $U_{\lambda,x_0}(x)$, and the free energy of initial data is smaller than that of $U_{\lambda,x_0}(x)$.

References

- L. Chen, J.-G. Liu and J. H. Wang, Degenerate Keller-Segel system, preprint (2011), to appear on SIAM Journal on Mathematical Analysis.
- [2] A. Blanchet, J. A. Carrillo, and P. Laurencot, Criticaor a Patlak-Keller-Segel model with degenerate diffusion in higher dimensions, *Calc. Var.*, **35** (2009), pp. 133–168.

Joint work with: Li Chen (Tsinghua University, Beijing, 100084, China) and Jian-Guo Liu (Duke University, Durham, NC 27708. USA)...

185

S53 – BioFluids Models III – Room H, 12.20--12.50

Numerical techniques for solving nonlinear kinetic rate model of reactive liquid Chromatography

Shumaila Javeed

Max Planck Institute for Dynamics of Complex Technical Systems, Sandtorstr. 1, 39106 Magdeburg, Germany, Institute for Analysis and Numeric, Otto-von-Guericke University, 39106 Magdeburg, Germany javeed@mpi-magdeburg.mpg.de

Chromatography is a versatile separation techniques widely used for analysis, purification, and for the production of fine chemicals in pharmaceutical and food industries. The technique is particularly useful for numerous complex processes, such as the separation of enantiomers and the isolation of proteins. Chromatographic models are useful for understanding, design and control of the process. The objective of this work is to numerically analyze the kinetic rate model of reactive liquid chromatography. The model contains coupled system of convection-diffusion-reaction partial differential equations with dominated convective terms, ordinary differential equations and algebraic equations. The simulation of multi-component chromatographic processes under nonlinear conditions and reaction kinetics require fast and accurate numerical methods. Shocks discontinuities may develop in the solution, causing numerical difficulties for the schemes. Thus, an efficient and accurate numerical method is needed for producing physically realistic solutions. A high resolution semi-discrete flux-limiting finite volume scheme is proposed for solving this model. The suggested scheme is capable to suppress numerical oscillations and, thus, preserves the positivity of numerical solutions. Moreover, the scheme has capability to accurately capture sharp discontinuities of chromatographic fronts on coarse grids. The method is robust and well suited for large-scale time-dependent simulations of chromatographic processes where accuracy is highly demanding. Several test problems of reactive chromatographic processes are investigated. The results of the current method are validated against other available schemes.

References

- S. Javeed, S. Qamar, A. Seidel-Morgenstern, G. Warnecke, Efficient and accurate numerical simulation of nonlinear chromatographic processes, *Computer and Chemical Engineering*, 35. (2011), pp. 2294-2305.
- [2] 2. S. Javeed, S. Qamar, A. Seidel-Morgenstern, G. Warnecke, A discontinuous Galerkin method to solve chromatographic models, *Journal of Chromatography A*, 40., (2011), pp. 7137-7146.
- [3] S. Javeed, S. Qamar, A. Seidel-Morgenstern, G. Warnecke, Parametric study of thermal effects in reactive liquid chromatography, (2012), to appear on *Chemical Engineering Journal*

Joint work with: Shamsul Qamar, (Max Planck Institute for Dynamics of Complex Technical Systems, Sandtorstr. 1, 39106 Magdeburg, Germany), Andreas Seidel-Morgenstern, (Max Planck Institute for Dynamics of Complex Technical Systems,

Sandtorstr. 1, 39106 Magdeburg, Germany), Gerald Warnecke, (Institute for Analysis and Numeric, Otto-von-Guericke University, 39106 Magdeburg, Germany)

10.9 Session 54 — Room I — Models and Simulations in Mechanics

S54 – Models and Simulations in Mechanics – Room I, 11.20–11.50

Detonation wave problems: modeling, numerical simulations and linear stability

Filipe Carvalho Centro de Matemática, Universidade do Minho, Braga, Portugal [filipecarvalho@esce.ipvc.pt]

In this talk, some recent studies arising in the area of the reactive Bolzmann equation for gaseous mixtures and its hydrodynamical limit will be presented, mainly addressed to the modeling of reactive gas systems and the propagation and stability of steady detonation waves [1].

Detonation waves are combustion fronts triggered by a strong shock and sustained by a chemical reaction [2]. They can be mathematically modeled by the reactive Euler equations, which express conservation of momentum and total energy (kinetic and chemical) as well as reaction rate of the constituents. Experimental studies show that the detonation waves tend to be structurally unstable and a first attempt to understand and describe the instabilities is a hydrodynamic stability analysis based on the linearization of the governing Euler equations and a normal-mode representation of the perturbations [2]. It is well known that the numerical analysis of detonation waves and its hydrodynamic stability is a rich and challenging problem with many engineering applications [3]. We investigate this problem starting by considering a binary mixture whose particles can undergo elastic collisions and collisions with chemical reaction of type $A + A \rightleftharpoons B + B$. We assume that the behavior of the mixture is modeled by a system of Boltzmann equations for the constituent distribution functions, with both elastic scattering and reactive collision terms. Then we pass to the hydrodynamic limit for an Eulerian regime and use the resulting macroscopic reactive equations to study the detonation wave problem and its hydrodynamic stability. The numerical technique is described and some representative computational results are presented and discussed.

References

- [1] F Carvalho and A J Soares, submitted paper.
- [2] W Fickett, Introduction to Detonation Theory, University of California (1986).
- [3] J H S Lee, The Detonation Phenomenon, Cambridge University Press (2008).
- [4] H. I. Lee and D. S. Stewart, J. Fluid Mech. 216, 103–132 (1990).

Joint work with: Ana Jacinta Soares (Centro de Matemática, Universidade do Minho, Braga, Portugal)

S54 – Models and Simulations in Mechanics – Room I, 11.50–12.20

Exact solutions to the Riemann problem for compressible isothermal Euler equations for two phase flows with and without phase transition

> Maren Hantke Otto-von-Guericke University Magdeburg maren.hantke@ovgu.de

We consider the isothermal Euler equations with phase transition between a liquid and a vapor phase. The mass transfer is modeled by a kinetic relation. We prove existence and uniqueness results. Further, we construct the exact solution for Riemann problems. We derive analogous results for the cases of initially one phase with resulting condensation by compression or evaporation by expansion. Further we present numerical results for these cases. We compare the results to similar problems without phase transition.

References

- [1] W. Dreyer, F. Duderstadt, M. Hantke, and G. Warnecke, On phase change of a vapor bubble in liquid water, WIAS Preprint no. 1424 (2009), to appear on *Continuum Mechanics and Thermodynamics*
- [2] M. Hantke, W. Dreyer, and G. Warnecke, Exact solutions to the Riemann problem for compressible isothermal Euler equations for two phase flows with and without phase transition, WIAS Preprint no. 1620 (2011), to appear on *Quarterly of Applied Mathematics*
- [3] I. Müller and W. Müller, Fundamentals of Thermodynamics and Applications, Springer-Verlag, (2009)
- [4] S. Müller and A. Voss, The Riemann Problem for the Euler equations with nonconvex and nonsmooth equation of state: Construction of wave curves, *SIAM J. Sci. Comput.*, **28** (2006), pp. 651-681
- [5] E. F. Toro, Riemann Solvers and Numerical Methods for Fluid Dynamics, Springer-Verlag, (1999)

Joint work with: Wolfgang Dreyer (WIAS Berlin), Gerald Warnecke (Otto-von-Guericke University Magdeburg)

S54 – Models and Simulations in Mechanics – Room I, 12.20–12.50

- * * * -

Modeling, simulation and optimisation of gas flow in an exhaust pipe

Martin Rybicki Universität Hamburg martin.rybicki@math.uni-hamburg.de

We study the gas flow in an exhaust pipe. In particular we focus on the requirement of heating up the catalytic converter.

Starting from one-dimensional hyperbolic balance laws for a pipe with variable cross section, we will derive by using a small Mach number limit an asymp totic model consisting of coupled ODEs and PDEs. For the description of the full exhaust pipe we use a network Ansatz connecting verious pipes with (different) constant cross section. This model still describes the main features and is computationally a few orders of magnitude faster the the original model.

Furthermore we will present a related optimization problem.

Joint work with: Prof. Dr. Ingenuin Gasser (Universität Hamburg)

11 Abstracts of contributed lectures — Thursday 14.55–15.55

11.1 Session 56 — Room A — Nonlinear Waves I

S56 – Nonlinear Waves I – Room A, $14.55{-}15.25$

An efficient splitting technique for two-layer Shallow-Water model

Françoise Foucher Laboratoire de Mathématiques Jean Leray University of Nantes, 2 rue de la Houssinière, 44300 Nantes, France

We consider the numerical system of the shallow-water equations for the one-dimensional flow of two superposed layers of immiscible fluids with different constant densities, over a bottom with non flat topography. We propose to extend a recent technique, introduced in [1], to approximate the solutions of the model under consideration. The benefits of this approach are twofold. In one hand, the resulting scheme turns out to be very easy to be implemented. Indeed, we obtain a relevant way to couple two Saint-Venant models associated with each layer. Despite the work by Bouchut-Morales [2] where splitting tentativeness are shown to fail, we here obtain fairly good approximations even if very severe regimes are considered. In the second hand, the scheme is proved to be well-balance preserving and layer height positive preserving. Several numerical experiments are performed and give improved results. For instance, considering lock-exchanged layers, the suggested method produces expected approximations while standard or sophisticated splitting approaches failed (see [2]).

References

- C. Berthon, F. Foucher, Hydrostatic Upwind Schemes for Shallow-Water Equations, Finite Volumes for Complex Application VI, Springer Proceedings in Mathematics, 4 (2011), pp. 97-106.
- [2] F. Bouchut, T. Morales, An entropy satisfying scheme for two-layer shallow water equations with uncoupled treatment, *Math. Model. Numer. Anal.*, 42 (2008), pp. 683-698.

Joint work with: Christophe Berthon (*Laboratoire de Mathématiques Jean Leray*, University of Nantes, 2 rue de la Houssinière, 44300 Nantes, France), Tomás Morales de Luna (*Dpto. de Matemáticas*, Universidad de Córdoba, Campus de Rabanales, 14071 Córdoba, Spain).

* * * -----

S56 – Nonlinear Waves I – Room A, 15.25–15.55

Stability of solitary waves in generalized Korteweg-de Vries and Euler-Korteweg / Boussinesq equations

Johannes Höwing University of Konstanz Johanneshoewing@gmail.com

We show that solitary waves for the generalized Korteweg-de Vries equation and for the generalized Boussinesq equation (the p-system endowed with capillarity) are stable if the flux function p satisfies

p'' > 0 and $p''' \le 0$.

While p'' > 0 alone suffices for the stability of waves of sufficiently small amplitude, obvious examples show that $p''' \leq 0$ cannot be omitted in the general case. In particular, the generalized Boussinesq equation with

$$p(v) = kv^{-\gamma}$$
 with $\gamma \ge 1, \ k > 0$

describes the flow of an inviscid isothermal ideal (barotropic) fluid with capillarity. In this talk, we present the following new stability results:

Theorem 1 Consider the generalized Korteweg-de Vries (gKdV) equation with a smooth function p satisfying p'' > 0 and $p''' \le 0$. Then any solitary wave is stable.

Theorem 2 Consider the generalized Boussinesq equation with $p : \mathbb{R} \to \mathbb{R}$ or $p : (0, \infty) \to \mathbb{R}$ satisfying p' < 0, p'' > 0 and $p''' \leq 0$. Then any solitary wave is stable.

These results complement the findings of Bona, Souganidis and Strauss [1], and Bona and Sachs [2], respectively; the only overlap of Theorems 1 and 2 with those consisting exactly of the quadratic nonlinearity $p''' \equiv 0$. Note, however, that Theorems 1 and 2 are not restricted to pure power laws.

Furthermore, we will extend some of our results to the case of non-constant capillarity in the so-called Euler-Korteweg equation.

References

- J. Bona, P.E. Souganidis and W.A. Strauss, Stability and Instability of Solitary Waves of Korteweg-de Vries Type, Proc. R. Soc. Lond. A. 411 (1987), pp. 395-412.
- [2] J. Bona and L. Sachs, Global existence of smooth solutions and stability of solitary waves for a generalized Boussinesq equation, *Comm. Math. Phys.* **118** (1988), pp. 15-29.

11.2 Session 57 — Room G — Numerical Methods XVII

S57 – Numerical Methods XVII – Room G, 14.55–15.25

Inflow-Implicit/Outflow-Explicit Finite Volume Methods for Solving Advection Equations

Karol Mikula

Slovak University of Technology, Faculty of Engineering, Radlinskeho 11, 81368 Bratislava, Slovakia karol.mikula@stuba.sk

We present new semi-implicit schemes for solving non-stationary advection equations. The basic idea is that outflow from a cell is treated explicitly while inflow is treated implicitly. This is natural, since we know what is outflowing from a cell at the old time step but we leave the method to resolve a system of equations determined by the inflows to a cell to obtain the solution values at the new time step. The matrix of the system in our inflow-implicit/outflow-explicit (I²OE) method is determined by the inflow fluxes which results in a M-matrix yielding favourable solvability and stability properties for the scheme. Since the explicit (outflow) part is not always dominated by the implicit (inflow) part and thus some oscillations can occur, we build a stabilization based on the upstream weighted averages with coefficients determined by the flux-corrected transport approach yielding high resolution versions of the basic scheme. We show that on uniform rectangular grids, the I²OE method is exact for any choice of time step in the case of constant velocity transport of quadratic functions in any dimension. We also show its formal second order accuracy in space and time for 1D advection problems with variable velocity and local mass conservativity in case of divergence free velocities. The unconditional L²stability for divergence free velocity in 1D on periodic domains and unconditional L[∞]-stability for the stabilized high resolution variants of the scheme is proved. Numerical results and comparisons with the well-known explicit and other implicit schemes are presented and discussed regarding stability and precision of computations in the case of time steps several times exceeding the CFL stability condition. The application of the new schemes in numerical modelling of forest fire front propagation and in image segmentation are also presented.

References

- K. Mikula, M. Ohlberger: Inflow-Implicit/Outflow-Explicit Scheme for Solving Advection Equations, in Finite Volumes in Complex Applications VI, Problems & Perspectives, Eds. J.Fořt et al. (Proceedings of the Sixth International Conference on Finite Volumes in Complex Applications, Prague, June 6-10, 2011), Springer Verlag, 2011, pp. 683-692.
- [2] K. Mikula, M. Ohlberger, J.Urbán: Inflow-Implicit/Outflow-Explicit Finite Volume Methods for Solving Advection Equations, Preprint, Angewandte Mathematik und Informatik, Universitaet Münster, February 2012.

Joint work with: Mario Ohlberger (University of Münster, Germany) and Jozef Urbán (Slovak University of Technology and TatraMed s.r.o., Bratislava, Slovakia).

* * * ------

S57 – Numerical Methods XVII – Room G, 15.25–15.55

A new multidimensional-type reconstruction and limiting procedure for unstructured cell-centered finite volumes solving hyperbolic conservation laws

Argiris I. Delis

Department of Sciences, Technical University of Crete, Chania, Greece 73100 adelis@science.tuc.gr

On unstructured meshes the cell-centered finite volume (CCFV) formulation where the control volumes are the mesh elements themselves is, probably, the most applied approach for numerically solving two-dimensional (2D) hyperbolic conservation laws. In this work, and within the CCFV framework, second-order spatial accuracy is achieved with a MUSCL-type linear reconstruction technique where a novel edge-based multidimensionaltype limiting procedure is derived for the control of the total variation of the reconstructed field. To this end, a relatively simple but very effective modification to the reconstruction procedure for CCFV schemes is introduced that takes into account geometrical characteristics of computational triangular (but not only) meshes. The proposed strategy is shown not to suffer from loss of accuracy on grids with poor connectivity and compares well, or in favor, to solution reconstructions that implement well-known multidimensional limiters. In addition, our approach can be applied using edge-type limiters thus, avoiding the procedure of solving any minimization problems. Although the proposed limited reconstruction is independent from the Riemann solver used, well-known approximate Riemann solvers are utilized to compute the numerical fluxes while the Green-Gauss (G-G) divergence formulation for gradient computations is implemented. Two different stencils for the G-G gradient computations are critically tested, in conjunction with the proposed limiting strategy, on various grid types, for smooth and non-smooth flow conditions. We apply this reconstruction to CCFV solutions on unstructured triangular meshes for the 2D Euler equations and in the development of a well-balanced scheme for the simulation of unsteady 2D shallow water flows over arbitrary topography. The proposed strategy is shown to retain the formal order of accuracy and produce accurate shock/bore computations on different mesh types, even for distorted and highly stretched ones.

References

 A.I. Delis, I.K. Nikolos and M.Kazolea. Performance and Comparison of Cell-Centered and Node-Centered Unstructured Finite Volume Discretizations for Shallow Water Free Surface Flows, Archives of Computational Methods in Engineering, 18 (2011) 57-118. [2] A.I. Delis and I.K. Nikolos, A novel multidimensional solution reconstruction and edge-based limiting procedure for unstructured cell-centered finite volumes with application to shallow water dynamics, preprint (2012).

Joint work with: Prof. Ioannis K. Nikolos, (Department of Production Engineering & Management, Technical University of Crete, Greece)

11.3 Session 59 – Room H – Mechanics and Fluids

S59 – Mechanics and Fluids – Room H, 14.55–15.25

Renormalization and universality of blowup in hydrodynamic flows and conservation laws

Alexei A. Mailybaev Instituto Nacional de Matemática Pura e Aplicada - IMPA, Rio de Janeiro, Brazil a.mailybaev@gmail.com, alexei@impa.br

We consider self-similar solutions describing intermittent bursts in shell models of turbulence, and study their relationship with blowup phenomena in continuous hydrodynamic models. First, we show that these solutions are very close to self-similar solution for the Fourier transformed inviscid Burgers equation corresponding to shock formation from smooth initial data. Then, the result is generalized to hyperbolic conservation laws in one space dimension describing compressible flows. It is shown that the renormalized wave profile tends to a universal function, which is independent both of initial conditions and of a specific form of the conservation law. This phenomenon can be viewed as a new manifestation of the renormalization group theory. Finally, we discuss possibilities for application of the developed theory for detecting and describing a blowup in incompressible flows.

S59 – Mechanics and Fluids – Room H, 15.25–15.55

* * *

Long waves on 3D shear flows: hyperbolicity and discontinuous solutions

Alexander Khe Lavrentyev Institute of Hydrodynamics alekhe@hydro.nsc.ru

A long wave approximation of the equation governing three dimensional flows of ideal incompressible fluid is considered. Theory of the generalized hyperbolicity introduced in [1] is applied to the integrodifferential system obtained.

The system in question cannot be written in the form of conservation laws. A special rearrangement is applied to the equations in order to consider discontinuous solutions. It is shown that the assumption made is consistent with the properties of the discontinuous solutions of the original Euler equations. Relations at the discontinuity front are derived, and stability conditions for the discontinuity are formulated. The problem of determining the flow parameters behind the discontinuity front from known parameters before the front and specified velocity of motion of the front are investigated.

It is shown that the flow parameters behind the jump are defined by a certain curve which is an analog of the (ϑ, p) diagram in gas dynamics. A shock polar and examples of flows with a hydraulic jump are constructed for a particular class of solutions.

A numerical method for integrodifferential systems introduced in [2] is developed and applied to the case of 3D equations. Some illustrative solutions are presented for smooth and discontinuous flows.

References

- V. M. Teshukov, Characteristics and Riemann Invariants of the Kinetic Integrodifferential Equations of Bubbly Flow, in: *Hyperbolic Problems: Theory, Numerics, Applications. International Series of Numerical* Mathematics. 141 (2001), 891–900, DOI: 10.1007/978-3-0348-8372-6_43.
- [2] V. Teshukov, G. Russo, A. Chesnokov, Analytical and Numerical Solutions of the Shallow Water Equations for 2D Rotational Flows, *Math. Models Methods Appl. Sci.* 14 (2004), No. 10, 1451–1479.

11.4 Session 60 — Room C — Wave Patterns Analysis I

S60 – Wave Patterns Analysis I – Room C, 14.55–15.25

The Stefan problem and the vanishing surface tension limit

Mahir Hadžić Massachusetts Institute of Technology hadzic@math.mit.edu

We develop a new unified framework for the treatment of well-posedness for the Stefan problem with and without surface tension. This is a well-known model describing solid-liquid phase transitions. Our approach yields new estimates for the regularity of the moving surface in the absence of surface tension, which allows us to prove that solutions of the Stefan problem with positive surface tension converge to solutions of the Stefan problem without surface tension. Our techniques rely on a fluid-mechanics inspired approach which, in a suitable sense, combines the Eulerian and the Lagrangian viewpoint.

Joint work with: Steve Shkoller (University of California Davis)

S60 – Wave Patterns Analysis I – Room C, 15.25–15.55

* * *

On the stability of degenerate viscous shock profiles

Ramón G. Plaza *IIMAS-UNAM* plaza@mym.iimas.unam.mx Sonic shock waves travel with speed equal to one of the characteristic speeds of the medium, making the stability problem degenerate from the dynamical systems viewpoint. In this talk, I show how to obtain sharp L^p -decay rates for perturbations of scalar sonic shocks using energy methods [4]. The analysis involves interpolation inequalities, the Matsumura-Nishihara weight function [3] and energy estimates. The analysis can be applied to shocks of all orders of degeneracy and provides sharp rates of decay. The method is independent from the pointwise Green's function method [1, 2]. In addition, I show how to extend the analysis to the system case to obtain stability results in L^2 [5].

References

- P. Howard, Local tracking and stability for degenerate viscous shock waves, J. Differential Equations, 186 (2002), pp. 440-469.
- [2] P. Howard, Pointwise estimates and stability for degenerate viscous shock waves, J. Reine Angew. Math. 545 (2002), pp. 19-65.
- [3] A. Matsumura, K. Nishihara, Asymptotic stability of traveling waves for scalar viscous conservation laws with non-convex nonlinearity, *Comm. Math. Phys.* 165 (1994), pp. 83-96.
- [4] R.G. Plaza, L^p-decay rates for perturbations of degenerate scalar viscous shock waves, J. Math. Anal. Appl. 382 (2011), pp. 864-882.
- [5] R.G. Plaza, Nonlinear stability of degenerate shock profiles for viscous systems of conservation laws. In preparation.

11.5 Session 61 — Room E — Theory of Conservation Laws VII

S61 - Theory of Conservation Laws VII - Room E, 14.55-15.25

Continuous solutions to a balance law

Laura Caravenna OxPDE, University of Oxford laura.caravenna@maths.ox.ac.uk

When interpreting in the sense of distribution a scalar, 1D-balance law

$$u_t(t,x) + f(u(t,x))_x = g(t,x), \qquad (t,x) \in \mathbb{R}^+ \times \mathbb{R},$$

the source term g is naturally defined only up to \mathcal{L}^2 -negligible sets. When u is continuous, characteristics are not unique but however classically defined. Notwithstanding that, without continuity assumptions on g it is not a priori clear how to see the PDE as a growth condition of u along characteristics, because in the equation

$$\dot{\gamma}(t) = f'(u(t,\gamma(t))) \qquad \frac{d}{dt}u(t,\gamma(t)) = g(t,\gamma(t))$$

the pointwise values of g on curves are involved.

When $f(u) = u^2/2$, under the assumption that u is continuous and g is bounded, we have proved that u is Lipschitz along characteristics; this extends the case considered by C. Dafermos where g was also continuous in x at each fixed time. Beside that, we show the remarkable fact that there exists a Borel pointwise representative \hat{g} of g which is the derivative of u along any absolutely continuous characteristic line. This regularity is part of the characterization of intrinsic Lipschitz graphs in the Heisenberg group (joint work with F. Bigolin and F. Serra Cassano, University of Trento). For general non convex fluxes this is no more true, we provide a counterexample. However, under a sharp assumption on f one can still pointwise define a Borel function which is the derivative of u along characteristics (forthcoming work jointly with G. Alberti, University of Pisa, and S. Bianchini, SISSA).

Joint works with: G. Alberti (University of Pisa), S. Bianchini (SISSA), F. Bigolin (University of Trento), F. Serra Cassano (University of Trento).

S61 - Theory of Conservation Laws VII - Room E, 15.25-15.55

* * *

SBV regularity results for Hamilton-Jacobi equations

Daniela Tonon Universit Pierre et Marie Curie, Paris, France tonondaniela83@gmail.com

We present two results on the regularity of viscosity solutions of Hamilton-Jacobi equations described in [1], [2].

When the Hamiltonian is strictly convex, viscosity solutions of Hamilton-Jacobi equations are semiconcave, hence their gradient is BV. It is therefore of interest to see when it is SBV. The first result in this direction was obtained by Cannarsa, Mennucci and Sinestrari in [3] but requires a very regular initial datum for the Hamilton-Jacobi equation. First we prove the SBV regularity of the gradient of a viscosity solution of the Hamilton-Jacobi equation

$$\partial_t u + H(t, x, D_x u) = 0$$
 in a open $\Omega \subset [0, T] \times \mathbb{R}^n$,

under the hypothesis of uniform convexity of the Hamiltonian H in the last variable. Thus the SBV regularity holds even in the case of a bounded Lipschitz initial datum. Secondly we remove the uniform convexity hypothesis on the Hamiltonian, considering a viscosity solution u of the Hamilton-Jacobi equation

$$\partial_t u + H(D_x u) = 0$$
 in $\Omega \subset [0, T] \times \mathbb{R}^n$,

where Ω is open and H is smooth and convex. In this case the viscosity solution is only locally Lipschitz. However when the vector field $d(t, x) := H_p(D_x u(t, x))$, here H_p is the gradient of H, is BV for all t in [0, T] and suitable hypotheses on the Lagrangian L hold, the divergence of d(t,) can have Cantor part only for a countable number of t's in [0, T].

These results extend a result of Bianchini, De Lellis and Robyr in [4] for a uniformly convex Hamiltonian which depends only on the spatial gradient of the solution.

References

- [1] Bianchini, S. and Tonon, D., SBV regularity for Hamilton-Jacobi equations with Hamiltonian depending on (t, x), preprint (2011), submitted
- [2] Bianchini, S. and Tonon, D., SBV-like regularity for Hamilton-Jacobi equations with a convex Hamiltonian, preprint (2011), to appear on *JMAA*
- [3] Cannarsa, P. and Mennucci, A. and Sinestrari, C., Regularity results for solutions of a class of Hamilton-Jacobi Equations, Arch. Ration. Mech. Anal., Volume no. 140, (1997), pp. 197-223
- [4] Bianchini, S. and De Lellis, C. and Robyr, R., SBV regularity for Hamilton-Jacobi equations in ℝⁿ, Arch. Ration. Mech. Anal., Volume no. 200, (2011), pp.1003-1021

Joint work with: Stefano Bianchini (SISSA, Trieste, Italy)



11.6 Session 62 — Room I — BioFluids Models III

S62 – BioFluids Models III – Room I, 14.55-15.25

Multi-scale tissular-cellular model for wound healing

Patrizia Bagnerini

DIME, Università degli Studi di Genova, P.le Kennedy-Pad D, 16129 Genova, Italy bagnerini@diptem.unige.it

Extension of an epithelial membrane to close a hole is a very widespread process both in morphogenesis and in tissue repair. Differently to embryos which make a perfect repair, adult wound healing can generate a mass of fibrotic tissue which can have serious clinical consequences, from heart attacks to burns and cirrhosis (for instance foot chronic ulcers occur in 15% of diabetical patients). A good understanding of regenerative healing mechanisms and of how we can re-activate or reproduce them in adult setting is an important public health issue. In embryo wound healing the main mechanism driving epithelial advance to cover a hole is the contraction of an actin cable composed by meshworks of actin filaments (a polymer) cross-linked by a molecular motor, Myosin II. We proposed (see e.g. [1,2]) various macroscopic (that do not take into account the positions of the individual cells) models for simulating the contraction of an acto-myosin cable in different applications. We propagated the wound contour by level set methods [3], a class of popular algorithms to evolve interfaces. The models were validated either by numerical simulations or by experimental works performed in collaboration with biologists.

We propose here to couple the previous macroscopic model (which furnishes the dynamics of the tissue) with a cellular one (composed by a network of cells discretized as polygons and interacting through their common boundaries). At each time step, we solve a PDE corresponding to the macroscopic model and we use the obtained vector field as a velocity vector both in level set methods (to deplace the actin cable) and also in a cellular model (to move individual cells). Stable network configurations are then computed by minimizing an energy functional which allows to take into account changes in interfacial tension at cell boundaries. Moreover, the actin cable formation involves the propagation of actin and myosin waves which will be presented. If time permits, I will also describe an interesting transdifferentiation phenomena taking place during normal development of Drosophila embryo: a cell of epidermis, that we called *chameleon*, transdifferentiates, i.e. changes its identity and intercalates with the cells of the other compartment [4].

- L. Almeida, P. Bagnerini, A. Habbal, S. Noselli, F. Serman, A Mathematical Model for Dorsal Closure, Journal of Theoretical Biology, Volume 268, n. 1 (2011), pp. 105-119
- [2] L. Almeida, P. Bagnerini, A. Habbal, Modeling Actin Cable Contraction, Computers and Mathematics with Applications, to appear (2012)

- [3] S. Osher, J. A. Sethian, Fronts propagating with curvature-dependent speed: Algorithms based on Hamilton-Jacobi formulations, *Journal of Computational Physics*, Volume 79, n.1, pp. 12-49 (1988)
- [4] M. Gettings, F. Serman, R. Rousset, P. Bagnerini, L. Almeida, S. Noselli, JNK Signalling Controls Remodelling of the Segment Boundary Through Cell Reprogramming During Drosophila Morphogenesis, *Plos Biology*, Volume 8, n. 6 (2010)

Joint work with: Luís Almeida (Laboratoire Jacques-Louis Lions, Université Paris 06, F-75005, Paris, France)

S62 – BioFluids Models III – Room I, 15.25–15.55

Coupling hydrodynamics and biology to model and simulate algae growth

Anne-Celine Boulanger Paris VI University anne-celine.boulanger@inria.fr

Recently, biofuel production from microalgae has proved to have a high potential for biofuel production [1,2]. Several studies have demonstrated that some microalgae species could store more than 50% of their dry weight in lipids under certain conditions of nitrogen deprivency [1,3,4] leading to productivities in a range of order larger than terrestrial plants. In this context, the coupling between hydrodynamics and biology has mostly been studied with simple hydrodynamics models in photobioreactors [5] that are narrow closed tubes in which a turbulent flow transports the algae. We are rather interested in situations where the algae culture takes place in small circular ponds called raceways, used for the intensive outdoor culture of algae. Carrying out experiments on raceways is both expensive and time consuming. A model is thus a key tool to help in the optimal design of the process but also in its operation. The one developped herein possesses hyperbolic features and is thus delicate to analyze and simulate.

Due to the heterogeneity of raceways along the depth dimension regarding temperature, light intensity or nutrients availability, we adopt a multilayer approach for hydrodynamics and biology. For free surface hydrodynamics, we use a multilayer Saint-Venant model that allows mass exchanges[6, 7, 8], forced by a simplified representation of the paddlewheel. For the biological part, we have to deal with a great amount of potential models and parameters. We choose to restrict ourselves to one particular species, the diatoms, and follow the evolution of three variables : the phytoplanctonic carbon X (of particular interest for biofuel production), the extracellular nutrients S, but also the intracellular nitrogen N, that can be seen as a nutrient storage pool in the algae. We then build an improved Droop model that includes light effect on algae growth. In the end, our biological system writes

$$\frac{\partial \rho X}{\partial t} + \nabla .(\rho \mathbf{\underline{u}} X) = \nu X \Delta X + \rho(\mu(q, I) X - RX)$$
(1)

$$\frac{\partial\rho S}{\partial t} + \nabla .(\rho \underline{\mathbf{u}}S) = \nu S \Delta S - \rho \lambda(S,q) X \tag{2}$$

$$\frac{\partial \rho N}{\partial t} + \nabla (\rho \underline{\mathbf{u}} N) = \nu N \Delta N + \rho (\lambda(S, q) X - RN)$$
(3)

$$q = \frac{N}{X}, \quad \underline{\mathbf{u}} = (u, w) \tag{4}$$

where $\underline{\mathbf{u}}$ is the water velocity, I is the light intensity, R is a respiration coefficient, and the growth coefficient write ($Q_0, Q_l, K_s, \tilde{\mu}$ being constants):

$$\mu(q, I) = \tilde{\mu} \frac{I}{I + K_{sI} + \frac{I^2}{K_{sI}}} (1 - \frac{Q_0}{q})$$
(5)

$$\lambda(S,q) = \overline{\lambda} \frac{S}{S+k_S} \left(1 - \frac{q}{Q_l}\right) \tag{6}$$

From those equations, we derive a similar multilayer system by performing a Galerkin approximation of every variable followed by an integration by layers of the equations. A kinetic interpretation of the whole system will result in an efficient numerical scheme.

We show through numerical simulations in 2D that our method is capable of distinguishing situations of moving water or calm ponds in terms of carbon productivity. Moreover, we exhibit that *a posteriori* treatment of our velocity fields can provide lagrangian trajectories which are of great interest to assess the actual light pattern perceived by the algal cells and therefore understand its impact on the photosynthesis process. Eventually, we consider the 3D case in its hydrodynamical part and focus on rendering an appropriate velocity field in the pond thanks to real data measurements.

References

- [1] Chisti, Y., Biodiesel from microalgae, Biotechnology Advances, Volume 25 (2007), pp. 294-306
- [2] Wijffels, R.H. and Barbosa, M.J., An outlook on microalgal biofuels., Science, Volume 329 (2010), pp. 796-799
- [3] Rodolfi, L. and Zittelli, G. C. and Bassi, N. and Padovani, G. and Biondi, N. and Bonini, G. and Tredici, M. R., Microalgae for Oil: Strain Selection, Induction of Lipid Synthesis and Outdoor Mass Cultivation in a Low-Cost Photobioreactor., *Biotechnol. Bioeng.*, Volume 102 (2009), pp. 100-112
- [4] Williams, P. J. B. and Laurens, L. M. L., Microalgae as biodiesel and biomass feedstocks: Review and analysis of the biochemistry, energetics and economics., *Biotechnol. Bioeng.*, Volume 3 (2010), pp. 554-590
- [5] Lardon, L. and Helias, A. and Sialve, B. and Stayer, J. P. and Bernard, O., Life-Cycle Assessment of Biodiesel Production from Microalgae, *Environmental Science & Technology*, Volume 43 (2010), pp. 6475-6481
- [6] Audusse, E. and Bristeau, M.-O. and Pelanti, M. and Sainte-Marie, J., Approximation of the hydrostatic Navier-Stokes system for density stratified flows by a multilayer model. Kinetic interpretation and numerical validation., J. Comp. Phys., Volume 230 (2011), pp. 3453-3478
- [7] Audusse, E. and Bristeau, M.-O. and Perthame, B. and Sainte-Marie, J., A multilayer Saint-Venant system with mass exchanges for Shallow Water flows. Derivation and numerical validation., *ESAIM:* M2AN, Volume 45 (2010), pp. 169-200
- [8] Sainte-Marie, J., Vertically averaged models for the free surface Euler system. Derivation and kinetic interpretation., M3AS, Volume 21 (2011), pp. 459-490

Joint work with: Jacques Sainte-Marie (Paris VI University and French Ministry for Ecology (CETMEF))

11.7 Session 63 — Room F — Mean Curvature Motion and Moving Interfaces

S63 – Mean Curvature Motion and Moving Interfaces – Room F, 14.55–15.25

A semi-Lagrangian scheme for mean curvature motion with nonlinear Neumann conditions

Yves Achdou Univ. Paris Diderot, Sorbonne Paris Cité, Laboratoire Jacques-Louis Lions, UMR 7598, UPMC, CNRS, F-75205 Paris, France. achdou@ljll.univ-paris-diderot.fr A numerical method for mean curvature motion in bounded domains with nonlinear Neumann boundary conditions is proposed and analyzed. It consists of a semi-Lagrangian scheme in the main part of the domain as proposed by Carlini, Falcone and Ferretti, combined with a finite difference scheme in small layers near the boundary to cope with the boundary condition. The consistency of the new scheme is proved for nonstructured triangular meshes in dimension two. The monotonicity of a regularized version of the scheme with some additional vanishing artificial viscosity is studied. Details on the implementation are given. Numerical tests are presented.

Joint work with: Maurizio Falcone Dipartimento di Matematica, Università Roma "La Sapienza"

S63 – Mean Curvature Motion and Moving Interfaces – Room F, 15.25-15.55

* * *

Flux-based level set method for implicitly defined interfaces

Peter Frolkovič Slovak University of Technology, Faculty of Engineering, Radlinskeho 11, 81368 Bratislava, Slovakia peter.frolkovic@gmail.com

We present recent results concerning a development of flux-based level set method for several problems involving hyperbolic equations. The flux-based level set method [1,2] is used to solve the problems of moving interfaces defined implicitly where the movement is prescribed by external velocity field and variable speed in normal direction including a dependence on a curvature.

In this talk we introduce new results concerning, firstly, computation of a signed distance function, and, secondly, solution of a sequence of linear hyperbolic equations to extrapolate data available only at the interface. In both cases we apply an immersed interface formulation that is well-known numerical technique for parabolic and elliptic type of PDEs, but rarely used for hyperbolic equations.

Some applications will be presented including groundwater flow with moving boundary [4].

References

- [1] P. Frolkovič and K. Mikula, Flux-based level set method: a finite volume method for evolving interfaces, Applied Numerical Mathematics, Volume no. 4 (2007), pp. 436-454
- [2] P. Frolkovič and K. Mikula, High-resolution flux-based level set method, SIAM J. Sci. Comp., Volume no. 29 (2007), pp. 579-597
- [3] Z. Li and K. Ito, The immersed interface method: numerical solutions of PDEs involving interfaces and irregular domains, SIAM, (2006)
- [4] P. Frolkovič, Application of level set method for groundwater flow with moving boundary, submitted to Adv. Wat. Res. (2012)

Joint work with: Karol Mikula (Slovak University of Technology).

12 Abstracts of contributed lectures — Thursday 17.00–19.30

12.1 Session 64 — Room D — Numerical Methods XVIII

S64 - Numerical Methods XVIII - Room D, 17.00-17.30

A Well-Balanced Multi-Dimensional Reconstruction Scheme for Hydrostatic Equilibria

Roger Käppeli Seminar for Applied Mathematics, ETH Zürich, Switzerland roger.kaeppeli@sam.math.ethz.ch

Conservation laws with source terms, i.e. balance laws, allow steady state solutions where the flux divergence is exactly balanced by the source term. Standard high-resolution finite volume schemes do not preserve a discrete version of this balance and generate spurious waves that can obscure waves of interest. The main reason for the failure of standard schemes to preserve this equilibrium is due to the fact that it cannot be represented by simple polynomial functions. Hence, standard reconstruction techniques lead to non-zero truncation errors inducing spurious waves. Schemes that preserve exactly some discrete version of this equilibrium are termed as well-balanced.

In this talk we consider the equations of gas dynamics with gravitational source terms:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla \cdot (\boldsymbol{v} \rho \boldsymbol{v}) + \nabla P = -\rho \nabla \phi$$

$$\frac{\partial E}{\partial t} + \nabla \cdot [(E+P) \boldsymbol{v}] = -\rho \boldsymbol{v} \cdot \nabla \phi,$$
(1)

expressing the conservation of mass, momentum and total energy. Here ρ is the mass density, \boldsymbol{v} the velocity and $E = \rho e + \frac{\rho}{2}v^2$ the total energy density, being the sum of internal and kinetic energy density. The equations of gas dynamics must be closed by an equation of state $p = p(\rho, e)$ describing the thermodynamic properties of the gas. The source terms on the right hand side stem from gravity, where the gravitational potential ϕ is either given analytically or is determined by the Poisson equation

$$\nabla^2 \phi = 4\pi G\rho,\tag{2}$$

where G is the gravitational constant. These equations are of practical importance in many applications and we shall focus here on an astrophysical context.

An interesting steady state arising in a multitude of (astrophysical) applications is the hydrostatic state, where the pressure gradient exactly balances the gravitational force. For now, the existing schemes in the literature have been developed essentially for a constant gravitational acceleration. These schemes rely on an analytic reconstruction by simply integrating the hydrostatic equilibrium on a zone-by-zone basis. However, such analytical formulas are generally not available for the realistic astrophysical case of a multi-dimensional hydrostatic equilibrium, potentially itself the outcome of a dynamical simulation. We present a second-order well-balanced multi-dimensional reconstruction technique based on a local discrete representation of the hydrostatic equilibrium. Numerical quadrature rules are then used to build the equilibrium reconstruction on a zone-by-zone basis.

We will demonstrate the performance of our well-balanced reconstruction on a large number of numerical experiments, including realistic simulations of neutron stars and core-collapse supernovae explosions. Moreover, if time permits, we will address the extension of the scheme to higher order and magneto-hydrostatic equilibria.

Joint work with: Siddhartha Mishra (Seminar for Applied Mathematics, ETH Zürich, Switzerland)

S64 – Numerical Methods XVIII – Room D, 17.30–18.00

Well-balanced bicharacteristic-based scheme for two-layer shallow water flows including wet/dry fronts

Michael Dudzinski

Institute of Numerical Simulation, Hamburg University of Technology, Schwarzenbergstraße 95 E, 21073 Hamburg, Germany michael.dudzinski@tu-harburg.de

We aim to present a new well-balanced finite volume scheme for multidimensional multilayer shallow water flows including wet/dry fronts. The method developed here is constructed in the framework of the Finite Volume Evolution Galerkin (FVEG) schemes. The FVEG methods couple a finite volume formulation with evolution operators. The latter are constructed using the bicharacteristics of multidimensional hyperbolic systems. However, in the case of multilayer shallow water flows the required eigenstructure of the underlying equations is not readily available. Thus we approximate the evolution operators numerically. This approximation procedure can be used for arbitrary hyperbolic systems. We derive a well-balanced approximation of the evolution operators and prove that the FVEG scheme is well-balanced for the multilayer lake at rest states even in the presence of wet/dry fronts.

Joint work with: M. Lukáčová-Medvidová (Institue of Mathematics, Johannes Gutenberg University, Staudingerweg 9, 55128 Mainz, Germany, e-mail: lukacova@mathematik.uni-mainz.de)

S64 – Numerical Methods XVIII – Room D, 18.00–18.30

Well-balanced and positivity preserving DG schemes for shallow water flows with shock capturing by adaptive filtering procedures

Sigrun Ortleb Faculty of Mathematics und Natural Sciences, University of Kassel, Germany ortleb@mathematik.uni-kassel.de

In this talk, we use a high order discontinuous Galerkin (DG) method on unstructured triangular grids to solve the shallow water equations (SWE).

Without an additional damping mechanism for the DG scheme, it is well-known that the numerical solution will suffer from Gibbs oscillations close to discontinuities of the exact entropy solution. Hence, in order to introduce a small but sufficient amount of numerical dissipation to the DG scheme, we developed a novel damping strategy based on spectral viscosity [1], see the work in [2,3]. More precisely, we derived a relationship between the introduction of spectral viscosity to discontinuous Galerkin methods on unstructured triangular grids and modal filtering. In our set-up, the high order viscosity term is based on the Sturm-Liouville operator corresponding to the chosen triangular-grid DG basis, i.e. the Proriol-Koornwinder-Dubiner(PKD) polynomials (see e.g. [4]). The designed artificial viscosity can then be efficiently implemented as a modal filter which is applied to the numerical solution after each time step of the basic DG scheme. This new strategy serves as a low cost stabilizing mechanism for solutions having strong shocks - as for each time step only the multiplication of the coefficients of the approximate solution with precomputed factors is necessary. With respect to an increasing polynomial degree N, high order accuracy of modal filtering applied to the PKD expansions of sufficiently smooth functions has been proven in [2]. Additionally, we developed adaptive modal filters to prevent order deterioration in case of grid refinement. The aim of modal filtering in general is to stabilize the numerical scheme, but by construction no effort is made to obtain an oscillation-free approximation. The damping strategy is therefore combined with a postprocessing technique in order to reconstruct a more accurate pointwise approximation from the available information. This postprocessing is only carried out at output times to visualize an essentially non-oscillatory numerical solution. Our results in [2,3] show that the application of the digital total variation (DTV) filter [5] as a postprocessor is a very promising approach in this situation. The DTV filter was originally developed in the context of image processing and has also been applied to the Chebychev pseudospectral method in [6].

In [2], our focus was put on scalar conservation laws in order to study the principle mechanisms of the proposed techniques. In [3], we then demonstrated the viability of this new damping strategy for the Euler equations of gas dynamics. Now, we focus on the application of our DG schemes with adaptive filtering routines to the shallow water equations. In this case, we have to deal with several new challanges: As in the case of the Euler equations, we have to preserve the positivity (or non-negativity) of certain physical quantities - in the context of the SWE this refers to the water height. Furthermore, in order to preserve certain steady state solutions with non-constant bottom topography, it is important that the scheme is well-balanced. In this talk, we will follow the ideas of Xing, Zhang and Shu in [7] regarding positivity preservation and well-balancedness but stay with the modal filtering procedures as shock capturing strategy. We will furthermore extend this approach to implicit time integration where the nonlinear systems are solved by the Jacobian-free Newton-GMRES method. As in the case of the TVB limiter used in [7], it is not a priori clear how to combine the requirements of positivity preservation and well-balancedness with the shock capturing mechanism. Hence, we will also give some guidelines for the case that shock capturing is done by adaptive modal filtering.

References

- E. Tadmor, Convergence of Spectral Methods for Nonlinear Conservation Laws, SIAM J. Numer. Anal., 26 (1989), pp. 30-44
- [2] A. Meister, S. Ortleb and Th. Sonar, Application of Spectral Filtering to Discontinuous Galerkin Methods on Triangulations, Numer. Methods Partial Differ. Equ., doi:10.1002/num.20705 (2011)
- [3] A. Meister, S. Ortleb and Th. Sonar, New Adaptive Modal and DTV Filtering Routines for the DG Method on Triangular Grids applied to the Euler Equations, Int. J. Geomath., doi: 10.1007/s13137-012-0030-9 (2012)
- [4] G. E. Karniadakis, and S. Sherwin, Spectral/hp Element Methods for Computational Fluid Dynamics, Oxford University Press, (2005)
- [5] T. F. Chan, S. Osher and J. Shen, The digital TV filter and nonlinear denoising, *IEEE Trans. Image Process.*, 10 (2001), pp. 231-241
- S. A. Sarra, Digital total variation filtering as postprocessing for Chebyshev pseudospectral methods for conservation laws, *Numerical Algorithms* 41 (2006), pp. 17-33
- [7] Y. Xing, X. Zhang and C.-W. Shu, Positivity-preserving high order well-balanced discontinuous Galerkin methods for the shallow water equations, Adv. Water Resour., 33 (2010), pp. 1476-1493

Joint work with: Andreas Meister (Faculty of Mathematics und Natural Sciences, University of Kassel, Germany), Thomas Sonar (Institute of Computational Mathematics, Technical University of Braunschweig, Germany)

S64 – Numerical Methods XVIII – Room D, 18.30–19.00

Order conditions on IMEX Runge-Kutta schemes for hyperbolic systems with stiff relaxation

Sebastiano Boscarino

Dipartimento di Matematica e Informatica, Università di Catania

boscarino@dmi.unict.it

IMEX schemes are very effective in the numerical solution of hyperbolic systems with relaxation, since they allow an explicit treatment of the flux (which is in general non linear), still avoiding an excessive restriction on the time step due to small relaxation time ε . The determination of the coefficients defining IMEX Runge-Kutta are based on order conditions for additive Runge-Kutta, which are more restrictive than the classical order conditions for Runge-Kutta methods, because of the coupling of the implicit and explicit scheme [1]. Such conditions, however, only guarantee the classical order of accuracy, i.e. the one obtained when the time scale is fully resolved ($\Delta t \ll \varepsilon$). For small values of ε , we distinguish between hyperbolic and parabolic relaxation. In the first case, as $\varepsilon \to 0$, the system relaxes to a reduced hyperbolic system, provided the so-called subcharacteristic condition is satisfied. The capability to capture such limiting behavior is obtained by adopting an L-stable implicit scheme, however a degradation of accuracy is usually observed for intermediate values of the relaxation parameter. If one wants to improve the accuracy for non negligible values of the relaxation parameter, then one has to perform an asymptotic expansion of the exact and numerical solution in terms of ε , and impose that the two expansions agree to a given order. Such procedure generates new order conditions, which allow the construction of uniformly accurate schemes in ε [2]. In the case of diffusive relaxation, the situation is even more complicated, since the small parameter appears both in the relaxation term and in the hyperbolic part. If one insists in treating the hyperbolic part by an explicit scheme, then more conditions arise when matching the various terms of the exact and numerical solution in the expansion in ε . A detailed analysis of such additional conditions is presented, and it is shown how to use them to construct a second order IMEX scheme for hyperbolic systems with diffusive relaxation, with explicit treatment of the hyperbolic part [3].

References

- M.H. Carpenter, C.A. Kennedy, Additive Runge-Kutta schemes for convection-diffusion-reaction equations, Appl. Numer. Math. 44, no. 1-2, 139–181, (2003).
- [2] S. Boscarino, G.Russo, On a class of uniformly accurate IMEX Runge-Kutta schemes and applications to hyperbolic systems with relaxation. SIAM J. Sci. Comput., 31, 3, pp. 1926-1945, (2009).
- [3] S. Boscarino, G. Russo, Flux-Explicit IMEX Runge-Kutta schemes for hyperbolic to parabolic relaxation problems. Submitted to SINUM.

Joint work with: Giovanni Russo (Dipartimento di Matematica e Informatica, Università di Catania)

12.2 Session 65 — Room A — Nonlinear Waves II

S65 – Nonlinear Waves II – Room A, 17.00–17.30

Well posedness for Schrödinger type equations

Alessia Ascanelli Dipartimento di Matematica, Universita' di Ferrara alessia.ascanelli@unife.it We are going to prove some results about well posedenss in L^2 and in the Sobolev space $H^{\infty} = \bigcap_s H^s$ of the Cauchy problem for the equation

$$P(t, x, D_t, D_x)u(t, x) = f(t, x),$$

where $t \in [0, T]$, $x \in \mathbb{R}$ and P is an ansotropic evolution differential operator of degree of evolution $p \ge 2$, p integer, with real characteristics; to have real characteristics is a necessary condition for well-posedness in Sobolev spaces of the Cauchy problem, by the Lax-Mizohata theorem.

Such a kind of operators are usually referred to as "p evolution operators of Schrödinger type", since the Schrödinger operator (p = 2) is the most famous representative of the class. Notice that for p = 1 we are dealing with hyperbolic equations. Existence and uniqueness of the solution of the Cauchy problem for Schrödinger type operators can be proved by adapting the energy method, which is a typically hyperbolic technique. Indeed, this class of operators can be also regards as a generalization of the hyperbolic class.

We are going to consider both operators P of the first order with respect to ∂_t , and operators of an arbitrary order m. We are going to give decay conditions on the coefficients of P as $|x| \to \infty$ that are sufficient to get existence and uniqueness of the solution in L^2 or H^{∞} .

References

[1] A.Ascanelli, C.Boiti, L.Zanghirati, Well-posedness of the Cauchy problem for *p*-evolution equations, preprint (2012), sottoposto per la pubblicazione

Joint work with: Chiara Boiti and Luisa Zanghirati, (Universita' di Ferrara, Italy).

* * * ------

S65 – Nonlinear Waves II – Room A, 17.30–18.00

Global solutions of the two-component Camassa–Holm system

Katrin Grunert Norwegian University of Science and Technology (NTNU) katring@math.ntnu.no

The two-component Camassa-Holm (2CH) system is given by

 $u_t - u_{txx} + \kappa u_x + 3uu_x - 2u_x u_{xx} - uu_{xxx} + \eta \rho \rho_x = 0,$ $\rho_t + (u\rho)_x = 0,$

with arbitrary $\kappa \in \mathbb{R}$ and $\eta \in (0, \infty)$. In the case $\eta = 1$, under the assumption that $u(t, x) \to 0$ and $\rho(t, x) \to 1$ as $x \to \pm \infty$ at any time t, the above system has been derived in the context of shallow water theory, where u(t, x) describes the horizontal velocity and $\rho(t, x)$ the horizontal deviation of the surface from equilibrium. Furthermore, it is one out of many generalizations of the famous Camassa–Holm (CH) equation, which has been studied intensively. Thus naturally the question arises which results derived for the CH equation are also valid for the 2CH system. In this talk we will show how to describe global weak solutions. This question is of special interest since the 2CH system, like the CH equation, enjoys wave breaking and in general there are two possibilities how to continue solutions thereafter. Namely, either the energy is preserved which yields conservative solutions or if energy vanishes from the system, we obtain dissipative solutions. Additionally, we will admit initial data and hence solutions with nonvanishing asymptotics.

- A. Constantin and R.I. Ivanov, On an integrable two-component Camassa-Holm shallow water system. Phys. Lett. A, 372 (48) (2008), pp. 7129–7132.
- [2] K. Grunert, H. Holden, and X. Raynaud, Global conservative solutions of the Camassa-Holm equation for initial data with nonvanishing asymptotics, to appear on *Discrete Cont. Dyn. Syst. Ser. A.*
- [3] K. Grunert, H. Holden, and X. Raynaud, Global solutions for the two-component Camassa-Holm system, Preprint (2011).
- [4] C. Guan and Z. Yin, Global existence and blow-up phenomena for an integrable two-component Camassa-Holm shallow water system, *J.Differental Equations*, 248 (2010), pp. 2003–2014.
- [5] D. Henry, Infinite propagation speed for a two component Camassa-Holm equation, Discrete Contin. Dyn. Syst. Ser. B, 12 (3) (2009), pp. 597–606.

Joint work with: Helge Holden (Norwegian University of Science and Technology (NTNU) and University of Oslo), Xavier Raynaud (University of Oslo).

S65 – Nonlinear Waves II – Room A, 18.00–18.30

- * * * -

Simulations of the Lifshitz-Slyozov equations: the role of coagulation terms in the asymptotic behavior

Frédéric Lagoutière Université Paris-Sud 11 (Orsay) frederic.lagoutiere@math.u-psud.fr

We consider the Lifshitz-Slyozov system that describes the kinetics of precipitation from supersaturated solid solutions. If we denote by f(t, x) the polymer density at time t and size x, and by c(t) the monomer density, the system reads

$$\begin{cases} \partial_t f + \partial_x (Vf) = 0, \quad t \ge 0, \quad x \ge 0, \\ V(t,x) = x^{1/3} c(t) - 1, \quad t \ge 0, \quad x \ge 0, \\ c(t) + \int_0^\infty x f(t,x) dx = \rho, \quad t \ge 0, \end{cases}$$

where ρ is the initial (given) total mass of monomer *and* polymer.

We design a specific Finite Volume scheme to investigate numerically the behavior of the solutions, in particular the large time asymptotics. Our purpose is two-fold: first, we introduce an adapted scheme based on downwinding techniques in order to reduce the numerical diffusion; second, we discuss the influence of coagulation effects on the selection of the asymptotic profile. This allows to understand better some conjectures by Lifshitz and Slyozov.

Some important references for this system are

- Lifshitz E. M. and Slyozov V. V., The kinetics of precipitation from supersaturated solid solutions, J. Phys. Chem. Solids 19, 35–50 (1961).
- [2] Niethammer B. and Pego R., On the initial-value problem in the Lifshitz-Slyozov-Wagner theory of Ostwald ripening, SIAM J. Math. Anal., 31, 467–485 (2000).
- [3] Sagalovich V. V. and Slyozov V. V., Diffusive decomposition of solid solutions, Sov. Phys. Usp, 30, 23–44 (1987).

 [4] Carrillo J.-A. and Goudon T., A numerical study on large-time asymptotics of the Lifshitz-Slyozov system, J. Scient. Comp, 18, 429–473 (2003).

Joint work with: Thierry Goudon, (INRIA Sophia Antipolis) and Léon Matar Tine (Université Paris Descartes)

S65 - Nonlinear Waves II - Room A, 18.30-19.00

* * * ·

The Generalized Buckley-Leverett System

Wladimir Neves UFRJ-Federal University of Rio de Janeiro wladimir@im.ufrj.br

We show the solvability of a proposed Generalized Buckley-Leverett System, which is related to the multidimensional Muskat Problem. Moreover, we discuss some important questions concerning singular limits of the proposed model.

Joint work with: Nikolai Chemetov (CMAF-University of Lisbon)

S65 – Nonlinear Waves II – Room A, 19.00–19.30

* * * -

Grassmannians and multisoliton KP-II solutions

Simonetta Abenda Department of Mathematics, University of Bologna simonetta.abenda@unibo.it

We consider a family of exact (N, M) soliton solutions to the KP-II equation introduced in [4]

 $(-4u_t + u_{xxx} + 6uu_x)_x + 3u_y y = 0.$

A (N, M) soliton solution is a real bounded regular solution u(x, y, t) which has N (resp. M - N) line soliton solutions in asymptotics $(y \to +\infty)$ (resp. $(y \to -\infty)$). These solutions may be classified in terms of the Schubert decomposition of the Grassmannian manifold Gr(N, M) where the solution of the KP-II equation is defined as a torus orbit (ref. [10], [3],[7]). To each point in the Grassmannian there is associated a real and totally positive $N \times M$ matrix A ([8],[9],[5] and ref. therein) which in turn in associated to a Darboux transformation ([6]).

In [1] we associate to each point in the Grassmannian a compatible set of divisors sitting on a m-curve and give an explicit representation of the matrix A. We discuss the relation of our results with a conjecture in [2].

References

[1] Abenda, S. and Grinevich, P.P., Grassmannians and multisoliton KP-II solutions, in press (2012).

- [2] Dubrovin, B. A. and Natanzon, S. M., Real theta-function solutions of the Kadomtsev-Petviashvili equation. Izv. Akad. Nauk SSSR Ser. Mat. 52 (1988), pp. 267–286.
- [3] Hirota, R. The direct method in soliton theory, Cambridge Univ. Press (2004).
- [4] Kadomtsev B.B. and Petviashvili V.I., On the stability of solitary waves in weakly dispersive media, Sov. Phys. Dokl., 15 (1970), pp. 539-541.
- [5] Kodama, Y. and Williams, L. K., KP solitons, total positivity, and cluster algebras. Proc. Natl. Acad. Sci. USA 108 (2011), pp. 89848989.
- [6] Matveev V.B. and Salle M.A., Darboux transformations and solitons, Berlin Springer (1991).
- [7] Miwa T., Jimbo M. and Date E. Solitons: Differential Equations, Symmetries and Infinite-Dimensional Algebras (Cambridge: Cambridge University Press) (2000). 53
- [8] Postnikov A., Total positivity, Grassmannians, and networks. arXiv:math.CO/060976v1
- [9] Postnikov A. and Speyer D, W. L., Matching polytopes, toric geometry, and the non-negative part of the Grassmannian. J Alg Combin 30, (2009) pp. 173191.
- [10] Sato M. Soliton equations as dynamical systems on an infinite dimensional Grassmannian manifold RIMS Kokyuroku (Kyoto University) 439 (1981), pp. 3046.

Joint work with: Petr. G. Grinevich (Landau Institute of Physics, Moscow (Russia))

12.3 Session 66 — Room G — Numerical Methods XIX

S66 – Numerical Methods XIX – Room G, 17.00–17.30

A positive, well-balanced and entropy-satisfying scheme for shallow water flows.

Jacques Sainte-Marie Univ. P.M. Curie, Paris 6 & CETMEF Jacques.Sainte-Marie@inria.fr

We consider the Saint-Venant system [2] for shallow water flows with non-flat bottom

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial x} (Hu) = 0 \tag{1}$$

$$\frac{\partial(Hu)}{\partial t} + \frac{\partial}{\partial x} \left(Hu^2 + \frac{g}{2}H^2 \right) = -gH\frac{\partial z_b}{\partial x} \tag{2}$$

where H(x,t) and u(x,t) respectively denote the water depth and the averaged velocity and $z_b(x)$ represents the bathymetry. The free surface is defined by $\eta = H + z_b$.

This system is a well-known hyperbolic system of conservation laws that approximately describes various geophysical flows, such as rivers, coastal areas, oceans when completed with a Coriolis term, and granular flows when completed with friction. Numerical approximate solutions to this system may be generated using conservative finite volume methods, which are known to properly handle any shocks and contact discontinuities.

Various efficient and stable numerical schemes have been proposed to solve (1)-(2) and preserve relevant equilibria. Among other, the hydrostatic reconstruction strategy [1] allows to derive so called well-balanced schemes but the numerical behavior of this technique e.g. when considering large bottom slopes and small water depth is not fully satisfactory. We propose a positive, consistent and well-balanced scheme that does not require any reconstruction of the variables at the interfaces. The scheme admits a discrete entropy and a second order extension in space and in time. The numerical treatment of the topography source term is the crucial point of this paper and is based on the Boltzmann-Vlasov equation describing, at the microscopic level, the shallow water equations.

The proposed scheme is confronted with classical test cases.

References

- E. Audusse, F. Bouchut, M.-O. Bristeau, R. Klein, and B. Perthame, A fast and stable well-balanced scheme with hydrostatic reconstruction for Shallow Water flows, SIAM J. Sci. Comput. 25 (2004), no. 6, 2050–2065.
- A.J.C. Barré de Saint-Venant, Théorie du mouvement non permanent des eaux avec applications aux crues des rivières et à l'introduction des marées dans leur lit, C. R. Acad. Sci. Paris 73 (1871), 147–154.

Joint work with: Emmanuel Audusse univ. Paris- North & Inria, France, Marie-Odile Bristeau Inria, France, Carlos Parès univ. Malaga, Spain.

S66 – Numerical Methods XIX – Room G, 17.30–18.00

A new model and numerical method for compressible two-fluid Euler flow

Barry Koren CWI, Amsterdam, The Netherlands Barry.Koren@cwi.nl

In this paper, a new formulation is derived for the five-equation model by Kapila [1] for compressible, twofluid Euler flow: a model that assumes pressure and velocity equilibrium across the two-fluid interface. The formulation does not explicitly consider the two-fluid interface; it assumes that the flow is a mixture of the two fluids. The formulation differs from that by Kapila in that its fifth equation is a conservation-law-like energyexchange equation for one of the two fluids. No equation is used to describe the topology of two-fluid interfaces. The complete system of equations is written in integral form, which directly allows for the application of a finite-volume method and Riemann solver. For the energy exchange, two terms are derived: a mechanical work term and a thermodynamic work term.

To evaluate the cell-face states for the fluxes of mass and energy (of both the bulk fluid and one of the two separate fluids), and for the flux of momentum (of the bulk fluid only), an approximate Riemann solver is constructed. Left and right cell-face states for the Riemann solver are constructed by a limited, higher-order accurate interpolation.

The Riemann solver is also used for the evaluation of the energy-exchange terms. Both terms contain firstorder spatial derivatives, and need to be integrated over the cells, including the cell faces. At the cell faces, the energy-exchange terms are not Riemann integrable. This difficulty is circumvented by integrating the terms at the cell faces in solution space, instead of in physical space. For this integration, consistent and practical use is also made of the approximate Riemann solver.

In numerical tests it is shown that the model and numerical method perform well for both two-fluid interface problems and two-fluid mixture problems. Physically correct solutions are obtained without any tuning or postprocessing. Particularly for standard shock-bubble-interaction problems, it appears that the physical model and numerical method accurately resolve detailed flow features (Figure 1).

- [1] A.K. Kapila, Two-phase modeling of deflagration-to-detonation transition in granular materials: Reduced equations, *Physics of Fluids*, **13** (2001), pp. 3002-3024.
- [2] J.F. Haas and B. Sturtevant, Interaction of weak shock waves with cylindrical and spherical gas inhomogeneities, *Journal of Fluid Mechanics*, 181 (1987), pp. 41-76.

Joint work with: Jasper Kreeft (Delft University of Technology, The Netherlands).

S66 – Numerical Methods XIX – Room G, 18.00–18.30

- * * *

A Multi-Scale Approach for Infiltration Processes in Porous Media

Frederike Kissling Institute of Applied Analysis and Numerical Simulation, University of Stuttgart, Germany Frederike.Kissling@mathematik.uni-stuttgart.de

We consider the infiltration of a wetting fluid (e.g. water) into a porous medium, $D \subseteq \mathbb{R}^2$, which is initially fully saturated by a nonwetting fluid (e.g. oil). Thereby different patterns can form. Either the solutions can consist of a planar front, or instabilities can occur and preferential flow paths (fingers) form. Both cases have in common that saturation overshoots can appear (DiCarlo [2]). We are interested in solutions which contain such saturation overshoots.

Neglecting gravitational forces and assuming the fluids to be immiscible and incompressible, the governing equations for this two-phase flow problem are given by the Darcy-law and the mass balance law for each phase. For the capillarity-free case these laws lead in the fractional flow formulation to a first-order nonlinear evolution equation

$$S_t + \operatorname{div}\left(\mathbf{v}f(S)\right) = 0 \quad \text{in} \quad D \times (0, T), \tag{1}$$

with the unknown phase saturation $S = S(\mathbf{x}, t) \in [0, 1]$ and a nonlinear flux function f = f(S). In the case of a heterogeneous porous medium the flux function $f = f(\mathbf{x}, t, S)$ is discontinuous. A coupled Darcy-type pressure velocity equation closes the system for the unknown velocity $\mathbf{v} = \mathbf{v}(\mathbf{x}, t) \in \mathbb{R}^2$. This system can have multiple weak solutions: there is a whole family of solutions involving non-Laxian so-called nonclassical transitional waves [9]. The saturation overshoots mentioned above can be identified as nonclassical waves. In contrast to the well understood case of classical Laxian waves nonclassical waves are not fully determined by the characteristic information.

We are interested in the unique physically relevant solution which is selected as the singular limit of solutions of regularized equations [5]. For the regularized equation capillary pressure effects have to be added. Our model depends not on the static pressure p_c alone, but involves a rate-dependent contribution, called dynamic pressure [4]. This leads to the regularization of (1) in $D \times (0, T)$

$$S_t^{\epsilon,\tau} + \operatorname{div}\left(\mathbf{v}f(S^{\epsilon,\tau})\right) = -\epsilon \operatorname{div}\left(\overline{\lambda}(S^{\epsilon,\tau})\nabla p_c^{\operatorname{static}}(S^{\epsilon,\tau})\right) + \tau\epsilon^2 \operatorname{div}\left(\overline{\lambda}(S^{\epsilon,\tau})\nabla S_t^{\epsilon,\tau}\right),\tag{2}$$

with $\epsilon, \tau > 0$ and $\overline{\lambda}(S^{\epsilon,\tau}) = \lambda^n(S^{\epsilon,\tau})f(S^{\epsilon,\tau})$, where λ^n is the mobility of the nonwetting phase.

In general nonclassical waves cannot be approximated numerically by conventional schemes for conservation laws like (1) since they converge against the classical Kruzkov solution. But for direct numerics it is too expensive to solve the regularized equation (2) on the whole domain, because a very fine grid and very small time steps are necessary. The solution of the regularized equation is only of importance close to a transitional wave. We are interested in the correct behaviour of (1) on the macroscale. Apart from a transitional wave it is adequate to use the non-regularized equation [7].

We analyze the singular limit in the case of a discontinuous flux function. We couple both models to overcome

the problems mentioned above and present a new multidimensional mass-conserving numerical method [6, 8] which belongs to the class of Heterogeneous Multiscale Methods (HMM) in the sense of E&Engquist [3]. For the regularized equation (2) we use a special solver on a small microscale space-time domain whereas we use a standard Finite Volume scheme for the conservation law (1) on the macroscale. The additional information which we gain on the microscale serves as an update with the help of a novel flux function, which is a key part of the approximation for the multidimensional setting and was introduced in 1D in [1]. We present numerical results of the HMM for homogeneous as well as for heterogeneous porous media and demonstrate the efficiency of the new method.

References

- B. Boutin, C. Chalons, F. Lagoutière, and P.G. LeFloch, Convergent and Conservative Schemes for Nonclassical Solutions Based on Kinetic Relations, *Interfaces and Free Boundaries*, 10 (2008), pp. 399-421.
- [2] D. DiCarlo, Experimental Measurements of Saturation Overshoot on Infiltration, Water Resour. Res., 40 (2004).
- [3] W. E and B. Engquist, The Heterogeneous Multiscale Methods, Comm. Math. Sci., 1 (2003), pp. 87-132.
- [4] S. M. Hassanizadeh and W. G. Gray, Thermodynamic Basis of Capillary Pressure in Porous Media, Water Resour. Res., 29 (1993), pp. 3389-3405.
- [5] F. Kissling and K. H. Karlsen, Singular Limits for Scalar Conservation Law in Porous Media with Discontinuous Flux Function, in preparation.
- [6] F. Kissling, R. Helmig, and C. Rohde, A Multi-Scale Approach for the Modelling of Infiltration Processes in the Unsaturated Zone, preprint (2011).
- [7] F. Kissling and C. Rohde, The Computation of Nonclassical Shock Waves with a Heterogeneous Multiscale Method, Networks and Heterogeneous Media, 3 (2010), pp. 661-674.
- [8] F. Kissling and C. Rohde, The Computation of Nonclassical Shock Waves with a HMM: The multidimensional case, in preparation.
- [9] P. G. LeFloch, Hyperbolic Systems of Conservation Laws, Lectures in Mathematics. Birkhäuser, (2002)

Joint work with: Christian Rohde (Institute of Applied Analysis and Numerical Simulation, University of Stuttgart), Kenneth H. Karlsen (Centre of Mathematics for Applications, University of Oslo)

S66 – Numerical Methods XIX – Room G, 18.30–19.00

Compactness of central schemes for 1D hyperbolic systems

Bojan Popov Texas A&M University popov@math.tamu.edu

In this talk we will address compactness and convergence issues for staggered central schemes in the context of 1D hyperbolic systems of conservation laws. Although an invariant domain and convergence results were proven for the LxF scheme for the *p*-system in Eulerian coordinates, in the Lagrangian case it is still not known whether we can prove compactness for bounded (or BV) initial data because the invariant domain of the schemes is unbounded. For the isothermal case ($\gamma = 1$), this is a well-known conjecture that vacuum cannot form. Assuming compactness at any given time and using a global CFL, we were able to prove that the total increase of the Riemann invariants per time step of the LxF scheme can be bounded by the entropy production of the scheme, similar bounds should hold for $\gamma > 1$. What we numerically observe is that we do have compactness for a large class of admissible initial data which we will describe in this talk. For the second order NT scheme we will show that the component-wise limiting can result in violation of invariant domain property and even blowup in the L^{∞} norm in some cases. In order to numerically observe invariant domain property for the NT scheme one has to consider characteristic decomposition and use characteristic-wise limiting. In this setting, for the two-component chromatography, we were able to prove that the NT scheme has an invariant domain property. However, the NT invariant domain is not the same as the LxF invariant domain in that case. This seems to be a generic property of central second order schemes. Numerical evidence and theoretical results will be presented.

S66 – Numerical Methods XIX – Room G, 19.00–19.30

* * * -

New Central-Upwind Schemes for Euler Equations of Gas Dynamics

Alexander Kurganov Tulane University kurganov@math.tulane.edu

Godunov-type schemes are projection-evolution numerical methods for hyperbolic systems of conservation laws. In these schemes, the computed quantities are the cell averages, which are used to construct a global piecewise polynomial approximation, which is then evolved in time to the next time level using the integral form of conservation laws. Since piecewise polynomial reconstructions are generically discontinuous at cell interfaces, one has to solve the (generalized) Riemann problems to incorporate upwinding needed to ensure nonlinear stability and high resolution achieved by the resulting Godunov-type upwind scheme.

Godunov-type central schemes offer a popular Riemann-problem-solver-free alternative to upwind schemes. The first high-resolution central scheme was constructed in [6] using the staggered grid, which allows one to evolve the solution in time using the control volumes that contain the Riemann fans generated at the cell interfaces. This makes the staggered central schemes simpler and less computationally expensive, yet more diffusive than their upwind counterparts.

In a series of our previous works ([1–5]), we have developed a new class of *nonstaggered* Godunov-type central schemes—*central-upwind schemes*—that combine the simplicity and universality of the central approach with high accuracy and low dissipation of upwind schemes. The key idea behind the derivation of the central-upwind schemes is to use the *upwinding* information to construct control volumes of variable size proportional to the one-sided *local* speeds of propagation of the waves emerging at cell interfaces. The solution at the new time level is then realized in terms of its cell averages over a strictly nonuniform grid with the twice larger number of cells than the original grid. Therefore, this intermediate solution must be projected back onto the original grid. To accurately project the intermediate data, we first reconstruct linear pieces over the nonuniform cells and then average the obtained reconstruction over the original (nonstaggered) grid. As it has been demonstrated in [1], a sharper projection procedure leads to the reduction of numerical dissipation and thus to enhanced resolution of contact waves.

However, the resolution of the contact waves achieved by the central-upwind schemes from [1] is still not as good as the resolution achieved by high-order upwind schemes. We therefore improve the central-upwind schemes by further reducing the amount of numerical diffusion, which in turns will lead to the enhanced resolution of contact discontinuities and other slow moving waves. The new method is based on a more accurate projection step, which is performed by taking into account a particular information on the waves to be captured by the central-upwind scheme. The new projection procedure hinges on specific properties of contact discontinuities. Therefore, the new central-upwind schemes cannot be viewed as a "black-box" solver any more. On the other hand, the enhanced resolution allows to outperform upwind schemes without solving (generalized) Riemann problems.

When the new central-upwind scheme is applied to an isolated contact wave, both the pressure and velocity remain constant. This is an extremely important feature of the new scheme as none of the existing central schemes is capable of satisfying this property.

Even though the idea behind the new central-upwind scheme is presented in the context of the Euler equations of gas dynamics, it gives rise to a general framework for designing central-upwind schemes for a wide range of nonlinear hyperbolic problems: One has to first identify quantities which remain continuous across contact waves and then to build a central-upwind scheme with the projection step adjusted accordingly.

References

- A. Kurganov and C.-T. Lin, On the reduction of numerical dissipation in central-upwind schemes, Commun. Comput. Phys., 2 (2007), pp. 141-163.
- [2] A. Kurganov, S. Noelle and G. Petrova, Semi-discrete central-upwind scheme for hyperbolic conservation laws and Hamilton-Jacobi equations, SIAM J. Sci. Comput., 23 (2001), pp. 707-740.
- [3] A. Kurganov and G. Petrova, Central-upwind schemes on triangular grids for hyperbolic systems of conservation laws, Numer. Methods Partial Differential Equations, 21 (2005), pp. 536-552.
- [4] A. Kurganov and E. Tadmor, New high resolution central schemes for nonlinear conservation laws and convection-diffusion equations, J. Comput. Phys., 160 (2000), pp. 241-282.
- [5] A. Kurganov and E. Tadmor, Solution of two-dimensional Riemann problems for gas dynamics without Riemann problem solvers, Numer. Methods Partial Differential Equations, 18 (2002), pp. 584-608.
- [6] H. Nessyahu and E. Tadmor, Nonoscillatory central differencing for hyperbolic conservation laws, J. Comput. Phys., 87 (1990), pp. 408-463.

12.4 Session 67 - Room B - Conservation Laws and Applications I

S67 - Conservation Laws and Applications I - Room B, 17.00-17.30

Mathematical modeling for a free boundary problem of hyperbolic type and properties of its solution

Kazuaki Nakane Faculty of Medicine Osaka University k-nakane@sahs.med.osaka-u.ac.jp

Tomoko Shinohara Tokyo Metropolitan College of Industrial Technology sinohara@s.metro-cit.ac.jp

In this talk, we treat the following physical phenomenon gA thin tape is pasted on a plate. The tape is peeled from the plate by lifting up one of the edge of the tapeh. We are interested in the behavior of the peeling front, especially, the phenomenon of self-excitation vibration. We assume that the movement of the tape is governed by a hyperbolic equation and is affected by the peeling front. The shape of the tape is described by the graph of a function $u : \Omega \times \{t > 0\} \to \mathbf{R}$, where Ω is an interval of **R**. The action of this phenomenon is given by

$$(J) \quad J(u) = \int_0^{T^*} \int_{u>0} \left(u_t^2 - u_x^2 - Q \right) dx dt \qquad u \in K,$$

where K is a suitable function space. The effect of the peeling front is described by a positive constant Q (adhesion).

Here, a stationary point is assumed to be smooth in order to derive the Euler-Lagrange equation. We have the following hyperbolic equation and the free boundary condition

$$(P) \quad \left\{ \begin{array}{ll} u_{tt} - u_{xx} = 0 & \text{in} & \{(x,t); x > -l_0, \ t > 0\} \cap \{u > 0\}, \\ u_t^2 - u_x^2 + Q = 0 & \text{on} & \{(x,t); x > -l_0, \ t > 0\} \cap \partial \{u > 0\}, \end{array} \right.$$

with the initial conditions

(I)
$$\begin{cases} u(x,0) = e(x) & \text{on} & (-l_0,0), \\ u_t(x,0) = g(x) & \text{on} & (-l_0,0), \end{cases}$$

and the boundary condition

(B) $u(-l_0, t) = f(t)$ on $[0, \infty)$,

where e(x), g(x) and f(t) are given functions, and l_0 is a positive constant. The initial condition (I) implies that the thin tape has been already peeled from the plate on the interval $(-l_0, 0)$, and the boundary condition (B) corresponds to the situation in which the edge of the tape is lifted up by f(t).

Remark (i) If we extent the integral region of (J) to the entire tape, i.e., $\{u \ge 0\}$, the non-linear term in the Lagrangian would be a characteristic function whose support was $\{u > 0\}$. The Euler-Lagrange equation would be a hyperbolic equation that, formally, includes $\delta_{\partial\{u>0\}}$ as a non-linear term. In this phenomenon, the existence of the outer force on the peeling front $\partial\{u>0\}$ appears to indicate this is a natural expression. If we regard this phenomenon as the propagation of discontinuity of the first differential, then the equation which includes δ -function can be justified in the distribution sense.

(ii) If the linear approximation of the asymptotic expansion of the solution holds, then the second equation of (P) means that the characteristic polynomial is not equal to zero. Therefore, we treat the case in which its characteristic condition does not hold.

(iii) From the derivation of the free boundary condition, we regard the second equation of (P) as a conservation law. Therefore, the quantity of energy passing out through the free boundary is Q. Based on the balance with the Q and the vibration of the tape, the movement of the free boundary is determined.

(iv) Adding physical parameters τ and ρ to the second equation of (P), we can derive

$$\left|\frac{dx}{dt}\right| = \sqrt{\frac{\tau}{\rho} - \frac{Q}{\rho u_x^2}},$$

where τ is the tension and ρ is the line density. This implies that, Q causes the speed of the free boundary to be slower than the propagation speed of the tape. This causes the vibration of the tape.

Theorem 1([3, Theorem 3.4]). We impose several assumptions on e, f and g. For any T > 0, there exists a unique solution to (P), (I) and (B).

Theorem 2([3, Theorem 4.1]). Let us assume that

$$f'(\eta) = 0$$
 for all $\eta > T_p$,

with a certain number T_p satisfying $T_p > l_0$. Then, the free boundary moves periodically.

- H. W. Alt and L. A. Caffarelli, Existence and regularity for a minimum problem with free boundary, J. Reine Angew. Math., 325 (1981), 105–144.
- [2] H. Imai, K. Kikuchi, K. Nakane, S. Omata and T. Tachikawa, A Numerical Approach to the Asymptotic Behavior of Solutions of a One-Dimensional Free Boundary Problem of Hyperbolic Type, J.J.I.A.M., 18-1 (2001), 43–58.
- [3] K. Nakane and T. Shinohara, Existence of periodic solutions for a free boundary problem of hyperbolic type, Journal of Hyperbolic Differential Equations, Vol. 5, No. 4, (2008), 785–806.

S67 - Conservation Laws and Applications I - Room B, 17.30-18.00

* * *

Penalty methods for edge plasma transport in a tokamak

Thomas Auphan Aix-Marseille Université, LATP tauphan@cmi.univ-mrs.fr

One of the main issue for the fusion by magnetic confinement is the wall-plasma interaction. The plasma transport essentially occurs along the magnetic field lines. To protect the wall from the high temperature plasma, an obstacle called limiter which interrupts the field lines is placed on the edge of the tokamak. The penalization method is used to take into account the limiter for numerical simulations of the edge plasma transport.

To construct our method, we first use a simplified 1D model considering only the plasma density and momentum. This nonlinear hyperbolic system is very similar to the shallow-water equations. At the plasma-limiter interface, there is a Dirichlet boundary condition on the Mach number due to the Bohm criterion [1]. To take into account the boundary conditions, we propose a penalty method, inspired from [2], which does not generate any spurious boundary layer. This is confirmed by a BKW asymptotic expansion and numerical tests which show an optimal convergence rate when the penalty parameter tends to 0 [3,4]. To perform the numerical experiments, we implement a second-order finite volume scheme using: VFRoe nev flux, MUSCL reconstruction with slope limiter and a semi-implicit time discretization.

The resulting penalty method and the asymptotic analysis is extended to more general quasi-linear hyperbolic boundary value problems.

- Isoardi, L. and Chiavassa, G. and Ciraolo, G. and Haldenwang, P. and Serre, E. and Ghendrih, Ph. and Sarazin, Y. and Schwander, F. and Tamain, P., Penalization modeling of a limiter in the Tokamak edge plasma, *Journal of Computational Physics*, Volume 229 no. 6 (2010), pp. 2220-2235.
- [2] Fornet, B. and Guès, O., Penalization approach of semi-linear symmetric hyperbolic problems with dissipative boundary conditions, *Discrete and Continuous Dynamical Systems*, Volume 23 no. 3 (2009), pp. 827-845.
- [3] Angot, Ph. and Auphan, T. and Guès, O., An optimal penalty method for the hyperbolic system modelling the edge plasma transport in a tokamak, preprint (2012), submitted to *Journal of Computational Physics*.
- [4] Angot, Ph. and Auphan, T. and Guès, O., Penalty Methods for the Hyperbolic System Modelling the Wall-Plasma Interaction in a Tokamak, in *Finite Volumes for Complex Applications VI - Problems & Perspectives*, Springer (2011), pp. 31-38.
Joint work with: Philippe Angot (Aix-Marseille Université, LATP) and Olivier Guès (Aix-Marseille Université, LATP)

S67 – Conservation Laws and Applications I – Room B, 18.00-18.30

A hyperbolic model for phase transitions in porous media

Andrea Corli Department of Mathematics, University of Ferrara, Italy andrea.corli@unife.it

We consider the following model for the isothermal and inviscid fluid flow through a porous medium, in presence of liquid-vapor phase changes:

 $\begin{cases} v_t - u_x = 0, \\ u_t + p(v, \lambda)_x = -\alpha u, \\ \lambda_t = \frac{1}{\tau} \left(p(v, \lambda) - p_e \right) \lambda(\lambda - 1), \end{cases}$

for t > 0 and $x \in \mathbb{R}$. Here v denotes the specific volume, u the velocity, p the pressure, $\lambda \in [0, 1]$ the mass-density fraction of the vapor in the fluid. The flow is hosted in a medium that induces a friction force proportional, with constant $\alpha > 0$, to the linear momentum, with opposite direction. The constants $\tau > 0$ and $p_e > 0$ are the characteristic reaction time and the equilibrium pressure, respectively. See [2] and [3] for related models.

We establish the existence and uniqueness of traveling waves in a wide range of situations. More precisely, the end states may be formed either by pure phases or mixtures; in the latter case, the pressure equals the equilibrium pressure. An interesting mathematical feature of the problem lies in the fact that the associated dynamical system turns out to be singular at points where the sonic and the equilibrium curves meet. Details are contained in [1].

References

- [1] A. Corli and H. Fan, Traveling waves of phase transitions in porous media, preprint (2012), to appear on *Appl. Anal.*
- [2] H. Fan, On a model of the dynamics of liquid/vapor phase transitions, SIAM J. Appl. Math., 60 (2000), pp. 1270–1301.
- [3] L. Hsiao and D. Serre, Global existence of solutions for the system of compressible adiabatic flow through porous media, SIAM J. Appl. Math., 27 (1996), pp. 70–77.

- * * * -

Joint work with: Haitao Fan (Department of Mathematics, Georgetown University, Washington, DC).

S67 - Conservation Laws and Applications I - Room B, 18.30-19.00

Lipschitz Semigroup for an Integro–Differential Equation for Slow Erosion

Graziano Guerra Dept. of Mathematics and Applications, Milano-Bicocca University, Italy graziano.guerra@unimib.it

We consider a model derived in [1] describing the changes for large times in the standing profile of a hill due to material sliding on it:

$$U_t(t,x) - \left(\exp\int_x^{+\infty} f\left(U_x(t,y)\right) \, dy\right)_x = 0.$$
⁽¹⁾

Here x is the space variable, U is the height of the profile, and t represents the total mass of moving layer that slid through. The slope of the profile U_x is assumed to remain strictly positive. The erosion function f, depends only on the slope and denotes the rate of mass being eroded or deposited per unit length and per unit mass passing through. There is a critical slope where no interaction happens and f vanishes. In a normalized model one could choose the critical slope to be 1. If the slope is bigger than 1, then f > 0 and erosion happens, so that the moving layer grows. If the slope is smaller than 1, then f < 0 and part of the moving layer deposits on the standing bed. In general, the erosion function f is non-linear, therefore the solutions of (1) may well lose regularity. Under suitable assumptions on f, the slope U_x remains uniformly bounded in t. We are here interested in the case where the erosion function f allows the development of singularities in the profile U. So we are faced with the problem of defining $f(U_x)$ where U_x could be a distribution. In [3] this problem was addressed and weak (possibly discontinuous) solutions to (1) were shown to exists. We are here interested in showing the existence of a Lipschitz continuous (with respect to time and the initial data) semigroup of such solutions.

The main point is to find out the variable which depends in a Lipschitz way from the initial data. Given a profile U(t, x) satisfying $U(t, x_2) - U(t, x_1) \ge \kappa (x_2 - x_1)$ for all $x_1, x_2 \in \mathbb{R}, x_1 \le x_2$, we introduce the (Lipschitz continuous) inverse function X = X(t, u) which is the graph completion of the inverse in space of U, X(t, u) = x iff $u \in [U(t, x-), U(t, x+)]$. Define the function z(t, u) to be the *u*-derivative of X(t, u), i.e., $z(t, u) \doteq X_u(t, u)$. In the case of a smooth function U, we can rewrite the integral in (1) as

$$\int_{x}^{+\infty} f(U_x(t,y)) \, dy = \int_{U(t,x)}^{+\infty} g(z(t,v)) \, dv \tag{2}$$

where the function g is defined by g(s) = s f(1/s) Remark that the right hand side in (2) is well defined also if $U(t) \in \mathbf{BV}(\mathbb{R};\mathbb{R})$. Therefore (2) can be used as a definition of the integral of the non linear function f of the measure U_x . Through a generalized wave front tracking algorithm, we will construct approximate solutions to (1), (2) which converge strongly to a Lipschitz semigroup $z(t, u) = (S_t z_o)(u)$ whose trajectories are solutions to (1), (2) and satisfies

$$\|S_t z_0 - S_{\bar{t}} z_1\|_{L^1} \le L \left(\|z_0 - z_1\|_{L^1} + |t - \bar{t}| \right).$$

- D. Amadori and W. Shen, The slow erosion limit in a model of granular flow, Arch. Ration. Mech. Anal. 199 (2011), pp. 1–31.
- [2] Colombo, Rinaldo M. and Guerra, Graziano and Shen, Wen, Lipschitz Semigroup for an Integro–Differential Equation for Slow Erosion, *Q. Appl. Math.* (To appear).
- [3] W. Shen and T.Y. Zhang, Erosion profile by a global model for granularflow, Arch. Ration. Mech. Anal. (To appear).

Joint work with: Rinaldo M. Colombo (Dept. of Mathematics, Brescia University, Italy), Wen Shen (Dept. of Mathematics, Penn State University, U.S.A.)

S67 - Conservation Laws and Applications I - Room B, 19.00-19.30

Nonlinear hyperbolic balance laws coupled with ordinary differential equations

Mauro Garavello Dipartimento di Matematica e Applicazioni, University of Milano Bicocca mauro.garavello@unimib.it

Systems composed by PDEs and ODEs can be used for the description of complex phenomena, whose evolution has both macroscopic and microscopic behaviors.

Similar multiscale models are used, for example, in the simulation of the blood flow in the human body: the main parts of the circulatory system are modeled by PDEs, while the remaining parts by ODEs. Other possible applications of such models are: traffic flows, supply chain, particles inside fluids.

In the talk we present a nonlinear hyperbolic system of balance laws coupled with a system of ordinary differential equations:

$$\begin{array}{ll} \langle \ \partial_t u(t,x) + \partial_x f(u(t,x)) = g(u(t,x)), & t > 0, x > \gamma(t), \\ \dot{w}(t) = F\left(t, w(t), u(t, \gamma(t)+)\right), & t > 0, \\ \dot{\gamma}(t) = \Pi(w(t)), & t > 0, \\ b\left(u(t, \gamma(t))\right) = B(t, w(t)), & t > 0, \\ u(0,x) = u_o(x), & x > \gamma(0), \\ w(0) = w_o, \\ \gamma(0) = x_o, \end{array}$$

More precisely, the ordinary differential equations influence the solution to the balance laws by means of the boundary term $b(u(t, \gamma(t))) = B(t, w(t))$, where $\gamma(t)$ describes a moving boundary, and, at the same time, the balance laws modify the vector field F of the ordinary differential equations. We discuss existence and well posedness of the Cauchy problem of this coupled system.

References

- R. Borsche, R.M. Colombo, M. Garavello, On the Coupling of Systems of Hyperbolic Conservation Laws with Ordinary Differential Equations, *Nonlinearity*, 23 (2010), pp. 2749-2770.
- [2] R. Borsche, R.M. Colombo, M. Garavello, Mixed Systems: ODEs-Balance Laws, J. Differential Equations 252 (2012), pp. 23112338.

Joint work with: Raul Borsche (Technische Universität Kaiserslautern), Rinaldo M. Colombo (Dipartimento di Matematica, University of Brescia).

12.5 Session 68 — Room H — Kinetic Models III

S68 – Kinetic Models III – Room H, 17.00–17.30

Time and space discrete scheme to suppress numerical solution oscillation for the neutron transport equations

Zhenying Hong

Institute of Applied Physics and Computational Mathematics, Department 6, NO.2. Fenghaodong Road, Haidian District, Beijing, 100094, China zyhong@iapcm.ac.cn

The neutron transport equations which are hyperbolic partial differential equations used in radiation shielding and nuclear reaction system, as well as medicine realm, are linearized version of the equation originally developed by Boltzmann for the kinetic theory of gases. There exit numerical solution oscillation for typical discrete scheme when solving multi-group multi media sophisticated time-dependent neutron transport equations which brings difficulty for mathematics and physics analysis. Especially for pivotal physical quantity, we can not take the key physical progress for the numerical solution oscillation.

In this paper, the numerical solution oscillation for sophisticated time dependent neutron transport equations is investigated. The influence of time discrete scheme and space discrete scheme on this oscillating phenomenon is analyzed for neutron transport equations. The typical time discrete scheme does not consider the adaptive time step. Therefore the physical curve about differential quantity for time variable exits numerical oscillation. The time step is given by physical progress in a general way and the time step change is very large(some magnitude difference) for the whole physical progress. The second-order time evolution scheme for time variable is very suit to adaptive time step problem and suppresses the numerical solution oscillation. Moreover, the linear discontinuous finite element method for space variable is an asymptotic preserving scheme and restricts the oscillation problem which exits in exponential method and diamond difference. Numerical experiments show that second-order time evolution scheme and linear discontinuous finite element method yield more accurate results and provide very smooth physical quantity curves.

- Adams M.L., Discontinuous Finite Element Transport Solutions in Thick Diffusive Problems, Nucl.Sci.Eng, 137. (2001), pp. 298-333
- [2] Coelho P. J., Bounded Skew High-Order Resolution Schemes for the Discrete Ordinates Method, J. Comput. Phys 175. (2002), pp. 412-437
- [3] Dedner A., Vollmöller P., An Adaptive Higher Order Method for Solving the Radiation Transport Equation on Unstructured Grids, J.Comput. Phys, 178. (2001), pp. 263-289
- [4] Frank M., Klar A., Larsen E.W., Yasuda S., Time-dependent Simplified Pn Approximation to the Equation of Radiative Transfer, *J.Comput.Phys*, 226. (2007), pp. 2289-2305
- [5] Hong Z. and Yuan G, A Parallel Algorithm with Interface Prediction and Correction for Spherical Geometric Transport Equation, *Prog. Nucl. Energy*, 51. (2009), pp.268-273.
- [6] Klose A.D. and Larsen E.W., Light Transport in Biological Tissue Based on the Simplified Spherical Harmonics Equations, *J. Comput. Phys*, 220. (2006), pp. 441-470
- [7] Lathrop K.D., A Comparison of Angular Difference Schemes for One-dimensional Spherical Geometry Sn Equations, Nucl.Sci.Eng, 134. (2000), pp. 239-264.
- [8] Lewis E.E. and Miller W.F., Computational Methods of Neutron Transport. La Grange Park. Amermos Nuclear Societ, Inc, (1993)

- [9] Machorro E., Discontinuous Galerkin Finite Element Method Applied to the 1-D Spherical Neutron Transport Equation, J. Comput. Phys, 223. (2007), pp. 67-81
- [10] Morel J.E. and Wareing.T.A., A Linear-Discontinuous Spatial Differencing Scheme for Sn Radiative Transfer Calculations, J.Comput.Phys, 128. (1996), pp. 445-462
- [11] Ryan G.M., James P.H., Thomas A.B., On Solutions to the Pn Equations for Thermal Radiative Transfer, J.Comput. Phys, 227. (2008), pp. 2864-2885
- [12] Sahni D.C.and Sharma A., Computation of Higher Spherical Harmonics Moments of the Angular Flux for Neutron Transport Problems in Spherical Geometry, Ann. Nucl. Energy, 27. (2004), pp. 411-433
- Siewert C.E., A Discrete-ordinates Solution for Multigroup Transport Theory with Upscattering, JQSRT,64. (2000), pp. 255-273
- [14] Wareing T.A., McGhee J.M, Morel, J.E., Discontinuous Finite Element Sn Methods on 3-D Unstructured Grids, Nucl.Sci.Eng, 138. (2001), pp. ,256-268
- [15] Hong, Z., Yuan, G., Fu, X., Methods of determining iterative initial value for time-dependent neutron transport equation, Journal On Numerical Methods and Computer Applications, 29. (2008), pp. 302-312
- [16] Hong, Z., Yuan, G., Fu, X., Oscillation of numerical for time-dependent particle transport equation, *Prog.Nucl.Energy*, 52. (2010), pp. 315-320
- [17] Hong, Z., Yuan, G., Fu, X., Yang, S., Modified time discrete scheme for time-dependent neutron transport equation, Nuclear Power Engineering, 31, S2. (2010), pp.34-37
- [18] Lathrop, K.D., Spatial differencing the transport equation: positivity vs. accuracy, J.Comp.Phys, 4. (1969), pp. 475-498
- [19] McClarren,G.G., A quasi-linear implementation of high-resolution time integration for P_N the equations, Nucl.Sci.Eng, 159. (2008), pp. 330-337
- [20] Morel, J.E., Wareing, T.A., A linear-discontinuous spatial differencing scheme for S_n radiative transfer calculations, *J. Comp. Phys*, **128.** (1996), pp. 445-462
- [21] Nkaoua, T., Sentis, R., A new time discretization for the radiative transfer equations: analysis and comparison with the classical discretization, SIAM J.NUMER.ANAL, 30. (1993), pp.733-748
- [22] Olson, G.L., Efficient solution of multi-dimensional flux-limited nonequilibrium radiation diffusion coupled to material conduction with second-order time discretization, *J. Comput. Phys*, **226.** (2007), pp. 1181-1195
- [23] Olson,G.L., Second-order time evolution of P_N equations for radiation transport, J.Comput.Phys, 228. (2009), pp. 3072-3083
- [24] Szilard, Rh.H., Pomraning, G.c., Numerical transport and diffusion methods in radiative transfer, Nucl. Sci. Eng, 112. (1992), pp.256-269
- [25] Larsen E.W., Morel J.E., Asymptotic solutions of numerical transport problems in optically thick, diffusive regimes II, J.Comput.Phys, 83. (1989), pp.212-236
- [26] Klar A., An asymptotic-induced scheme for nonstationary transport equations in the diffusive limit, SIAM J. NUMER. ANAL ,3. (1998), pp.1073-1094
- [27] Jin S., Efficient asymptotic-preserving(AP) schemes for some multiscale kinetic equations, SIAM J.Sci. Comp, 21. (1999), pp.441-454

Joint work with: Guangwei, Yuan, (Institute of Applied Physics and Computational Mathematics, Department 6, NO.2. Fenghaodong Road, Haidian District, Beijing, 100094, China), xuedong, fu(Institute of Applied Physics and Computational Mathematics, Department 6, NO.2. Fenghaodong Road, Haidian District, Beijing, 100094, China)

S68 – Kinetic Models III – Room H, 17.30-18.00

Microscopically Implicit-Macroscopically Explicit schemes for kinetic models

Gabriella Puppo Politecnico di Torino gabriella.puppo@polito.it

In this talk a new class of numerical methods for the BGK model of kinetic equations is presented. The schemes proposed are implicit with respect to the distribution function, while the macroscopic moments are evolved explicitly. In this fashion, the stability condition on the time step coincides with a macroscopic CFL, evaluated using estimated values for the macroscopic velocity and sound speed. Thus the stability restriction does not depend on the relaxation time and it does not depend on the microscopic velocity of energetic particles either.

This generalizes previous results presented in [4], where only the stiffness due to the relaxation time was addressed, as in [1] and [2]. With the technique proposed here, the updating of the distribution function requires the solution of a *linear* system of equations, even though the BGK model is highly non linear. Thus the proposed schemes are particularly effective for high or moderate Mach numbers, where the macroscopic CFL condition is comparable to accuracy requirements.

We show results for schemes of order 1 and 2, and the generalization to higher order is sketched, [3]. We also show that these schemes enjoy the Asymptotic Preserving (AP) property of [1], that is they recover the correct asymptotic limit for small and vanishing relaxation times. Moreover, the schemes proposed also preserve the opposite free molecular flow limit of very large relaxation times.

This technique can be generalized to other models, since it relays on the possibility of decomposing a model into fast and slow modes.

References

- F. Filbet S. Jin, A class of asymptotic-preserving schemes for kinetic equations and related problems with stiff sources, J. Comput. Phys., 229 (2010), pp. 7625–7648.
- [2] M. Lemou and L. Mieussens, A new asymptotic preserving scheme based on micro-macro formulation for linear kinetic equations in the diffusion limit, SIAM Journal on Scientific Computing, 31 (2008), pp. 334–368.
- [3] S. Pieraccini, G. Puppo Microscopically Implicit-Macroscopically Explicit schemes for the BGK equation. J. Comp. Phys, 231, (2012), pp. 299-327.
- [4] S. Pieraccini, G. Puppo, Implicit-Explicit schemes for BGK kinetic equations, J. Sci. Comput. 32 (2007), pp. 128.

* *

Joint work with: Sandra Pieraccini (Politecnico di Torino)

S68 – Kinetic Models III – Room H, $18.00{-}18.30$

Global weak solution for kinetic models of active swimming and passive suspensions

Xiuqing Chen Beijing Univ of Posts and Telecommunications tsinghuacxq@yahoo.com.cn

Bacterium swimming, acting as a force dipole, in fluid is modeled by a coupled Fokker-Planck equation and incompressible (Navier-)Stokes equation. According to the mechanism for swimming, a bacterium can be classified into pusher and puller. The local flow generated by the pusher outward force dipole increases the local straining flow, and hence reduces the effective viscosity and enhance flow-mixture. Therefore, a kind of instability appears for pusher swimming, which has been extensively studied numerically in physics literature. This instability can be explained by the fact that there is no entropy-dissipation relation for the pusher suspensions of coupled Fokker-Planck-(Navier-)Stokes system. Nonetheless, with some careful estimates, we are able to control the entropy and obtain the existence of global weak entropy solution for both pusher and puller systems.

Polymer suspensions is commonly modeled by suspension of extensible rods. There exists a spring force resisting to the rod extension. It is well known for this system that there is a relative entropy-dissipation relation with maxwellian weight. The main difficult of establishing global weak solution is the weak compactness of stress tensor exerted by the rod particles on the fluid. We will present a compactness embedding theorem

 $H^1_M(\mathbb{R}^d) \hookrightarrow \hookrightarrow L^2_{M(1+|n|^2)}(\mathbb{R}^d) \quad (M = Ce^{-U}, U \text{ is the spring potential}),$

provided that the spring potential is super-linear at far field. This compactness embedding theorem enables us to establish the existence of global weak solution to the coupled Fokker-Planck-(Navier-)Stokes equations for polymers. We will also prove that this compactness embedding theorem does not hold for linear spring potential, which indicating that the super-linear condition is sharp.

References

- Xiuqing Chen and Jian-Guo Liu, Global Weak Entropy Solution to Doi-Saintillan-Shelley Model for Active and Passive Rod-like and Ellipsoidal Particle Suspensions, preprint (2012)
- [2] Xiuqing Chen and Jian-Guo Liu, Global Weak Entropy Solution to Hookean Dumbbell Model, preprint (2012)

Joint work with: Jian-Guo,Liu (Duke University)

S68 - Kinetic Models III - Room H, 18.30-19.00

Asymptotic stability of kinetic plasmas for general collision potentials

Renjun Duan Department of Mathematics, The Chinese University of Hong Kong rjduan@math.cuhk.edu.hk An important physical model describing the dynamics of ionized plasmas in the collisional kinetic theory is the Vlasov-Poisson-Boltzmann system for which the plasma responds strongly to the self-consistent electrostatic force. This talk is concerned with the global close-to-equilibrium dynamics of kinetic plasmas in the whole space. We establish the global existence and optimal convergence rates of solutions near a global Maxwellian to the Cauchy problem on the Vlasov-Poisson-Boltzmann system for angular cutoff potentials with $-2 \le \gamma \le 1$. The main idea is to introduce a time dependent weight function in the velocity variable to produce a new dissipative mechanism so as to capture the kinetic singularity of the cross-section at both zero and infinity relative velocities. The approach that we have developed can be applied to several other physical situations such as the Landau collision for the Coulomb potential, the Boltzmann collision without angular cutoff, and even the appearance of the coupled Maxwell equations through the Lorentz force.

Joint work with: Tong Yang (City University of Hong Kong), Huijiang Zhao (Wuhan University)

S68 – Kinetic Models III – Room H, 19.00–19.30

On the asymptotics of solutions to resonator equations

Buğra Kabil Institut für Angewandte Analysis und numerische Simulation, University of Stuttgart Bugra.Kabil@mathematik.uni-stuttgart.de

We consider a system of micro-beam resonators within the thermoelastic theory of Lord and Shulman [4]. It is a particular case of a thermoelastic system given by a coupling of a plate equation to a hyperbolic heat equation arising from Cattaneo's law of heat conduction. Micro-resonators have high sensitivity at room temperature. Thermoelastic damping is one of the reasons for the dissipation or loss of energy from the system to its surroundings, see [7, 1, 2]. We model the problem of thermoelastic damping in micro-resonators by coupling the plate equation to a modified heat equation with one relaxation parameter proposed by Lord and Shulman.

The system of equations reads as

$$\begin{aligned} a\Delta^2 u + \Delta\theta + u_{tt} &= H & (x,t) \in \mathbb{R}^n \times (0,\infty), \\ \Delta\theta - m\theta + d\Delta\hat{u}_t &= c\hat{\theta}_t + G & (x,t) \in \mathbb{R}^n \times (0,\infty), \end{aligned}$$

where $\hat{f} = f + \tau f_t$. The initial conditions are given by

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad \theta(x,0) = \theta_0(x), \quad \theta_t(x,0) = \theta_1(x).$$

The constant $\tau > 0$ represents the relaxation parameter. We assume first that a, d and c are positive constants. The constant m plays an important role and it is assumed to be non-negative. The term $-m\theta$ can be seen as a physical damping. H and G correspond to external forces and heat supply.

The system in a bounded domain $B \subset \mathbb{R}^n$ was partly considered in [5, 6]. It was shown that the associated semigroup for $\tau > 0$ is not exponentially stable. We introduce in the first equation an additional term, called damping, to assure exponential stability. The first equation of the damped system has the form

$$a\Delta^2 u + \Delta\theta + u_{tt} + \gamma u_t = 0 \qquad (x,t) \in B \times (0,\infty),$$

where $\gamma > 0$ is a damping factor. The natural energy is given by

$$E(t) = \int_{B} (d\hat{u}_t^2 + ad|\Delta\hat{u}|^2 + c\hat{\theta}^2 + \tau(|\nabla\theta|^2 + m\theta^2)) \mathrm{d}B.$$

Using a suitable Lyapunov functional, we can show that the energy of the damped system is exponentially stable.

We consider the resonator equations in the whole space which have not been studied in the literature before. We want to study the asymptotic behaviour of the solutions to the Cauchy problem. Our aim is to determine for different values of m the decay rates for $t \to \infty$. We will see that the absence of the term $-m\theta$ will change the behaviour of the asymptotics. Using Fourier transform we get a system of ordinary differential equations. The solution of this system is explicitly given. Now one can determine L^1-L^∞ estimates for the solutions. Using interpolation techniques we obtain the following result for suitable constants p, q and $N_p > (1 - 2/q)(3n + 3)$.

Theorem 1. Let $m \neq 0$. Then there is a constant c(n,q) > 0, $\forall V(0) \in W^{N_p,p}(\mathbb{R}^n)$, $\forall t \ge 0$:

$$||V_t(t)||_q \leq c(1+t)^{-\frac{n}{4}(1-\frac{2}{q})}||V(0)||_{N_n,p},$$

where $V(t) = (\hat{u}(t), \hat{u}_t(t), \theta(t), \theta_t(t)).$

Letting m = 0, we observe that the homogenized system can be rewritten as

$$a\Delta^2 u + \Delta\theta + u_{tt} = 0,$$

$$c\theta_t + \nabla' q - d\Delta u_t = 0,$$

$$\tau q_t + q + \nabla\theta = 0,$$

where q is the heat flux. Analogously, we can study the asymptotics of this system. As mentioned before, the system changes its decay rates for m = 0. We can obtain an estimate for the second time derivative of the solution. So we have the following theorem for suitable constants p, q and $N_p > (1 - 2/q)(3n + 5)$.

Theorem 2. Let m = 0. Then there is a constant c(n,q) > 0, $\forall V(0) \in W^{N_p,p}(\mathbb{R}^n)$, $\forall t \ge 0$:

$$||V_{tt}(t)||_q \leqslant c(1+t)^{-\frac{n}{2}(1-\frac{2}{q})}||V(0)||_{N_p,p},$$

where $V(t) = (\hat{u}(t), \hat{u}_t(t), \theta(t), \theta_t(t)).$

We remark that one can get estimates for the vector V(t) by putting some conditions on the space dimension n. It should be mentioned that the constant m does not play a role in a bounded domain.

- [1] Akhiezer, A.I., Beretetskii, V.B., Quantum Electrodynamics, Interscience Publishers, pp. 23-25, 1965
- [2] Bahaa, E. A. S., Malvin, T. C., Grundlagen der Photonik, Wiley-VHC, 2008
- [3] Kabil B., On the asymptotics of solutions to resonator equations, Konstanzer Schriften in Mathematik Nr. 281, ISSN 1430-3558
- [4] Lord, H.W., Shulman, Y., A generalized dynamical theory of thermoelasticity, J. Mech. Phy. Solids, Vol. 15, pp. 299-309, 1967
- [5] Racke, R., Quintanilla, R., Qualitative aspects in resonators, Arch. Mech. 60, pp.345-360, 2008
- [6] Racke, R., Quintanilla, R., Addendum to: Qualitative aspects in resonators, Konstanzer Schriften in Mathematik Nr. 277, ISSN 1430-3358.
- [7] Sun, Y., Fang, D., Soh, A.K. Thermoelastic damping in micro-beam resonators, Int. J. Solids Structures 43, pp. 3213-3229, 2006

12.6 Session 69 — Room C — Wave Patterns Analysis II

S69 – Wave Patterns Analysis II – Room C, 17.00–17.30

Multi-dimensional rarefaction waves

Denis Serre ENS, Lyon denis.serre@ens-lyon.fr

For a self-similar solution to a system of conservation laws, genuine nonlinearity allows the regularity to reduce to Lipschitz continuity where the type changes. This is well-known in one space dimension, where a constant state \bar{u} bifurcates towards a rarefaction wave at a point x/t that equals an eigenvalue $\lambda_j(\bar{u})$. We extend this observation to several space dimensions. The result generalizes a calculation made by Bae, Chen and Feldman in their paper about irrotational gas dynamics

Joint work with: Heinrich Freistühler (University of Konstanz, Germany).

S69 – Wave Patterns Analysis II – Room C, 17.30–18.00

- * * * -

Stability of small shocks associated with Metivier-convex modes

Heinrich Freistühler University of Konstanz, Germany heinrich.freistuehler@uni-konstanz.de

Consider a hyperbolic system

$$\partial_t U + \partial_{x_1} F_1(U) + \partial_{x_2} F_2(U) = 0 \tag{1}$$

of (at least two) conservation laws in two space variables and a corresponding piecewise constant Laxian shock wave

$$U(x,t) = \begin{cases} U^{-}, & x_1 < 0, \\ U^{+}, & x_1 > 0, \end{cases}$$
(2)

of speed 0. The Kreiss-Majda Lopatinski determinant

$$\Delta(\tau,\xi) = \det(R_1^-(\tau,\xi),\dots,R_{p-1}^-(\tau,\xi),\tau[U] + i[F^{\xi}(U)],R_{p+1}^+(\tau,\xi),\dots,R_n^+(\tau,\xi))$$

of (2) is defined on

$$S \equiv \{(\tau,\xi) \in \mathbb{C} \times \mathbb{R} : \operatorname{Re} \tau \ge 0, |\tau|^2 + \xi^2 = 1\},\$$

with $F^{\xi} \equiv \xi F_2$ and

$$\{R_1^-(\tau,\xi),\ldots,R_{p-1}^-(\tau,\xi)\}, \{R_{p+1}^+(\tau,\xi),\ldots,R_n^+(\tau,\xi)\}$$

continuous bases for the (extensions to S of) the stable/unstable spaces $E^{-}(\tau,\xi), E^{+}(\tau,\xi)$ of

 $A(\tau,\xi) \equiv (\tau I + iDF^{\xi}(U^{\mp}))(DF_1(U^{\mp}))^{-1}.$

We call a simple mode $\Lambda = \Lambda(U, N, \xi)$ of a system (1) *Métivier convex* if

- (a) $D_U \Lambda(U, N, 0) \notin \text{left-Im}(D(FN)(U) \Lambda(U, N, 0)I)$ ("genuine nonlinearity") and
- (b) $D_{\xi}^{2}\Lambda(U, N, 0) > 0.$

It has been proved in [1] (and reproved in [2]) that sufficiently small Laxian shock waves associated with a Métivier convex mode satisfy the uniform Kreiss-Lopatinski condition,

$$\Delta(\tau,\xi) \neq 0 \quad \text{for all } (\tau,\xi) \in S, \quad \text{if } p = 1 \text{ or } p = n.$$
(3)

Among other things, this talk shows that there exist symmetric constant-multiplicity hyperbolic systems of conservation laws with a Métivier convex mode such that, in contrast to the above, for any sufficiently small shock wave associated with that mode,

$$\Delta(i\sigma,\xi) = 0 \quad \text{for some } \sigma, \xi \in \mathbb{R} \setminus \{0\} \text{ with } \sigma^2 + \xi^2 = 1.$$
(4)

This unexpected finding has interesting consequences.

References

- G. Métivier, Stability of multi-dimensional weak shocks, Comm. Partial Differential Eqs. 15 (1990), pp. 983-1028.
- [2] H. Freistühler & P. Szmolyan, Spectral stability of small-amplitude viscous shock waves in several space dimensions, Arch. Ration. Mech. Anal. 195 (2010), pp. 353-373.

S69 – Wave Patterns Analysis II – Room C, 18.00–18.30

The onset of instability for quasi-linear systems

Benjamin Texier Université Paris-Diderot et Ecole Normale Supérieure, Paris texier@math.jussieu.fr

A recent "Lax-Mizohata" theorem of Guy Métivier [1] states that for quasi-linear first-order systems, the Cauchy problem is well-posed only if the principal symbol is hyperbolic. For complex scalar equations, Lerner Morimoto and Xu considered in [2] the limiting case in which the system is initially hyperbolic, but as time increases hyperbolicity is lost.

I will discuss joint results with Nicolas Lerner, Toan Nguyen, Lu Yong and Marta Strani, that extend the results of [1] and [2] in various directions. Our approach consists in detecting instabilities from growth properties of the micro-local linear flow. Examples based on the Burgers, Van der Waals, and Klein-Gordon-Zakharov equations illustrate the results.

- G. Métivier, Remarks on the well-posedness of the nonlinear Cauchy problem, Geometric analysis of PDE and several complex variables, Contemp. Math., vol. 368, Amer. Math. Soc., Providence, RI, 2005, pp. 337-356.
- [2] N. Lerner, Y. Morimoto, C. J. Xu, Instability of the Cauchy-Kovalevskaya solution for a class of non-linear systems, American J. Math., 132 (2010), 1, 99-123.

[3] N. Lerner, T. Nguyen, B. Texier, Loss of hyperbolicity and ill-posedness for quasi-linear first-order systems, preprint, 2012.

Joint work with: Nicolas Lerner, (Université Pierre-et-Marie-Curie), Lu Yong (Université Paris-Diderot), Marta Strani (La Sapienza)

S69 – Wave Patterns Analysis II – Room C, 18.30–19.00

Stability of supersonic flow onto a wedge with the attached weak shock under the fulfillment of the weak Lopatinski condition

Dmitry Tkachev Sobolev Institute of Mathematics, Novosibirsk State University, Russia tkachev@math.nsc.ru

As is well known, the classical problem of supersonic stationary inviscid and non-heatconducting gas flow onto a planar infinite wedge (when the vertex angle σ is smaller than a limiting value σ_{max}) theoretically has two solutions. The first solution corresponds to the case of a strong shock (the components of the velocity vector $U = (u_0, v_0)$ behind the shock wave satisfy the inequality $u_0^2 + v_0^2 < c_0^2$, where c_0 is the sound speed), and the second one correspond to the case of a weak shock when $u_0^2 + v_0^2 > c_0^2$ [1].

Paradoxically, but in practice, in physical and numerical experiments, only one of two theoretically admissible solutions is realized. This is the case of a weak shock. R. Courant and K.O. Friedrichs set up the hypothesis that the flow with a weak shock is stable (by Lyapunov) whereas the flow with a strong shock is unstable. This is a lead for understanding this phenomenon.

On the linear level the Courant-Friedrich's hypothesis was justified in [2-4] for the case of a strong shock and in [5,6] for the case of a strong shock. Moreover, in [2-4] the instability nature when the wave comes to the wedge's vertex was clarified. In [5,6], under the assumption that the uniform Lopatinski condition holds an exact solution of the problem for the case of compactly supported initial data was found and the proof that this solution becomes stationary for a finite time (!) was given.

In our report we consider the linearized problem for the case of a weak shock but, unlike the study in [5,6], we assume that on the shock front the Lopatinski condition is satisfied only in a weak sense, i.e., the uniform Lopatinski condition is violated. Such a case takes place, for example, for a normal gas under certain assumptions on the equation of state [7]. We find a representation for the classical solution. Unlike the case of the uniform Lopatinski condition in [5,6], there appear additional disturbances (plane waves) in this representation. It turns out that the Courant-Friedrichs hypothesis in this situation is true as well, and the analysis of the obtained solution enables one to conclude that for compactly supported initial data the solution of the linear problem becomes stationary for a finite time.

This work was supported by RFBR (Russian Foundation for Basic Research) grants No. 10-01-00320-a and 11-08-00286-a.

- [1] R. Courant, K.O. Friedrichs, Supersonic flow and shock waves, Intersc. Publ., New York, 1948.
- [2] A.M. Blokhin, D.L. Tkachev, L.O. Baldan, Study of the stability in the problem on flowing around a wedge. The case of strong wave, J. Math. Anal. Appl., 319(2006), 248-277.
- [3] A.M. Blokhin, D.L. Tkachev, Yu.Yu. Pashinin, Stability condition for strong shock waves in the problem of flow around an infinite plane wedge, *Nonlinear Analysis: Hybrid Systems*, 2(2008), 1-17.

- [4] A.M. Blokhin, D. L. Tkachev, Yu.Yu. Pashinin, The Strong Shock Wave in the Problem on Flow Around Infinite Plane Wedge, in *Hyperbolic Problems. Theory, Numerics, Applications, Proceedings of the 11th International Conference on Hyperbolic Problems (Ecole Normale Superieure, Lyon, France, 2006)*, Springer - Verlag, Berlin, 2008, 1037-1044.
- [5] A.M. Blokhin, D.L. Tkachev, Stability of a supersonic flow about a wedge with weak shock wave, Sbornik: Mathematics, 200(2) (2009), 157-184.
- [6] D.L. Tkachev, A.M. Blokhin, Courant Friedrich's hypothesis and stability of weak shock, in Hyperbolic Problems. Theory, Numerics, Applications, Proceedings of the 12th International Conference on Hyperbolic Problems (Center for Scientific Computation and Mathematical Modeling, University of Maryland, College Park, 2008), 67, no. 2, Amer. Math. Society (2009), 958 - 966.
- [7] A.M. Blokhin, R.S. Bushmanov, D.L. Tkachev, On the fulfillment of the Lopatinski condition in the problem of normal gas flow onto a wedge, *Preprint no.* 71, Sobolev Institute of Mathematics, Novosibirsk, 2011.

Joint work with: Alexander Blokhin (Sobolev Institute of Mathematics, Novosibirsk State University, Russia)

S69 – Wave Patterns Analysis II – Room C, 19.00–19.30

Glancing weak Mach reflection

Allen M. Tesdall City University of New York, College of Staten Island allen.tesdall@csi.cuny.edu

We study the glancing limit of weak Mach reflection, in which the wedge angle approaches zero as the Mach number is held fixed. Lighthill showed using linearized theory that the strength of the reflected shock approaches zero at the triple point in glancing reflections. Therefore, to understand the nonlinear structure of the solution near the triple point in a glancing reflection, one needs to understand how the reflected shock diffracts nonlinearly into the Mach shock as its strength approaches zero. To this end, we formulate a half-space initial boundary value problem for the unsteady transonic small disturbance equations in y > 0. In this problem, the strong shock is approximated by a "soft" boundary y = 0 on which the pressure is constant. We solve this IBVP numerically using high resolution methods. The numerical solutions show a complex reflection pattern similar to the one that occurs in the Guderley Mach reflection of weak shocks. This is joint work with John Hunter.

References

- [1] M. J. Lighthill, The diffraction of blast. II, Proc. R. Soc. Lond. A, 200 (1950), pp. 554–565.
- [2] A. M. Tesdall and J. K. Hunter, Self-similar solutions for weak shock reflection, SIAM J. Appl. Math., 63 (2002), pp. 42–61.

Joint work with: John K. Hunter (University of California, Davis)

12.7 Session 70 - Room E - Conservation Laws and Applications II

S70 - Conservation Laws and Applications II - Room E, 17.00-17.30

Traveling wave solutions in scalar conservation laws with anomalous diffusion

Franz Achleitner Vienna University of Technology franz.achleitner@tuwien.ac.at

We consider scalar conservation laws with anomalous diffusion,

$$\partial_t u + \partial_x f(u) = \partial_x \mathcal{D}^{\alpha} u, \quad (t, x) \in \mathbb{R}_+ \times \mathbb{R}, \tag{1}$$

for a density $u: \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$, $(t, x) \mapsto u(t, x)$, a smooth flux function f(u) and a non-local operator

$$(\mathcal{D}^{\alpha}u)(x) = \frac{1}{\Gamma(1-\alpha)} \int_{-\infty}^{x} \frac{u'(y)}{(x-y)^{\alpha}} dy$$
(2)

with $0 < \alpha < 1$.

We prove the global solvability for the Cauchy problem to (1) in L^{∞} , i.e. the existence of a unique mild solution for the Cauchy problem with essentially bounded initial datum, by following the analysis of Droniou, Gallouet and Vovelle [DGV] in case of an anomalous diffusion realized by a fractional Laplacian. The crucial property is the non-negativity of the semigroup generated by $\partial_x \mathcal{D}^{\alpha}$, which is a consequence of its interpretation as an infinitesimal generator of an $(\alpha + 1)$ -stable Levy process [S], and allows to prove a maximum principle for solutions of the Cauchy problem.

To analyze the existence of traveling wave solutions connecting different far-field values, we work with the original representation (2) of \mathcal{D}^{α} , and obtain the traveling wave equation associated to (1) in form of a nonlinear Volterra integral equation. Assuming (even a bit less than) convexity of the flux function and that the solutions of the associated linear Volterra integral equation form a one-dimensional subspace of $H^2(\mathbb{R}_-)$, we can show the existence and uniqueness of monotone solutions satisfying the entropy condition for classical shock waves of the underlying inviscid conservation law. This requires to extend the well known results for the existence of viscous shock profiles, which solve (local) ordinary differential equations.

Moreover, we prove the dynamic nonlinear stability of the traveling waves under small perturbations, similarly to the case of the standard diffusive regularization, by constructing a Lyapunov functional.

For more details, we refer to our article [AHS].

Finally, we will provide an example of a single layer shallow water flow, where the pressure is governed by a nonlinear conservation law with the aforementioned nonlocal diffusion term and additional dispersion term [KCEG], and report on our recent progress in the analysis of smooth shock profiles.

References

- [AHS] F. Achleitner, S. Hittmeir and Ch. Schmeiser, On nonlinear conservation laws with a nonlocal diffusion term, *Journal of Differential Equations*, **250(4)** (2011), pp. 2177 2196.
- [DGV] J. Droniou, T. Gallouet and J. Vovelle, Global solution and smoothing effect for a non-local regularization of a hyperbolic equation, J. Evol. Equ., 3 (2003), pp. 499–521.
- [KCEG] A. Kluwick, E. A. Cox, A. Exner and C. Grinschgl, On the internal structure of weakly nonlinear bores in laminar high Reynolds number flow, Acta Mech., 210 (2010), pp. 135–157.
 - [S] Ken-iti Sato, Lévy processes and infinitely divisible distributions, Cambridge University Press, (1999).

Joint work with: Sabine Hittmeir (Vienna University of Technology), Christian Schmeiser (University of Vienna)

* * * -----

S70 – Conservation Laws and Applications II – Room E, 17.30-18.00

On the Doi model for the suspensions of rod-like molecules in compressible fluids

Hantaek Bae CSCAMM, University of Maryland hbae@cscamm.umd.edu

In this talk, we introduce a new model, *compressible Doi model* for suspensions of rod-like molecules in a dilute regime. This model is 5-dimensional (three-dimensions in physical space and two degrees of freedom on the sphere) and it describes the interaction between

- the orientation of rod-like molecules at the microscopic scale and
- the macroscopic properties of the fluid in which these molecules are contained.

The aim of this talk is to present the following subjects:

- new set of equations,
- definition of weak solution and main result,
- construction of approximate sequence of solutions.

This is a joint work with K. Trivisa (University of Maryland).

S70 - Conservation Laws and Applications II - Room E, 18.00-18.30

The Cauchy Problem for a conservation law with a multiplicative stochastic perturbation

Caroline Bauzet UMR-CNRS 5142, IPRA BP 1155, 63013 Pau Cedex France caroline.bauzet@etud.univ-pau.fr

We are interested in the formal non linear conservation law with a multiplicative stochastic perturbation of type:

$$du - \operatorname{div}(\vec{\mathbf{f}}(u))dt = h(u)dw \text{ in } \Omega \times \mathbb{R}^d \times]0, T[, \tag{1}$$

with an initial condition $u_0 \in L^2(\mathbb{R}^d)$ and $d \ge 1$. We consider a positive number T and $W = \{w_t, \mathcal{F}_t; 0 \le t \le T\}$ denotes a standard adapted one-dimensional continuous Brownian motion, defined one the classical Wiener space (Ω, \mathcal{F}, P) . Let us assume that

H₁: $\vec{\mathbf{f}} = (f_1, ..., f_d) : \mathbb{R} \to \mathbb{R}^d$ is a Lipschitz-continuous function and $f_i(0) = 0, \forall i = 1, ..., d$.

H₂: $h : \mathbb{R} \to \mathbb{R}$ is a Lipschitz continuous function with h(0) = 0.

We propose to prove a result of existence and uniqueness of the stochastic entropy solution for (1) in the set of predictable processes $\mathcal{N}^2_w(0, T, L^2(\mathbb{R}^d))$ [1]. A method of artificial viscosity is proposed to prove the existence of a solution. First we are interested in the parabolic problem :

 $du_{\epsilon} - [\epsilon \Delta u_{\epsilon} - \operatorname{div}(\vec{\mathbf{f}}(u_{\epsilon}))]dt = h(u_{\epsilon})dw \text{ in } \Omega \times \mathbb{R}^{d} \times]0, T[,$

with $u_0^{\epsilon} \in \mathcal{D}(\mathbb{R}^d)$, studied in [2]. Applying the It formula [1] to the process u_{ϵ} we obtain a viscous entropic formulation, and passing to the limit on the parameter ϵ , we get an entropic formulation for (1). The compactness properties used are based on the theory of Young measures and on measure-valued solutions. An appropriate adaptation of Kruzhkov's doubling variables technique is proposed to prove the uniqueness of the measurevalued entropy solution. Then standard arguments allow us to deduce existence (and uniqueness) of stochastic weak entropy solutions.

References

- G. Da Prato and J. Zabczyk, Stochastic equations in infinite dimensions, Encyclopedia of Mathematics and its Applications, Cambridge University Press, (1992).
- [2] G. Vallet, Stochastic perturbation of nonlinear degenerate parabolic problems, *Differential and integral equation 21*, Volume no.11-12, (2008), pp.1055-108.

Joint work with: Guy Vallet (UMR-CNRS 5142, IPRA BP 1155, 63013 Pau Cedex France), Petra Wittbold (Fakultät für Mathematik Universitätsstr. 2 45141 Essen).

S70 – Conservation Laws and Applications II – Room E, 18.30–19.00

Entropy decreasing in resonant contact discontinuities

Nicolas Seguin UPMC Univ Paris 06 & CNRS, UMR 7598, Laboratoire J.-L. Lions, F-75005, Paris, France nicolas.seguin@upmc.fr

We address the problem of the analysis and of the numerical approximation of resonant systems in different applications: trafic flows, fluid-particle interaction, two-phase flows, shalow water equations, coupling problems... By "resonant", we mean that the matrix $A(U) \in \mathbb{R}^{N \times N}$ in a quasilinear system of the form

$$\partial_t U + A(U)\partial_x U = 0 \tag{1}$$

is diagonalizable in \mathbb{R}^N for all $U \in \Omega \subset \mathbb{R}^N$ (Ω being the set of admissible states) except in a manifold $\Omega_r \subset \Omega$, for which $A(U_r)$, $U_r \in \Omega_r$, still admits real eigenvalues but it is no longer diagonalizable in \mathbb{R}^N . This implies that the multiplicity of at least one eigenvalue of $A(U_r)$ is greater than one. We focus more precisely on the case where it corresponds to the superposition of a linearly degenarate field with a genuinely nonlinear field.

A general form for such an issue is

$$\begin{cases} \partial_t U_l + \partial_x F_l(U_l) = 0, & t > 0, x < 0, \\ \partial_t U_r + \partial_x F_r(U_r) = 0, & t > 0, x > 0, \end{cases}$$
(2)

where F_l , F_r are two classical nonlinear $\mathcal{C}^2(\mathbb{R}^N; \mathbb{R}^N)$ fluxes and we supplement systems (2) by coupling conditions, which can be written in the abstract form

$$(U_l(t, 0^-), U_r(t, 0^+)) \in \mathcal{G}$$
 (3)

where $\mathcal{G} \in \mathbb{R} \times \mathbb{R}$ is called the *germ* and denotes the compatibility conditions we want to impose to connect the two half problems.

According to the problem we are faced with, the germ \mathcal{G} (and of course the fluxes F_l, F_r) may be very different and this leads to different theoretical problems and their numerical approximation has to be adapted. Nevertheless, in each cases, the eigenvalues of the Jacobian matrix $F'_l(U), F'_r(U)$ can vanish, leading to a *resonant* problem.

We present in the scalar case two cases where the analysis is fully achieved, respectively in [1] and [2]: the first one corresponds to the account for a less permeable slice in a porous medium, giving

$$F_l(U) = F_r(U) = U(1 - U) \text{ and } \mathcal{G} = \{(U, V) \in \mathbb{R}^2, F_l(U) = F_r(V) \le \overline{F}\}$$

where \overline{F} is computed from the permeability of the slice and U is the saturation, while the second case comes from the modelling of the friction due to a pointwise grid in a pipe: $F_l(U) = F_r(U) = U^2/2$ and \mathcal{G} is defined through the limit of

$$\partial_t U + \partial_x (U^2/2) = -\lambda U \rho_\epsilon(x)$$

where λ is a positive friction coefficient and $\rho_{\epsilon}(x) = \rho(x/\epsilon)/\epsilon$, ρ being a classical approximation of the unit.

In the case of systems, typical examples are the gas dynamics equations in a discontinuous duct, the shallow water equations with a discontinuous bottom and two-phase flows governed by the Baer-Nunziato model [3]. Following [4] and [5], we also provide a careful theoretical and numerical study of such resonant systems. On may also extend the scalar model of pointwise friction to the Euler equations.

In all these cases, we will show that the solution is discontinuous at the interface $\{x = 0\}$ and that the entropy may (and actually has to) decrease through it for some resonant solutions. It will be justified by regularization effects, leading to well-posedness results and convergence of adapted finite volume schemes.

References

- B. Andreianov, P. Goatin, N. Seguin, Finite volume schemes for locally constrained conservation laws, Num. Math. 115 (2010), pp. 609–645.
- B. Andreianov, N. Seguin, Analysis of a Burgers equation with singular resonant source term and convergence of well-balanced schemes, accepted for publication in *Discrete and Continuous Dynamical Systems* - Series A (DCDS-A).
- [3] M. R. Baer, J. W. Nunziato, A two phase mixture theory for the deflagration to detonation (DDT) transition in reactive granular materials, Int. J. Multiphase Flow 12 (1986), pp. 861–889
- C. Cancès, N. Seguin, Error estimate for Godunov approximation of locally constrained conservation laws, submitted.
- [5] P. Goatin, P. G. LeFloch, The Riemann problem for a class of resonant hyperbolic systems of balance laws, Ann. Inst. H. Poincaré Anal. Non Linéaire 21 (2004), pp. 881–902.
- [6] E. Isaacson, B. Temple, Convergence of the 2 × 2 Godunov method for a general resonant nonlinear balance law, SIAM J. Applied Math. 55 (1995), pp. 625–640.

Joint work with: Boris Andreianov (LMB, Université de Franche-Comté), Clément Cancès (LJLL, UPMC-Paris 6), Frédéric Coquel (CNRS), Paola Goatin (Inria), Jean-Marc Hérard (EDF R&D), Khaled Saleh (EDF R&D and LJLL, UPMC-Paris 6).

* * * —

S70 - Conservation Laws and Applications II - Room E, 19.00-19.30

Spectral Stability of Small-Amplitude Traveling Waves via Geometric Singular Perturbation Theory

Johannes Wächtler Universität Konstanz johannes.waechtler@uni-konstanz.de

We study the spectral stability of small-amplitude traveling waves in two different systems: First, in a system of reaction-diffusion equations where the reaction term undergoes a pitchfork bifurcation; second, in a strictly hyperbolic system of viscous conservation laws where the k-th characteristic family is not genuinely nonlinear. In either case, there exist families ϕ_{ε} of small-amplitude traveling waves. The eigenvalue problem associated with the linearization at ϕ_{ε} is a system of ordinary differential equations depending on two parameters, the amplitude ε and the spectral value κ . Suitably scaled, the system reveals a slow-fast structure. Using methods from geometric singular perturbation theory, this will be exploited to thoroughly describe the dynamics of the eigenvalue problem in the zero-amplitude limit. We will prove that the eigenvalue problem converges, in the limit $\varepsilon \to 0$, to the well-understood eigenvalue problem associated with a traveling wave ϕ_0 in a certain scalar equation. The profiles ϕ_{ε} then inherit the spectral stability from the respective limit profile ϕ_0 .

Following the line of thought of Freistühler and Szmolyan [1], the proofs rely on concepts from dynamical system theory, most notably on invariant manifold theory and geometric singular perturbation theory.

References

 H. Freistühler, P. Szmolyan, Spectral stability of small shock waves, Arch. Ration. Mech. Anal., 164 (2002), pp. 287 – 309.

12.8 Session 71 — Room I — Phase Field Models

S71 – Phase Field Models – Room I, 17.00–17.30

The Riemann problem for three-phase flow with quadratic permeabilities

Frederico Furtado University of Wyoming, USA furtado@uwyo.edu

We focus on a particular system of two conservation laws which models immiscible flow in porous media relevant for petroleum engineering. The Riemann solutions are found for a range of initial conditions important in applications, representing the injection of two fluids (water, gas) into a horizontal reservoir containing a third fluid (oil) to be displaced.

Despite loss of strict hyperbolicity, the solution for each data exists and is unique. Also, it depends L_1 continuously on the Riemann data. Such solutions always display a lead shock involving one of the injected fluids and the fluid already present. There is a threshold solution separating solutions according to which of the injected fluids is present in the lead shock.

In this particular model the permeability of each of the three phases depends quadratically on its own saturation. This simplification is conducive to explicit calculations, allowing the proof of many facts needed for establishing the structure of the solution, its existence and uniqueness. This particularization, however, preserves all essential topological features of the Riemann solution for a large class of permeability models typically used in petroleum engineering applications, which is described in another work [1].

[1] Pablo Castañeda, F. Furtado, D. Marchesin, The Riemann problem for three-phase flow in virgin reservoirs for general permeabilities, *Submitted to hyp2012*.

Joint work with: Arthur Azevedo (University of Brasília, Brazil), Aparecido de Souza (Federal University of Campina Grande, Brazil), and Dan Marchesin (IMPA, Brazil).

Numerical simulation of wave propagation in three-phase flows in porous media with spatially varying flux functions

Eduardo Abreu

Universidade Estadual de Campinas (University of Campinas) Department of Applied Mathematics eabreu@ime.unicamp.br

Distinct hyperbolic-parabolic models have been proposed for three-phase immiscible displacement problems in porous media (e.g., [3,4,5,7,8] and references cited therein), such as the scalar case of immiscible two-phase flow (the classical Buckley-Leverett waterflood problem) and the system case to simultaneous immiscible threephase flow (the classical oil/water/gas flow problem).

We describe a computational method [1] intended for simulating three-phase immiscible incompressible flow system in porous medium in two space dimensions, which is an extension of the scheme of [2] to a two-dimensional case with gravity and spatially varying flux functions:

$$\frac{\partial}{\partial t} [\phi(\mathbf{x})\mathbf{S}] + \nabla \cdot [\mathbf{vF}(\mathbf{S}) + k(\mathbf{x})\mathbf{H}(\mathbf{S})] = \nabla \cdot [\epsilon \, k(\mathbf{x}) \, \mathbf{D}(\mathbf{S})\nabla\mathbf{S}],\tag{1}$$

 $\mathbf{x} \in \Omega \subset \Re^2, t \ge 0, \epsilon > 0$, where $\mathbf{F}(\mathbf{S}) = [f_w, f_g]^\top$ and $\mathbf{H}(\mathbf{S}) = [h_w, h_g]^\top$, are nonlinear flux quantities with $i = w, g, f_i = f_i(\mathbf{S}), h_i = h_i(\mathbf{S}), \mathbf{S} = [S_w, S_g]^\top$ and $\mathbf{S}(\mathbf{x}, t) \in \mathbb{T} \subset \Re^2$. The vector quantity $\mathbf{v} = (v_x, v_y)$ is the total velocity (associated with an elliptic boundary value problem for the incompressible three-phase problem at hand). The quantities $\phi(\mathbf{x})$ and $k(\mathbf{x})$ are properties of the rock, porosity and permeability, respectively. Thus, the porous medium may be heterogeneous. For certain models of three-phase flow of type (1) used in mathematics and widely used in applications [3,4,7,8] is that the 2×2 system of first-order hyperbolic partial differential equations for S_w (water phase), S_w (gas phase) and $S_o = 1 - S_w - S_g$ (oil phase), which results when the diffusive (parabolic) matrix $\mathbf{D}(\mathbf{S})$ is neglected [3] is that it fails to be strictly hyperbolic somewhere in the state space $\mathbb{T} = \{(S_w, S_g) \mid 0 \le S_i \le 1; S_w + S_g + S_o = 1\}$; i.e., the characteristic speeds coincide, or resonate [3,4,8].

Our new computational method [1] is an operator-splitting procedure for decoupling the nonlinear hyperbolicparabolic three-phase flow equations (1) with mixed discretization methods, leading to purely hyperbolic, parabolic and elliptic subproblems. The computational procedure in [2,5] has been used to numerically investigate the existence of nonclassical waves in heterogeneous porous media of a simplified three-phase flow equations without gravity. Following [2,5], our hyperbolic solver is also based on the central scheme introduced by E. Tadmor [6]. Specifically, the resulting numerical formulation can handle the computation of spatially varying flux functions and the variable porosity in the transport system (1). We use locally conservative mixed finite elements to handle the associated parabolic and elliptic subproblems [1,2,5].

The numerical simulation of the differential equations (1) corroborates that the heterogeneity has a distinct effect in the simulation of wave propagation in porous media. In particular, the numerical simulations were able to qualitatively reproduce semi-analytical results for three-phase flows [7,8] in homogeneous media. Although not exhaustive, our numerical experiments [1] show some evidence of wave with nonclassical structure for the simulation of two-dimensional equations of three-phase flow (1) taking into account gravity and spatially varying flux functions.

- [1] E. Abreu, Numerical modelling of three-phase immiscible flow in heterogeneous porous media with gravitational effects, *Submitted*, (2012).
- [2] E. Abreu and J. Douglas Jr. and F. Furtado and D. Marchesin and F. Pereira, Three-phase immiscible displacement in heterogeneous petroleum reservoirs, *Mathematics and Computers in Simulation*, 73 (2006), 2-20.
- [3] A. Azevedo and D. Marchesin and B. J. Plohr and K. Zumbrun, Capillary instability in models for threephase flow, *Zeitschrift fur Angewandte Mathematik und Physik*, 53 (2002), 713-746.
- [4] E. Isaacson and D. Marchesin and B. Plohr and and J. B. Temple, Multiphase flow models with singular Riemann problems, *Comput. Appl. Math.*, **11** (1992), 147-166.
- [5] E. Abreu and J. Douglas Jr and F. Furtado and F. Pereira, Operator splitting based on physics for flow in porous media. *International Journal of Computational Science*, 2 (2008), 315-335.
- [6] N. Nessyahu and E. Tadmor. Non-oscillatory central differencing for hyperbolic conservation laws. *Journal of Computational Physics*, 87 (1990), 408-463.
- [7] A. Azevedo and A. Souza and F. Furtado and D. Marchesin and B. Plohr, The solution by the wave curve method of three-phase flow in virgin reservoirs, *Transport in Porous Media*, 83 (2010), 99-125.
- [8] D. Marchesin and B. Plohr, Wave structure in wag recovery, Society of Petroleum Engineering Journal, 71314 (2001), 209-219.

S71 – Phase Field Models – Room I, 18.00–18.30

*

Adaptive two-three layer modelling of stratified flows

Sebastian Noelle *RWTH Aachen University* noelle@igpm.rwth-aachen.de

We consider stratified shallow water flow, for which layers of different density can be identified. Such flow occurs in oceans at sea gates or at river mouths, where water of different salinity and temperature and thus different density flows together.

Balance laws such as the shallow water equations present notorious difficulties due to a large variety of source terms and their well-balancing, and because of wet/dry fronts. For the two-layer equations, an additional challenge is presented by non-conservative products, which require a sophisticated analytical and numerical framework (see Dal Maso, Murat and LeFloch, Castro, Parés and collaborators [1] as well as work by Abgrall and Karni, Bouchut and Morales, Kurganov and others). While there is a lively debate on the issue of nonconservative products, we consider this issue to be settled for the sake of this presentation.

Here we focus on another difficulty, the possible loss of hyperbolicity due to moderate or large shear velocities between the layers. For flows which are not depth-averaged, this corresponds to the development of a Kelvin-Helmholtz instability and a mixing layer. We will discuss two approaches to overcome the breakdown of the two-layer model. Both involve the introduction of viscosity, but in rather different ways.

The first approach, due to Castro, Fernández-Nieto, González-Vida, and Parés [2] adds just enough viscosity to the algorithm to move the numerical solution out of the elliptic region up to the boundary of the hyperbolic region. The second approach, which we began to study in [3], introduces more vertical structure into the solution: first, a third layer is introduced locally before the hyperbolicity of the two-layer equation breaks down. This layer realizes an instantaneous mixing, and it is removed when (and where) the shear has decreased and the two-layer model would be hyperbolic again. These steps are highly nontrivial, because they require an understanding of the eigenvalues of the 6x6 three-layer system near the degenerate points where the eigenvalues coincide, see [3,4]. In his dissertation [4], Frings developed a finite volume scheme to test this strategy. Interestingly, this approach was not sufficient to maintain hyperbolicity of the adaptive 2/3-layer model.

In order to stabilize the 2/3 layer model further, Frings introduced *sublayers* within each of the two respectively three *macro-layers*. Similar to a model proposed earlier by Audusse [5], there is interlayer friction between the sublayers. But in the present work, the sublayer equations are coupled more weakly. This avoids the high-dimensional eigenspaces of [5]. Currently, we are also extending the direct approach of [2] to stabilize the 2/3-layer model.

We prove that this concept is robust by computing several underwater dambreak flows, and comparing them with laboratory experiments.

References

- M.Castro, P.LeFloch, M.-L.Muñoz-Ruiz and C.Parés, Why many theories of shock waves are necessary: convergence error in formally path-consistent schemes. J. Comput. Phys. 227 (2008), pp. 8107-8129.
- [2] M.Castro, E.D.Fernández-Nieto, J.M.González-Vida, C.Parés, Numerical treatment of the loss of hyperbolicity of the two-layer shallow-water system. J. Sci. Comput. 48 (2011), pp. 16-40.
- [3] M. Castro, J.T. Frings, S. Noelle, C. Parés, G. Puppo, On the hyperbolicity of two- and three-layer shallow water equations. IGPM report 314 (2010), RWTH Aachen University. Submitted to *Proceedings of the* 13th International Conference on Hyperbolic Problems (Peking, June 15-19, 2010) (Sept.30, 2010).
- [4] J.T.Frings, An adaptiv multilayer model for density layered shallow water flows. *Dissertation*, RWTH Aachen University, 2011.
- [5] E.Audusse, A multilayer Saint-Venant model: derivation and numerical validation. Discrete Contin. Dyn. Syst. Ser. B 5 (2005), pp. 189-214.

Joint work with: Jörn Thies Frings (*RWTH Aachen University*), Manuel Castro(*University of Malaga*), Carlos Parés (*University of Malaga*) Gabriella Puppo (*Politecnico di Torino*)

S71 – Phase Field Models – Room I, 18.30–19.00

Stone-Marchesin Model Equations of Three-Phase Flow in Oil Reservoir Simulation

Fumioki Asakura Department of Asset Management, Osaka Electro-Communication Univ. asakura@isc.osakac.ac.jp

We are concerned with the Cauchy problem for a system of conservation laws

$$\begin{cases} \frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left[\frac{\alpha u^2}{\alpha u^2 + \beta v^2 + \gamma (1 - u - v)^2} \right] = 0, \\ \frac{\partial v}{\partial t} + \frac{\partial}{\partial x} \left[\frac{\beta v^2}{\alpha u^2 + \beta v^2 + \gamma (1 - u - v)^2} \right] = 0 \end{cases}$$

where α, β, γ are positive constants and $(u, v) \in \Omega$: 0 < u + v < 1, u, v > 0.

This is a non-strictly hyperbolic system: at $U^* = \frac{\gamma}{\beta\gamma + \gamma\alpha + \alpha\beta} t(\beta, \alpha)$, the characteristic speeds coincide and the linearization is a symmetric hyperbolic system. [2] is a good over-view and the approximation around U^* is fully studied in [4].

We study global shock structures of solutions. The Hugoniot locus of U_0 is a plane cubic curve and the 2-phase like flow curves ([3]) are straight lines.

Theorem 1. Suppose that the following (all) three inequalities hold

 $\alpha\beta < \beta\gamma + \gamma\alpha, \quad \beta\gamma < \gamma\alpha + \alpha\beta, \quad \gamma\alpha < \alpha\beta + \beta\gamma.$

Then on each 2-phase like flow curve, there exist undercompressive shock waves connecting two states on the curve.

Theorem 2. Suppose that one of the following three inequalities holds

 $\alpha\beta > \beta\gamma + \gamma\alpha, \quad \beta\gamma > \gamma\alpha + \alpha\beta, \quad \gamma\alpha > \alpha\beta + \beta\gamma.$

Then on one of the three 2-phase like flow curve, there exist overcompressive shock waves connecting two states on the curve.

If U_0 is on a 2-phase like flow curve \mathcal{L} , the Hugoniot curve is a hyperbola plus \mathcal{L} and there is a *secondary* bifurcation point. We get a precise condition so that two states can be connected by an under or overcompressive shock wave.

References

- F. Asakura and M. Yamazaki, Viscous shock profiles for 2 × 2 systems of hyperbolic conservation laws with an umbilic point, J. Hyperbolic Differential Equations, 6, (2009), pp. 483-524
- [2] D. Marchesin and B. Plohr, Theory of three-phase flow applied to water-alternating-gas enhanced oil recovery, Hyperbolic Problems; Theory, Numerics, Applications, Proceedings of the 8th International Conference in Magdeburg (Vol.II), Birkhäuser Verlag, (2001), pp. 693-702
- [3] H. B. Medeiros, Stable hyperbolic singularities for three-phase flow models in oil reservoir simulation, Acta Applicandae Mathematicae, 28, (1992) pp. 135-159
- [4] D. Schaeffer and M. Shearer, The classification of 2×2 systems of non-strictly hyperbolic conservation laws, with applications to oil recovery, *Comm. Pure Appl. Math.*, **40**, (1987) pp. 141-178.

S71 – Phase Field Models – Room I, 19.00–19.30

Exact solutions to ideal hydrodynamics of inelastic gases: global existence and singularities

Olga S. Rozanova Moscow State University rozanova@mech.math.msu.su

The motion of the dilute gas where the characteristic hydrodynamic length scale of the flow is sufficiently large and the viscous and heat conduction terms can be neglected is governed by the equations of ideal granular hydrodynamics [1], [2]. This system is given in $\mathbb{R} \times \mathbb{R}^n$, $n \geq 1$, and has the following form:

$$\partial_t \rho + \operatorname{div}_x(\rho u) = 0,$$

$$\partial_t(\rho u) + \operatorname{div}_x(\rho u \otimes u) = -\nabla_x p,$$

$$\partial_t T + (u, \nabla_x T) + (\gamma - 1)T \operatorname{div}_x u = -\Lambda \rho T^{3/2},$$

237

where ρ is the gas density, $u = (u_1, ..., u_n)$ is the velocity, T is the temperature, $p = \rho T$ is the pressure, the constants γ and $\Lambda > 0$ are chosen for physical reasons. The only difference between the system above and the standard ideal gas dynamic equations (where the elastic colliding of particles is supposed) is the presence of the inelastic energy loss term $-\Lambda\rho T^{3/2}$.

First we prove that the solutions with conserved mass, total energy and finite momentum of inertia generically lose their initial smoothness within a finite time in any space dimension n and $1 < \gamma \leq 1 + \frac{2}{n}$. In the one-dimensional case we introduce a solution depending only on the spatial coordinate outside of a ball containing the origin and prove that this solution under rather general assumptions on initial data cannot be global in time too [3].

Further, we construct a large family of exact solutions to the system in several dimensions for arbitrary γ . In dependence on initial conditions these solutions can keep smoothness for all t > 0 or develop singularities. In the latter case we show that the singularity in the component of density is integrable for a spatial dimension greater than one. A special attention we pay to 2D case, where the singularity can be formed either in a point or along a line. We show that an initial vorticity prevents the formation of singularity. We consider a special case of the Chaplygin gas ($\gamma = -1$), where a special solution satisfies a couple of equations and therefore in 1D case the system can be written in the Riemann invariant and can be treated in a standard way (the criterion of the singularity formation can be found and the Riemann problem can be solved). We also construct an exact axially symmetric solution with separable time and space variables having a strong singularity in the density component beginning from the initial moment of time, whereas other components of solution are initially continuous.

References

- [1] Brilliantov N V and Pöschel T, Kinetic theory of granular gases, Oxford: Oxford University Press, 2004.
- [2] Fouxon I, Meerson B, Assaf M and Livne E, Formation and evolution of density singularities in ideal hydrodynamics of freely cooling inelastic gases: A family of exact solutions *Physics of Fluids*, 19(2007), 093303.
- [3] Rozanova O, Formation of singularities in solutions to ideal hydrodynamics of freely cooling inelastic gases, *Nonlinearity*, 25 (2012), to appear.

12.9 Session 72 — Room F — Numerical Methods for Hamilton-Jacobi Equations

S72 - Numerical Methods for Hamilton-Jacobi Equations - Room F, 17.00-17.30

Monotone numerical approximations for optimal control of hybrid systems

Roberto Ferretti Dipartimento di Matematica, Università di Roma Tre ferretti@mat.uniroma3.it

Hybrid systems are a general framework which can model a large class of control systems arising whenever a collection of continuous- and discrete-time dynamics are put together in a single model [1, 2, 3]. We consider here hybrid systems in the form:

$$X(t) = f(X(t), q(t), u(t)),$$

with an initial condition X(0) = x, where $X \in \mathbb{R}^d$ is the continuous state and $q \in \mathbb{I}$ is the discrete one (this meaning that by changing q the system switches between different dynamics). The control set \mathcal{U} is the set of measurable function $u: (0, \infty) \to U$, where $U \in \mathbb{R}^m$ is compact.

The trajectory undergoes discrete jump when it hits two predefined sets A (the autonomous jump set) and C (the controlled jump set) of \mathbb{R}^d . More precisely, at each commutation the trajectory can jump to a predefined set D and a possibly different discrete state $q' \in \mathbb{I}$, and

- on hitting A, the jump is given by a prescribed transition map $g : \mathbb{R}^d \times A \times \mathcal{V} \to D \times \mathbb{I}$, where \mathcal{V} is the discrete control set. We denote by τ_i an arrival time to A, and by $(X(\tau_i^-), q(\tau_i^-))$ the point before a jump, while the state after the jump will be denoted by $(X(\tau_i^+), q(\tau_i^+)) = g(X(\tau_i^-), w)$;
- when the trajectory evolves in the set C, the controller can choose either to jump or not. By ξ_i we denote a switching time. The controlled jump destination of $(X(\xi_i^-), q(\xi_i^-))$ is $(X(\xi_i^+), q(\xi_i^+)) \in D \times \mathbb{I}$.

Classical assumptions will be made on the sets A, C, D and on the functions f and g. For every strategy $\theta := (u(\cdot), v(\cdot), (\xi_i), (\tau_k))$, we associate the cost defined by:

$$J(x,\theta) := \int_{0}^{+\infty} \ell(X(t), q(t), u(t)) e^{-\lambda t} dt + \sum_{k=0}^{\infty} C_a(X(\tau_k^-), v) e^{-\lambda \tau_k} + \sum_{i=0}^{\infty} C_c(X(\xi_i^-), \xi_i^-) e^{-\lambda \xi_i}$$
(1)

where λ is the discount factor, $\ell : \Omega \times \mathbb{I} \times U \to \mathbb{R}_+$ is the running cost, $C_a : A \times \mathbb{I} \times \mathcal{V} \to \mathbb{R}_+$ is the autonomous jump cost and $C_c : C \times \mathbb{I} \times D \to \mathbb{R}_+$ is the controlled jump cost. The value function V is then defined as:

$$V(x,q) := \inf_{\theta \in \mathcal{U} \times \mathcal{V} \times [0,\infty) \times D} J(x,\theta).$$
⁽²⁾

The functions ℓ , C_c and C_a are assumed to be nonnegative and Lipschitz continuous. Moreover, $C_a(x, v)$ and $C_c(x, x')$ are uniformly bounded from below by some C' > 0. By using viscosity arguments as in [1], it can be proved that V is the unique bounded continuous solution of the quasi-variational inequality:

$$\begin{cases} V(x,q) - \mathcal{M}V(x,q) = 0 & x \in A, \\ \max(V(x,q) - \mathcal{N}V(x,q), \lambda V(x,q) + H(x,q, D_x V(x,q)) = 0 & x \in C, \\ \lambda V(x,q) + H(x, D_x V(x,q)) = 0 & \text{else}, \end{cases}$$
(3)

where

$$H(x,q,p) := \sup_{u \in U} \{ -\ell(x,q,u) - f(x,q,u) \cdot p \},$$

$$\mathcal{M}\phi(x,q) := \inf_{v \in \mathcal{V}} \{ \phi(g(x,q,v)) + C_a(x,v) \quad (x \in A),$$

$$\mathcal{N}\phi(x,q) := \inf_{x' \in D, q' \in \mathbb{I}} \{ \phi(x',q') + C_c(x,x') \} \quad (x \in C).$$

We consider monotone schemes approximating (3) in the fixed point form:

$$V^{h}(x,q) = T^{h}(x,q,V^{h}) = \begin{cases} M^{h}V^{h}(x,q) & x \in A\\ \min\{N^{h}V^{h}(x,q), S^{h}(x,q,V^{h})\} & x \in C\\ S^{h}(x,q,V^{h}) & \text{else}, \end{cases}$$
(4)

where V^h is the numerical approximation of the value function V, indexed by the discretization parameter h. We shall first discuss the convergence issues related to this class of approximation schemes, and discuss the reconstruction of optimal trajectories for the underlying control problem. Moreover, we will provide some numerical examples to show the efficiency of the proposed method.

- G. Barles, S. Dharmatti and M. Ramaswamy, Unbounded viscosity solutions of hybrid control systems, ESAIM:COCV, 16 (2010), pp. 176–193
- [2] M.S. Branicky, V. Borkar and S. Mitter, A unified framework for hybrid control problem, *IEEE Transac*tions on automated control, 43 (1998), pp. 31–45
- [3] S. Dharmatti and M. Ramaswamy, Hybrid control system and viscosity solutions, SIAM J. on Contol and Optimization, 44 (2005), pp. 1259–1288

[4] R. Ferretti and H. Zidani, Numerical Hamilton–Jacobi approach for solving some optimal control problems governed by hybrid systems, preprint (2012)

Joint work with: Hasnaa Zidani (ENSTA Paristech), Jun-Yi Zhao (ENSTA Paristech)

S72 – Numerical Methods for Hamilton-Jacobi Equations – Room F, 17.30-18.00

* * *

Semi-Lagrangian discontinuous Galerkin schemes for first and second order partial differential equations

Olivier Bokanowski

Laboratoire Jacques-Louis Lions, Univ. Pierre et Marie Curie 75252 Paris Cedex 05 France UFR de Mathématiques, Site Chevaleret, Université Paris-Diderot, 75205 Paris Cedex France Inria Saclay Commands / Ensta Paris Tech, boka@math.jussieu.fr

In this work [1], we propose explicit, CFL-free, high-order schemes for the approximation of some first and second order time-dependant partial differential equations. The scheme is based on a weak formulation of a semi-lagrangian scheme using discontinous Galerkin elements, together with a systematic use of the splitting strategy. It follows the idea of the recent works of Qiu and Shu [2] and of Crouseilles, Mehrenberger and Vecil [3] for first order advection equation, based on exact integration and splitting techniques. We prove high order behavior, stability as well as convergence of our scheme in the case of linear second order PDEs with constant coefficients, or linear first order PDEs with nonconstant coefficients. The obtention of general high-order schemes for nonconstant coefficients is the subject of ongoing works that will be discussed, as well as some extentions to nonlinear obstacle problems [4,5]. The schemes are illustrated on several numerical examples, including the Black and Scholes PDE in finance.

References

- [1] O. Bokanowski, G. Simarmata, Preprint.
- [2] J-M. Qiu and C-W. Shu, "Positivity preserving semi-Lagrangian discontinuous Galerkin formulation: theoretical analysis and application to the Vlasov-Poisson system", J. Comput. Phys., 230 (2011), no. 23, pp. 8386–8409.
- [3] N. Crouseilles, M. Mehrenberger and F. Vecil, "Discontinuous- Galerkin semi-Lagrangian method for Vlasov-Poisson" *ESAIM: proceedings*, Volume no. 32 (2011), pp. 211-230.
- [4] O. Bokanowski, Y. Cheng, C.-W. Shu, "A discontinuous Galerkin scheme for front propagation with obstacles", *Preprint*.
- [5] O. Bokanowski, F. Bonnans, in preparation.

Joint work with: G. Simarmata (Rabobank, Netherlands).

This work was (partially) supported by the European Union under the 7th Framework Programme "FP7-PEOPLE-2010-ITN" Grant agreement number 264735-SADCO, and by the Inria Saclay & Ensta Paris-Tech project "Commands".

- * * * ------

S72 – Numerical Methods for Hamilton-Jacobi Equations – Room F, 18.00–18.30

Semi-Lagrangian schemes for linear and fully non-linear Hamilton-Jacobi-Bellman equations

Kristian Debrabant Department of Mathematics and Computer Science, University of Southern Denmark debrabant@imada.sdu.dk

In this talk we consider the numerical solution of diffusion equations of Hamilton-Jacobi-Bellman type

$$u_t - \inf_{\alpha \in \mathcal{A}} \left\{ L^{\alpha}[u](t,x) + c^{\alpha}(t,x)u + f^{\alpha}(t,x) \right\} = 0 \qquad \text{in} \quad (0,T] \times \mathbb{R}^N,$$
$$u(0,x) = g(x) \qquad \text{in} \quad \mathbb{R}^N,$$

where

$$L^{\alpha}[u](t,x) = \operatorname{tr}[a^{\alpha}(t,x)D^{2}u(t,x)] + b^{\alpha}(t,x)Du(t,x).$$

The solution of such problems can be interpreted as value function of a stochastic control problem. We introduce a class of monotone approximation schemes relying on monotone interpolation. Besides providing a unifying framework for several known first order accurate schemes [2,3,5], the presented class of schemes includes new methods that are second order accurate in space and converge for essentially monotone solutions. Some stability and convergence results are given and the method is applied to a super-replication problem from finance [1]. The results are mainly taken from paper [4].

References

- B. Bruder, O. Bokanowski, S. Maroso, and H. Zidani. Numerical approximation for a superreplication problem under gamma constraints. SIAM J. of Numer. Anal. 47 (2009), no. 3, 2289-2320.
- [2] F. Camilli and M. Falcone, An approximation scheme for the optimal control of diffusion processes, RAIRO Modél. Math. Anal. Numér. 29 (1995), no. 1, 97–122.
- [3] M. G. Crandall and P.-L. Lions. Convergent difference schemes for nonlinear parabolic equations and mean curvature motion. Numer. Math. 75 (1996), no. 1, 17–41.
- [4] K. Debrabant and E. R. Jakobsen, Semi-Lagrangian schemes for linear and fully non-linear diffusion equations, to appear in *Mathematics of Computation*.
- [5] M. Falcone, A numerical approach to the infinite horizon problem of deterministic control theory, Appl. Math. Optim. 15 (1987), no. 1, 1–13.

Joint work with: Espen R. Jakobsen (Department of Mathematical Sciences, Norwegian University of Science and Technology, Trondheim, Norway)

- * * *

S72 – Numerical Methods for Hamilton-Jacobi Equations – Room F, $18.30{-}19.00$

An approximation scheme for an Eikonal Equation with discontinuous coefficient

Adriano Festa Imperial College, London a.festa@imperial.ac.uk

We will present a numerical scheme for a class of non classical Hamilton-Jacobi equations where we can have discontinuous viscosity solutions, following the definition of Ishii [7]. In this case, through a particular condition on the discontinuities [4, 10] we can preserve a comparison principle for the solutions and so uniqueness. We will introduce and study a semi Lagrangian numerical scheme and also deriving some error bounds which will be, because of the discontinuity on the viscosity solutions, in L^1 norm. We will finally present some applicative situations where we have good performances of our methods, in particular Shape-from-Shading and a optimal navigation problem.

References

- M. Bardi, I. Capuzzo-Dolcetta, Optimal Control and Viscosity Solution of Hamilton-Jacobi-Bellman Equations, Birkhauser, Boston Heidelberg (1997).
- [2] E. N. Barron and R. Jensen, Semicontinuous viscosity solutions for Hamilton-Jacobi equations with convex Hamiltonians, Comm. Partial. Diff. Eq. 15 (1990), pp. 1713-1742.
- [3] O. Bokanowski, N. Forcadel and H. Zidani, L1-error estimates for numerical approximations of Hamilton-Jacobi-Bellman equations in dimension 1, Math. Comp. 79 (2010), pp. 13951426
- [4] K. Deckelnick and C. Elliott, Uniqueness and error analysis for Hamilton-Jacobi equations with discontinuities, *Interfaces and free boundaries.*, 6 (2004), pp. 329-349.
- [5] M. Falcone, R. Ferretti, Semi-Lagrangian Approximation Schemes for Linear and Hamilton-Jacobi Equations, SIAM, 2011.
- [6] H. Frankowska, Lower semicontinuous solutions of Hamilton-Jacobi-Bellman equations, SIAM J. Control Optim., 31 (1993), pp. 257-272
- [7] H. Ishii, Hamilton-Jacobi equations with discontinuous Hamiltonians an arbitrary open sets, Bull. Fac. Sci. Engrg. Chuo. Univ., 28 (1985), pp. 33-77.
- [8] D. Ostrov, Viscosity solutions and convergence of monotone schemes for synthetic aperture radar shapefrom-shading equations with discontinuous intensities, SIAM J. Appl. Math. 59 (1999), pp. 2060-2085.
- [9] P. Soravia, Boundary Value Problems for Hamilton-Jacobi Equations with Discontinuous Lagrangian, Indiana Univ. Math. J., 51 (2002), pp. 451-477.
- [10] P. Soravia, Degenerate eikonal equations with discontinuous refraction index, ESAIM Control Optim. Calc. Var. 12 (2006), pp. 216-23.

Joint work with: Maurizio Falcone ("Sapienza University of Rome")

241

S72 - Numerical Methods for Hamilton-Jacobi Equations - Room F, 19.00-19.30

Approximation of the Effective Hamiltonian Through a Degenerate Elliptic Problem

Martin Nolte University of Freiburg nolte@mathematik.uni-freiburg.de

Given a strongly convex Hamiltonian $H \in C^2(\mathbb{T}^d, \mathbb{R}^d)$, H = H(x, p), we seek a numerical approximation of its effective Hamiltonian \overline{H} given by

$$\bar{H}(P) = \inf_{u \in \mathcal{C}^{\infty}(\mathbb{T}^d)} \max_{x \in \mathbb{T}^d} H(x, P + \nabla u).$$

In [1] it was shown that $\overline{H}(P)$ can be approximated by

$$\bar{H}_k(P) := \inf_{u \in \mathcal{C}^{\infty}(\mathbb{T}^d)} \frac{1}{k} \ln \left[\int_{\mathbb{T}^d} e^{kH(x, P + \nabla u))} \, dx \right] \stackrel{k \to \infty}{\longrightarrow} \bar{H}(P),$$

leading us to the nonlinear elliptic Euler-Lagrange equation

$$\nabla \cdot \left(e^{kH(x,P+\nabla u_k)} H_p(x,P+\nabla u_k) \right) = 0 \quad \text{in } \mathbb{T}^d.$$

In [3], a finite difference discretization for this Euler-Lagrange equation is applied to simple Hamiltonians of the form $H(x,p) = \frac{1}{2}|p|^2 + E(x)$. We extend this scheme to general strongly convex C²-Hamiltonians. Unfortunatly, this scheme is unstable unless $k \leq C(\Delta x)$. Adding numerical viscosity, we stabilize the scheme and pass to the limit $k \to \infty$ in the discretization. This results in a numerical scheme for the equation

$$-H_p(x,\nabla u)H_p^T(x,\nabla u):\nabla^2 u = H_x^T(x,\nabla u)H_p(x,\nabla u) \quad \text{in } \mathbb{T}^d.$$

As shown in [1], $u_k \to u \in C(\mathbb{T}^d)$ and u is a viscosity solution of this equation.

Numerical experiments indicate that it is possible to recover $\overline{H}(P)$ from u through the formula

$$\bar{H}(P) = \sup_{x \in \mathbb{T}^d} H(x, \nabla u).$$

- L.C. Evans, Some New PDE Methods for Weak KAM Theory, Calc. Var. Partial Differ. Equ, 17 (2003), pp. 159-177
- [2] M. Nolte, Efficient Numerical Approximation of the Effective Hamiltonian, Doctoral Dissertation, University of Freiburg (2011)
- [3] M. Rorro, Numerical Methods for the Effective Hamiltonian on Serial and Parallel Machines, Doctoral Dissertation, University of Rome (2007)

13 Abstracts of posters

13.1 Monday, 14.00–16.35, Via Bassi Rooms, first floor

Global existence and energy decay of solutions for a nondissipative wave equation with a time varying delay term

Abbes Benaissa Laboratory of Mathematics, Djillali Liabes University, P. O. Box 89, Sidi Bel Abbes 22000, ALGERIA. benaissa_abbes@yahoo.com

We consider the energy decay for nondissipative wave equation in a bounded domain with a time varying delay term in the internal feedback. We use an approach introduced by Guesmia which leads to decay estimates (known in the dissipative case) when the integral inequalities method due to Haraux-Komornik [4] cannot be applied due to the lack of dissipativity. First we study the stability of a nonlinear wave equation of the form

 $u_{tt}(x,t) - \Delta_x u(x,t) + \mu_1 \sigma(t) u_t(x,t) + \mu_2 \sigma(t) u_t(x,t-\tau(t)) + \theta(t) h(\nabla_x u) = 0$

in a bounded domain. We consider the general case with a nonlinear function h satisfying a smallness condition, and obtain the decay of solutions under a relation between the weight of the delay term in the feedback and the weight of the term without delay. We impose no control on the sign of the derivative of the energy related to the above equation.

In the second case we consider the case $\theta \equiv const$ and $h(\nabla u) = -\nabla \Phi \nabla u$. We prove an exponential decay result of the energy without any smallness condition on the function h.

References

- A. Benaissa, S. Benazzouz, Energy decay of solutions to the Cauchy problem for a nondissipative wave equation, J. Math. Phys., 51, (2010)-12, 123504.
- [2] R. Datko, J. Lagnese & M.P. Polis, An example on the effect of time delays in boundary feedback stabilization of wave equations, SIAM J. Control Optim. 24 (1986), 152-156.
- [3] A. Guesmia, A new approach of stabilization of nondissipative distributed systems, SIAM J. Control Optimization 42, (2003)-1, 24-52.
- [4] V. Komornik, Exact Controllability and Stabilization. The Multiplier Method, Masson-John Wiley, Paris, 1994.
- [5] S. Nicaise, & C. Pignotti, Stability and instability results of the wave equation with a delay term in the boundary or internal feedbacks, SIAM J. Control Optim. 45 (2006)-5, 1561-1585.

Joint work with: Salim. A. Messaoudi (Department of Mathematics and Statistics, KFUPM, Dhahran 31261, Saudi Arabia).

On the Thermostatted Kinetic Models

Carlo Bianca Dipartimento di Scienze Matematiche, Politecnico di Torino carlo.bianca@polito.it Different approaches inspired to equilibrium or non equilibrium statistical mechanics have been developed in an attempt to describe collective behaviors and macroscopic features of complex phenomena in nature and society as the result of microscopic interactions. Kinetic theory for active particles models have been developed in space homogeneity for complex systems where macroscopic external effects are neglected [2]. Accordingly the random interactions among individuals will eventually move the system towards equilibrium. If, on the other hand, an external force field acts on the system, the applied field does work on the system thereby moving it away from equilibrium [1]. The excess energy needs to be removed so as to achieve a steady state. A method, which is common in nonequilibrium molecular dynamics simulations, is the use of deterministic thermostats, which consists by introducing a damping term into the equations of motion [5].

This talk is concerned with the mathematical modelling of complex systems subjected to external force fields whose magnitude exerts an action on the particles. A Gaussian isokinetic thermostat is introduced in order to keep constant the energy of the system. The resulting model is expressed by means of nonlinear hyperbolic partial integro-differential equations [3]. The global in time existence and uniqueness of the solution to the relative Cauchy problem is shown for which the density and the energy of the solution are preserved [4].

References

- C. Bianca, On the mathematical transport theory in microporous media: The billiard approach, Nonlinear Analysis: Hybrid Systems, 4 (2010), pp. 699-735
- [2] C. Bianca, Mathematical modelling for keloid formation triggered by virus: Malignant effects and immune system competition, Math. Models Methods Appl. Sci., 21 (2011), pp. 389-419
- [3] C. Bianca, Kinetic theory for active particles modelling coupled to Gaussian thermostats, Applied Mathematical Sciences, 6 (2012), pp. 651-660
- [4] C. Bianca, An existence and uniqueness theorem for the Cauchy problem for thermostatted-KTAP models, Int. Journal of Math. Analysis, 6 (2012), pp. 813-824
- [5] O.G. Jepps and L. Rondoni, Deterministic thermostats, theories of nonequilibrium systems and parallels with the ergodic condition, J. Phys. A: Math. Theor., 43 (2010), 133001

Continuum Models of a Limit Order Book

*

Giancarlo Facchi The Pennsylvania State University facchi@math.psu.edu

An external buyer asks for a random amount X > 0 of a certain asset. X is distributed according to a probability distribution μ on \mathbb{R}_+ . We assume that this external buyer will buy the amount X at the lowest available price, as long as this price does not exceed a given upper bound.

Different agents are willing to sell various quantities of the asset at different prices, competing against each other in order to fulfill the random incoming market order. We denote by $u_i(p)$ the density of sell limit orders at price p, placed by agent i. In other words, for any p > 0,

$$U_i(p) = \int_0^p u_i(s) \, ds = \begin{bmatrix} \text{total amount of asset offered} \\ \text{for sale at price} \leq p \text{ by agent } i \end{bmatrix}.$$
(1)

We consider two types of agents. "Small agents" own a very small amount of assets, compared to the total amount of asset on sale, therefore a single agent cannot influence the probability of execution of the sell orders of the other agents. On the other hand, a "large agent" can significantly influence the probability of execution of the limit orders placed by other competing agents.

Our main goal is to study the existence, uniqueness and stability of a Nash equilibrium, where each agent follows his best strategy, in reply to the actions of all other agents. We seek to describe the optimal strategies for the various agents and estimate their expected payoffs.

We consider several market models where different types of agents interact. The Nash equilibrium is determined as the asymptotic limit for $t \to \infty$ of the solution to a system of conservation laws with nonlocal flux:

 $u_{i,t} + F_i (p, u_1, \dots, u_m, U)_p = 0, \quad i = 1, \dots, m,$

where $U(p) = \sum_{i=1}^{m} U_i(p)$, and u_i and U_i are as in (1).

Joint work with: Alberto Bressan (*The Pennsylvania State University*), Giuseppe Maria Coclite (*Università degli Studi di Bari*)

Bryan's effect and nonlinear damping

Temple H Fay

Department of Mathematics and Statistics, Tshwane University of Technology, South Africa thfay@hotmail.com

Modern vibratory gyroscopes [1] are designed using the fact that when a near-perfectly manufactured vibrating structure is subjected to a rotation in three-dimensional space (referred to as inertial rotation), the vibrating pattern rotates (within the structure) at a rate proportional to the inertial angular rate. This effect, known as "Bryan's effect", was first observed by G.H. Bryan in 1890 [2]. For the constant of proportionality, Bryan made the following calculation for a body consisting of a ring or cylinder

 $BF = \frac{\text{Rate of rotation of the vibrating pattern}}{\text{Inertial rate of rotation of the vibrating structure}}$

for various modes of vibration. This constant of proportionality BF has come to be known as "Bryan's factor". In 2011 a slowly rotating, fluid-filled sphere undergoing light anisotropic viscously damped vibrations was considered by Joubert, Shatalov and Coetzee [3]. Some structures have particles that vibrate at a high frequency and hence particle velocity is expected to be high at various points in time and consequently, a viscous damping model for Bryan's effect might not be ideal. Indeed, Sir Isaac Newton stated in Principia: Vol. 1: The Motion of Bodies (1687), that viscous damping "is more a mathematical hypothesis than a physical one." In this paper we tentatively introduce nonlinear damping into the equations of motion of a vibrating, slowly rotating ring or shell and show that the rate of rotation of the vibrating pattern is affected strongly by such damping.

- [1] E.J. Loper Jr. and D.D. Lynch, Hemispherical resonator gyroscope, U.S. Patent No. 4,951,508 (1990).
- [2] G.H. Bryan, On the beats in the vibrations of a revolving cylinder or bel, Proceedings of the Cambridge Philosophical Society 7 (1890), pp. 101-111.
- [3] S.V. Joubert, M.Y. Shatalov and C.E. Coetzee, Analysing manufacturing imperfections in a spherical vibratory gyroscope, in *Proceedings of the 4th IEEE International Workshop on Advances in Sensors and Interfaces*, IEEE Catalog Number: CFP11IWI-USB (2011), pp. 165-170.

Joint work with: Stephan V. Joubert (Department of Mathematics and Statistics, Tshwane University of Technology, South Africa)

Optimal control of level set dynamics via a finite-dimensional approximation scheme

Mauro Gaggero

ISSIA-National Research Council of Italy, Via De Marini 6, 16149 Genova, Italy mauro.gaggero@ge.issia.cnr.it

Optimal control of systems that describe the dynamics of level sets is investigated by using a methodology for approximate feedback control design. The design of the proposed regulators is achieved by resorting to an approximation scheme based on the extended Ritz method [1]. Such a scheme, which we first investigated in [2] and [3], consists in constraining the manipulable terms of the model equation to take on a fixed structure, where a finite number of free parameters can be suitably chosen. The original infinite-dimensional optimization problem is then reduced to a mathematical nonlinear programming one, in which the parameters have to be optimized. The proposed methodology is general since it allows one to deal with problems with different types of control (distributed or boundary control, control in the coefficients) within the same approximation framework.

In [2] and [3], we successfully applied the above-described method to a distributed optimal control problem for the Burgers' equation. As another application of the proposed approach, we present here an example based on the normal flow equation. The goal is to control the velocity term of the equation in order to track a desired closed curve on \mathbf{R}^2 associated with the zero level set of the solution of the normal flow equation. The exact solution of such a problem is very difficult to find, and thus the need of searching for approximate solutions arises. The velocity term is then replaced by a parametrized control law that depends on the values of the zero level set of the unknown function, thus turning out to be a feedback control law. The parametrization is based on radial basis functions with variable centers and widths. By substituting such a structure into the model equation and cost functional to minimize (given by the integral squared difference between the reference and actual zero level sets), we obtain a nonlinear programming problem, which is solved by using a sequential quadratic programming algorithm. At each iteration of such an algorithm we have to numerically solve the model equation. Simulation results are presented to show the effectiveness of the proposed approach as to both accuracy of suboptimal solutions and required computational effort.

References

- R. Zoppoli, M. Sanguineti, and T. Parisini, Approximating networks and extended Ritz method for the solution of functional optimization problems, *Journal of optimization theory and applications*, vol. 112 (2002), pp. 403-440.
- [2] A. Alessandri, R. Cianci, M. Gaggero, and R. Zoppoli, Approximate solution of feedback optimal control problems for distributed parameter systems," in *Proc. 8th IFAC Symposium on Nonlinear Control Systems* (2010), pp. 987–992.
- [3] A. Alessandri, M. Gaggero, and R. Zoppoli, Feedback optimal control of distributed parameter systems by using finite-dimensional approximation schemes, accepted for publication on *IEEE Transactions on Neural Networks and Learning Systems* (2012).

Joint work with: Angelo Alessandri (DIME-University of Genova, P.le Kennedy Pad. D, 16129 Genova, Italy), Patrizia Bagnerini (DIME-University of Genova, P.le Kennedy Pad. D, 16129 Genova, Italy), Marco Ghio (DIME-University of Genova, P.le Kennedy Pad. D, 16129 Genova, Italy).

* * * -

Compressible modeling of a cloudy atmosphere using a general pressure evolution equation

Michael Jaehn Leibniz-Institute for Tropospheric Research, Permoserstr. 15, 04318 Leipzig, Germany jaehn@tropos.de

An accurate formulation of a pressure evolution equation that is valid for cloud processes and rainfall is presented by using an approach of Fedkiw and Osher. This equation is coupled with a conservative prognostic total energy equation. Although the description of dissipative heating and moist processes will yield higher complexity, it is a desired criterion to ensure global energy conservation, because simulation results can differ significantly if traditional prognostic equations are used (e.g. dry potential temperature). For numerical reasons, it could be more effective to use an additional pressure tendency equation than a diagnostic relation, where pressure has to be a function of the thermodynamical variable and the additional moisture variables. Spatial discretization is realized by standard finite-volume methods. For the time integration we outline an implicit procedure by Rosenbrock time integrators and a special adapted split-explicit method. Results of different model setups will be illustrated by simulating idealized test cases.

Joint work with: Oswald Knoth (Leibniz-Institute for Tropospheric Research, Permoserstr. 15, 04318 Leipzig, Germany)

An Operator-difference Scheme for Hyperbolic PDEs with Significant First-order Derivative Term

- * * * ----

Mehmet Emir Koksal Mevlana University, Konya, Turkey mekoksal@mevlana.edu.tr

An abstract Cauchy problem for hyperbolic equations containing the operator A(t) with significant first-order derivative term is considered. This operator is unbounded self-adjoint positive linear operator with domain in an arbitrary Hilbert space. A second-order absolutely stable difference scheme is developed for solving the problem. The stability estimates for the solution of this difference scheme is presented. To support the theoretical statements for the solution of this difference scheme, various numerical examples are tested and the results are compared with other published numerical solutions obtained via a variety of methods. The modified difference scheme is applied for finding the transient response of a single phase lossy transmission line.

The Closest Point Method for Surfaces PDEs and Applications to Thin Film Flow

- * * * -

Thomas A. März University of Oxford maerz@maths.ox.ac.uk Partial differential equations (PDEs) are essential for modeling and understanding processes in all areas of science. The specialty of the PDEs considered here is that the differential operators involved are intrinsic to a curved surface. Such differential operators are used to model for example the flow of thin liquid films on a curved substrate (e.g., in industrial coating) [2].

The Closest Point Method [1,3] is a set of mathematical principles and associated numerical techniques for solving partial differential equations (PDEs) posed on surfaces. In this talk we present a calculus on surfaces based on closest point functions [4]. This calculus then forms the theoretical basis of the Closest Point Method and we show how to use it to set up a numerical method. Finally, we demonstrate the performance of the method on the hyperbolic thin film model [2]

$$\partial_t h + \operatorname{div}_S\left(\frac{h^3}{3}\nabla_S\kappa\right) = 0$$

which applies in situations where the mean curvature κ of the substrate is not negligible.

References

- Ruuth S., Merriman B., A Simple Embedding Method for Solving Partial Differential Equations on Surfaces, *Journal of Computational Physics*, 227 (2008), pp. 1943-1961.
- Howell P., Surface-tension-driven flow on a moving curved surface, *Journal of Engineering Mathematics*, 45 (2003), pp. 283-308.
- [3] Macdonald C., Ruuth S., Level set equations on surfaces via the Closest Point Method, Journal of Scientific Computing, 35 (2008), pp. 219-240.
- [4] März T., Macdonald C., Calculus on surfaces with general closest point functions, preprint (2012), arXiv:1202.3001

Joint work with: Colin B. Macdonald (University of Oxford)

Wave propagation in discrete heterogeneous media

Aurora Marica Basque Center for Applied Mathematics, Bilbao, Basque Country, Spain marica@bcamath.org

In this talk, we describe the propagation properties of the one and two-dimensional wave and transport equations with variable coefficients semi-discretized in space by finite difference and P_1 -finite element schemes on nonuniform meshes obtained as diffeomorphic transformations of uniform ones. In particular, we introduce and give a rigorous meaning to notions like the principal symbol of the discrete wave operator or the corresponding bi-characteristic rays. The main mathematical tool we employ is the discrete Wigner transform, which, in the limit as the mesh size parameter tends to zero, yields a measure propagating along curves which are solutions of a Hamiltonian system. Of course, due to dispersion phenomena, the high frequency dynamics does not coincide with the continuous one. Our analysis holds for $C^{1,1}(\mathbb{R}^d)$ -coefficients and $C^{2,1}(\mathbb{R}^d)$ -diffeomorphic transformations defining the grid. We also present several numerical simulations that confirm the predicted paths of the space-time projections of the bi-characteristic rays. Based on the theoretical analysis and simulations, we describe some of the pathological phenomena that these rays might exhibit as, for example, their reflection before touching the boundary of the space domain. This leads, in particular, to the failure of the classical properties of boundary observability of continuous waves, arising in control and inverse problems theory. Joint work with: Enrique Zuazua (Basque Center for Applied Mathematics and Basque Foundation for Science, Bilbao, Basque Country, Spain).

Indirect internal stabilization of the thermoelastic Bresse system

Nadine Najdi Beirut Arab University najdi_nadine@hotmail.com

In this work, we study the energy decay rate for a thermoelastic Bresse system. The system consists of three wave equations and two heat equations coupled in certain pattern. The two wave equations about the longitudinal displacement and shear angle displacement are effectively damped by the dissipation from the two heat equations, however. The system is governed by the following differential partial equations:

$$\rho h \varphi_{tt} - G h (\varphi_x + \psi + k\omega)_x - kE h (\omega_x - k\varphi) = 0, \qquad (1)$$

$$\rho I\psi_{tt} - EI\psi_{xx} + Gh(\varphi_x + \psi + k\omega) + \alpha\xi_x = 0, \qquad (2)$$

$$\rho h\omega_{tt} - Eh(\omega_x - k\varphi)_x + kGh(\varphi_x + \psi + k\omega) + \alpha\theta_x = 0, \qquad (3)$$

$$\rho c\theta_t - \theta_{xx} + \alpha T_0 \omega_{tx} = 0, \tag{4}$$

$$\rho c \xi_t - \xi_{xx} + \alpha T_0 \psi_{tx} = 0, \qquad (5)$$

where φ , ψ , ω are the vertical, rotation angle and longitudinal displacements; θ and ξ are the temperature deviations from the reference temperature T_0 along the longitudinal and vertical directions; E, G, ρ , I, h, k, c, are positive constants for the elastic and thermal material properties. The notation u_t (respectively u_x) indicate the partial derivatives with respect to time $t \geq 0$ (respectively with respect to spatial location $x \in [0, L]$). In this thesis, we study the energy decay rate for the thermoelastic Bresse system (1)-(5) with the boundary conditions

$$\omega_x(t,x) = \varphi(t,x) = \psi_x(t,x) = \theta(t,x) = \xi(t,x) = 0, \text{ for } x = 0, L,$$
(6)

or

$$\omega(t,x) = \varphi(t,x) = \psi(t,x) = \theta(t,x) = \xi(t,x) = 0, \text{ for } x = 0, L,$$
(7)

and initial conditions

$$\begin{aligned}
\omega(0,x) &= \omega_0(x), \ \omega_t(0,x) = \omega_1(x), \ \psi(0,x) = \psi_0(x), \ \psi_t(0,x) = \psi_1(x), \\
\varphi(0,x) &= \varphi_0(x), \ \varphi_t(0,x) = \varphi_1(x), \ \theta(0,x) = \theta_0(x), \ \xi(0,x) = \xi_0(x).
\end{aligned}$$
(8)

There are number of publications concerning the stabilization of the Bresse system [3], [4], [2] and [1]. In particular, in [3], Liu and Rao studied the stabilization of the Bresse system with two different temperature dissipation law effective on the equations about the longitudinal displacement and shear angle displacement. Under the equal speed wave propagation condition, they established an exponential energy decay rate. Otherwise, they showed that the smooth solution decays polynomially to zero with rates $\frac{1}{t^{1/2}}$ or $\frac{1}{t^{1/4}}$ provided the boundary conditions is Dirichlet-Neumann-Neumann or Dirichlet-Dirichlet-Dirichlet type respectively. In [2], Fatori and Rivera consider the Bresse system with one globally temperature dissipation law effective on the equation about the shear angle displacement. They established the same exponential energy decay rate in the case of equal speed wave propagation condition. Otherwise, they showed that the smooth solution decays polynomially to zero with rates $\frac{1}{t^{1/2}}$.

In this work, we consider the thermoelastic Bresse system damped by two locally internal distributed temperature dissipation laws with Dirichlet-Neumann-Neumann or Dirichlet-Dirichlet-Dirichlet boundary conditions type. Under the equal speed wave propagation condition, E = G, we establish the same exponential energy decay rate for usual initial data. On the contrary, when $E \neq G$, we first prove the non-exponential decay rate for the Bresse system with Dirichlet-Neumann-Neumann condition type. Therefore, we establish a new polynomial energy decay rate of type $\frac{1}{t}$ for the smooth solution.

References

- Alabau-Boussouira, Fatiha and Muoz Rivera, Jaime E. and Almeida Jnior, Dilberto da S, Stability to weak dissipative Bresse system, J. Math. Anal. Appl., Vol. 347, No. 2, (2011), pp. 481-498.
- [2] Fatori, Luci Harue and Rivera, Jaime E. Muoz, Rates of decay to weak thermoelastic Bresse system, IMA J. Appl. Math., Vol. 75, No. 6, (2010), pp. 881–904.
- [3] Z. Liu and B. Rao, Energy Decay of the Thermoelastic Bresse System, Z. angew. Math.Phys., Vol. 60, No. 1 (2009), pp. 54-69.
- [4] A. Wehbe and W. Youssef, Exponential and polynomial stability of an elastic Bresse system with two locally distributed feedback, *Journal of Mathematical Physics*, Vol. 51, No. 10, (2010), pp. 1067-1078.

Boundary Controllability of a Hyperbolic PDE with ODE Boundary Conditions Modeling a One-Dimensional Flow

Gilbert Raras Peralta Institut für Mathematik und Wissenschaftliches Rechnen Karl-Franzens-Universität Graz, Heinrichstrasse 36, A-8010 Graz, Austria gilbert.peralta@edu.uni-graz.at

We study a model that describes the flow of an incompressible fluid in an elastic tube that is connected to two tanks. The model is based on Euler's continuity equation and the law of balance of momentum; it is a system of quasilinear PDEs with nonlinear ODE boundary conditions. The system is linearized about the equilibrium state and semigroup theoretic proofs of well-posedness and stability of this linear system are sketched. Based on this model, a boundary control system is considered. Using some perturbation results for discrete spectral operators and operators in finite-dimensional spaces, it is shown that the normalized eigenvectors of the operator associated to the system form a Riesz basis of the state space, provided that the viscosity is sufficiently small. This induces a proof that the system is exactly controllable. Using a generalized Kadec's one-quarter theorem, a minimal time of controllability is given for single input controls. The control can be obtained by minimizing a cost functional with PDE constraints.

Joint work with: Georg Propst (Institut für Mathematik und Wissenschaftliches Rechnen, Karl-Franzens-Universität Graz, Heinrichstrasse 36, A-8010 Graz, Austria)

* *
Riemann solvers for compressible isothermal Euler equations for two phase flows with phase transition

Ferdinand Thein Otto-von-Guericke University Magdeburg ferdinand.thein@st.ovgu.de

We consider the isothermal Euler equations with phase transition between a liquid and a vapor phase. The mass transfer is modeled by a kinetic relation. Existence and uniqueness results were proven in [1]. We construct a Riemann solver to obtain the numerical solution for associated Riemann problems. This solver generalizes the HLLC solver such that it can take into account mass transfer between the phases. The calculated results will be compared to the exact solutions. Therefor we will highlight the major difficulties and propose possible strategies to overcome these problems. A talk held by Maren Hantke will give further insight to this topic, especially considering the exact solution.

References

- M. Hantke, W. Dreyer, and G. Warnecke, Exact solutions to the Riemann problem for compressible isothermal Euler equations for two phase flows with and without phase transition, WIAS Preprint no. 1620 (2011), to appear on *Quarterly of Applied Mathematics*
- [2] E. F. Toro, Riemann Solvers and Numerical Methods for Fluid Dynamics, Springer-Verlag, (1999)
- [3] C. Küchler, *Riemannlöser für isotherme Eulergleichungen in zwei Phasen*, Diploma Thesis, Otto-von-Guericke Universität, (2011)

Joint work with: Maren Hantke, Gerald Warnecke (Otto-von-Guericke University Magdeburg)

Analytical Solution of Second-Order Hyperbolic Telegraph Equation by Homotopy Analysis Method

Amit Tomar Indian Institute of Technology Roorkee,India amitmath.140gmail.com

In this Letter, the homotopy analysis method is applied to obtain the solutions of the initial value problem of hyperbolic type which is called telegraph equation. This analytic technique is valid for dealing with the nonlinearity and provides a convenient way of controlling the convergence region and rate of the series solution. The results obtained by the present method are compared with exact solutions. The results reveal that the implemented technique is very effective and convenient for solving nonlinear partial differential equations. Some illustrative examples are presented to show the efficiency of the method.

- M. Dehghan, A. Shokri, A numerical method for solving the hyperbolic telegraph equation, Numer. Methods Partial differ. Equ. Math., 24(4) (2007), pp. 1080-1093
- [2] SJ. Liao, Beyond perturbation: introduction to the homotopy analysis method. Boca Raton, Chapman & Hall/CRC Press, (2003)

[3] F. Gao, C.Chi, Unconditionally stable difference schemes for a one-space- dimensional linear hyperbolic equation, Appl. Math. Comput, 187 (2007), pp. 1272-1276

Joint work with: Rajan Arora (Indian Institute of Technology Roorkee, India), V.P.Singh (Indian Institute of Technology Roorkee, India)

13.2 Tuesday, 14.00–16.35, Via Bassi Rooms, first floor

Shallow water equations for horizontal-shear flows: characteristics, analytical and numerical solutions

Alexander Chesnokov Lavrentyev Institute of Hydrodynamics chesnokov@hydro.nsc.ru

The talk focuses on the theoretical analysis of the equations

$$\begin{cases} u_t + uu_x + vu_y + gh_x = 0, \quad h_y = 0, \\ h_t + (uh)_x + (vh)_y = 0, \quad uY'_i(x) - v\big|_{y=Y_i} = 0, \quad (i = 1, 2), \end{cases}$$

describing open channel flows of ideal incompressible fluid with horizontal velocity shear in the long wave approximation [1]. Here (u, v) is the fluid velocity; h is the free-surface height over the flat bottom z = 0; $y = Y_1(x)$ and $y = Y_2(x)$ are the lateral channel walls; and g is the constant gravity acceleration. In the semi-Lagrangian frame of reference this model transforms to the integrodifferential equations. Theoretical analysis of the model is based on the proposed by V. M. Teshukov concept of hyperbolicity for systems of equations with operator coefficients. A distinctive feature of integrodifferential models is the presence of both discrete and continuous spectrum of characteristic velocities.

Necessary and sufficient conditions of generalized hyperbolicity for the equations of motion are formulated. An example of verification of the hyperbolicity conditions is given, and an analogy with the well-known stability criterion for shear flows is noted. Exact (in particular, periodical) solutions of the model are constructed and interpreted physically for the class of traveling waves. It is shown that traveling waves are stable in the linear approximation only in the case of an insignificant change in the fluid depth. Differential balance laws approximating the basic integrodifferential equation are proposed. These equations are used to perform numerical calculations of the waves propagation.

The concepts of sub- and supercritical flows are introduced for the model describing the steady-state horizontal-shear shallow flows of an ideal incompressible fluid with a free boundary in a channel of variable cross-section [2]. Internal structure of flow developed in a local channel contraction or expansion is analyzed. Continuous and discontinuous exact solutions describing different flow regimes are constructed and their properties are studied. Analytical solutions for flows with the formation of recirculation zones are obtained.

References

- Chesnokov A.A., Liapidevskii V.Yu., Wave motion of an ideal fluid in a narrow open channel, J. App. Mech. Tech. Phys., 50 (2009), pp. 220–228.
- [2] Liapidevskii V.Yu., Chesnokov A.A., Sub- and supercritical horizontal-shear flows in an open channel of variable cross-section, *Fluid Dynamics*, 44 (2009), pp. 903–916.

Joint work with: Valery Liapidevskii (Lavrentyev Institute of Hydrodynamics).

* *

Hyperbolic problems in the theory of longitudinal vibrations of non-thin rods

Igor Fedotov Tshwane University of Technology, Department of Mathematics and Statistics fedotovi@tut.ac.za

The longitudinal vibration of rods is normally considered in mathematical physics in terms of the classical model described by the wave equation under assumptions that the rod is thin and relatively long. More general theories were formulated by taking into consideration the effect of the lateral motion effects of shear stress of relatively thick rod and was considered by Rayleigh in 1894 and Bishop in 1952. The Rayleigh-Bishop model is described by a fourth order partial differential equation not containing the fourth time derivative. This model was generalized by Mindlin and Hermann. They considered the lateral displacement proportional to an independent function of time and the longitudinal coordinate. This result can be formulated as an equation of forth order resolved with respect to the highest order time derivative. To obtain more general class of equations, the displacements of rods are expressed in the form of a power series expansion in the lateral coordinate. We wish to classify all of the above mentioned equations within the frame of the general theory of hyperbolic equations (are they strictly hyperbolic, hyperbolic or pseudohyperbolic). The Study of General hyperbolic equations was launched in 1937 by I.G. Petrovsky in his paper on Cauchy problems where he gave a general definition of the hyperbolicity. The initial Petrovsky's results are complete. Further development of the theory was concerned not with obtaining new profound results but rather with the improvement of methods of proofs and the application of modern tools such as Distribution Theory. The Monograph of Leray (1952) can be considered as the next step in this direction. Further substantial progress was made by Garding (1957). In 1938 Petrovsky extended his theory to general systems of partial differential equation not resolved with respect to the highest time derivative. The interest in such problems returned after S.L. Sobolev's paper appeared in 1954. Following Sobolev's investigations S.A. Galpern (1960 and 1963) considered differential operators not resolved with respect to highest time-derivative. A detailed survey of such problems can be found in the monograph of Demidenko and Uspensky. We use the approach on the theory of Hyperbolic equations developed by L.R. Volevich. and S.G. Gindikin (1967,1996 and 1999). They obtained deep results concerning mixed problems for general hyperbolic equations. This talk is about a comment on recent findings of I. Fedotov and L.R. Volevich (2006) which should provide a thorough understanding of the hyperbolic and pseudohyperbolic operator arising in the theory of longitudinal vibrations of elastic bars. Invertibility of some hyperbolic problems is discussed.

Joint work with: Herve Michel Tenkam (Tshwane University of Technology, Department of Mathematics and Statistics), Micheal Shatalov (Tshwane University of Technology, Department of Mathematics and Statistics, CSIR, MSM).

Exact Riemann solutions to compressible Euler equations in ducts with discontinuous cross–section

Ee Han Otto-von-Guericke-Universität Magdeburg han.ee@st.ovgu.de

We determine completely the exact Riemann solutions for the system of Euler equations in a duct with discontinuous varying cross–section. The crucial point in solving the Riemann problem for hyperbolic system is the construction of the wave curves. To address the difficulty in the construction due to the nonstrict hyperbolicity of the underlying system, we introduce the L–M and R–M curves in the velocity–pressure phase plane. The behaviors of the L–M and R–M curves for six basic cases are fully analyzed. We observe that in certain cases the L–M and R–M curves contain the bifurcation which leads to the non–uniqueness of the Riemann solutions. The physically relevant solution is singled out among all the possible exact solutions by comparing them with the numerical results of the axisymmetric Euler equation model.

References

- T. P. Liu, Nonlinear resonance for quasilinear hyperbolic equation, J. Math. Phys., 28, (1987), pp. 2593-2602.
- [2] D. Marchesin and P. J. Paes-Leme, A Riemann problem in gas dynamics with bifurcation, Comp. Maths. Appls., 12, (1986), pp. 433-455.
- [3] E. Han, M. Hantke, and G. Warnecke, Exact Riemann solutions in ducts with discontinuous cross-section. Preprint submitted.
- [4] E. Han, M. Hantke, and G. Warnecke, Criteria for non uniqueness of Riemann solutions to compressible duct flows. Preprint submitted.

Joint work with: Maren, Hantke (Otto-von-Guericke-Universität Magdeburg), Gerald Warnecke (Otto-von-Guericke-Universität Magdeburg)

Viscous Profiles for Shock Waves in Isentropic Magnetohydrodynamics

> Andreas Klaiber University of Konstanz Andreas.Klaiber@uni-konstanz.de

Standing planar waves in isentropic magnetohydrodynamics (IMHD) are governed by the autonomous system

$$\mu v' = mv + p(v) + \frac{1}{2} |\mathbf{b}|^2 - j,$$

$$\nu \mathbf{w}' = m\mathbf{w} - a\mathbf{b},$$

$$\eta \mathbf{b}' = v\mathbf{b} - a\mathbf{w} - \begin{pmatrix} c \\ 0 \end{pmatrix},$$

(Σ)

of ordinary differential equations in \mathbb{R}^5 , where the usual physical notations have been adopted; the longitudinal components of momentum $m := \rho v$ and magnetic field a are known to be constant. The constants $\mu - \nu > 0$, $\nu > 0$, $\eta > 0$ represent the fluid's longitudinal and transversal viscosity, and electrical resistivity, respectively. Furthermore, $p(v) = \hat{p}(m/v)$ is derived from a general barotropic pressure law $\hat{p} = \hat{p}(\rho)$. Only two constants of integration have to be considered, namely $j \in (-\infty, +\infty)$ and $c \in [0, +\infty)$.

A heteroclinic orbit connecting two rest points \mathbf{u}^{\pm} of Σ corresponds to a viscous profile for the standing shock wave with the states \mathbf{u}^{-} and \mathbf{u}^{+} .

We present results from [3,4] which show that (i) system Σ is gradient-like, (ii) there are up to four isolated rest points, (iii) heteroclinic orbits between the rest points do or do not exist depending on the respective ratios of (μ, ν, η) . For the proof of (iii) we use the Conley index as in [1] and geometric singular perturbation theory as in [2].

- Conley, C. C. and Smoller, J. A., On the structure of magnetohydrodynamic shock waves, *Communications on Pure and Applied Mathematics*, 27 (1974), pp. 367-375
- [2] Freistühler, H. and Szmolyan, P., Existence and bifurcation of viscous profiles for all intermediate magnetohydrodynamic shock waves, SIAM Journal on Mathematical Analysis, 26 (1995), pp. 112-128
- [3] Klaiber, A., On the existence of viscous profiles in isothermal magnetohydrodynamics (in German), Diploma Thesis, 2009, University of Leipzig
- [4] Klaiber, A., Viscous Profiles for Shock Waves in Isothermal Magnetohydrodynamics, preprint (2011), submitted

Zero dissipation limit to rarefaction wave with vacuum for 1-D compressible Navier-Stokes equations

Mingjie Li College of Science, Minzu University of China, Beijing, China. lmjmath@gmail.com

In this talk, we show the zero dissipation limit to rarefaction wave with vacuum for the compressible Navier-Stokes equations. It is well-known that one-dimensional compressible heat-conductive gas dynamics has three elementary waves, i.e., shock wave, contact discontinuity wave and rarefaction wave. Among the three waves, only the rarefaction wave can be connected to vacuum. Given a rarefaction wave with one-side vacuum state to the compressible Euler equations, we can construct a sequence of solutions to one-dimensional compressible Navier-Stokes equations which converge to the above rarefaction wave with vacuum as the viscosity tends to zero. Moreover, the uniform convergence rate is obtained. The proof consists of a scaling argument and elementary energy analysis based on the underlying rarefaction wave structures.

References

- [1] Feimin Huang, Mingjie Li and Yi Wang, Zero dissipation limit to rarefaction wave with vacuum for 1-D compressible Navier-Stokes equations, *Accepted in SIAM J. Math. Anal.*, (2012).
- [3] Mingjie Li and Yi Wang, Zero dissipation limit to rarefaction wave with vacuum for 1-D compressible viscous heat-conductive flows, preprint (2012).

Joint work with: Feimin Huang (Institute of Applied Mathematics, AMSS, Academia Sinica, Beijing, China.), Yi Wang(Institute of Applied Mathematics, AMSS, Academia Sinica, Beijing, China.)

Boundary layer solution to systems of viscous conservation laws in half line

Tohru Nakamura Faculty of Mathematics, Kyushu University, Japan tohru@math.kyushu-u.ac.jp We consider the large-time behavior of solutions to the symmetric hyperbolic-parabolic system in the half line. We show the existence and asymptotic stability of the stationary solution (boundary layer solution) under the smallness assumption on the initial perturbation and the strength of the stationary solution. The key to proof is to derive the uniform a priori estimates by using Matsumura–Nishida's energy method under the stability condition of Shizuta–Kawashima type.

Joint work with: Shinya Nishibata (Tokyo Institute of Technology).

Large Time Behavior of Solutions for the Navier-Stokes equations for compressible fluid in three dimension

* * * -

Se Eun Noh Department of Mathematics, Myoungji University senoh@mju.ac.kr

In this talk, we study the pointwise estimate of Green's function and coupling of nonlinear waves to the isentropic Navier-Stokes equations for compressible fluid in three dimension. Singular waves in the Green's function dominates short time behaviors. The explicit form of leading low frequency waves representing large time behavior of linearized equations is obtained to analyze nonlinear interactions of dissipation waves and pointwise estimates of the time-asymptotic behavior of the solutions which shows dissipation and generalized Huygens' principle.

References

- Hoff, D and Zumbrun, K., Multi-dimensional diffusion wave for the Navier-Stokes equations of compressible flow. *Indiana. Univ. J.* Volume no. 44 (1995), No. 2, pp. 603-676.
- [2] Hoff, D and Zumbrun, K., Pointwise decay estimates for multidimensional Navier-Stokes diffusion waves. Z. Angew. Math. Phys. Volume no. 48 (1997), pp. 1-18.
- [3] Kawashima, S, System of a hyperbolic-parabolic composite type, with applications to the equations of magnetohydrodynamics. Thesis, Kyoto Univ. (1983).
- [4] Li, D., Green's function of the Navier-Stokes equations for gas dynamics in R³. Commun. Math. Phys. Volume no. 257 (2005), No. 3, pp. 579-619.
- [5] Liu, T.-P. and Yu, S.-H., Green's function of Boltzmann eauation 3-D waves. Bull. Inst. Math. Acad. Sin. (N. S.) Volume no. 1 (2006), No. 1, pp. 1-78.
- [6] Liu, T.-P. and Wang, W., The pointwise estimates of diffusion waves for the Navier-Stokes systems on odd multi-dimensions. *Commun. Math. Phys.* Volume no. 196 (1998), No. 1, pp. 145-173.
- [7] Liu, T.-P. and Zeng, Y., Large time behavior of solutions for general quasilinear hyperbolic-parabolic systems of conservation laws. *Mem. Amer. Math. Soc.* 125 (1997), no. 599.

Joint work with: Tai-Ping Liu (Institute of Mathematics, Academia Sinica).

*

On hyperbolic equations describing longitudinal vibration of accreting rods

Michael Y. Shatalov

Department of Mathematics and Statistics, Tshwane University of Technology, Private Bag X680, Pretoria 0001, South Africa, and

Sensor Science and Technology (SST) of CSIR Material Science and Manufacturing, P.O. Box 395, Pretoria 0001. CSIR. South Africa

mshatlov@csir.co.za

Presented by: Igor Fedotov

In this paper we analyse longitudinal vibration of a thin rod which is fixed at the left end and free at the right end. It is assumed that the rod is growing at its right end, i.e. its length is increasing according to a special law and hence it is a known function of time. This problem is described it terms of the linear classical, Rayleigh-Love and Rayleigh-Bishop models. For solution of this problem we make a special change of variables which transforms the original equations into new non-autonomous equations. It is shown that these equations are hyperbolic and possess several interesting and important properties. First of all, the amplitudes of vibration of the rod are growing with time. For example, if the rod length is increasing proportionally to time the amplitudes are also growing proportionally to time. Secondly, if a particular mode is excited it excites other modes. In this case the mechanism of the modes excitation is asymmetric, which means that the low frequency modes possess higher amplitudes compared to the higher frequency modes. A physical explanation of the above mentioned phenomena is proposed and a simplified model describing these effects is analysed.

Joint work with: Igor A. Fedotov (Department of Mathematics and Statistics, Tshwane University of Technology, Private BagX680, Pretoria 0001, South Africa).

Delta-shocks in the Navier-Stockes system of granular hydrodynamics

V. M. Shelkovich

St.-Petersburg State Architecture and Civil Engineering University, Russia shelkv@yahoo.com

1. Strong singular solutions and physical models. It is well known that there are "nonclassical" situations where the Cauchy problem for a system of conservation laws admits δ -shocks, which are solutions whose components contain Dirac delta functions. In contrast to the classical shock wave discontinuities, δ -shocks carry mass, momentum and energy and are related with transport and concentration processes. In numerous papers, δ -shocks were studied in the zero-pressure gas dynamics. This system was used to describe the formation of large-scale structures of the universe, for modeling "dusty" media and double-fluid mixtures of gas and solid particles. Systems of conservation laws admitting δ -shocks were used for modeling the formation and evolution of traffic jams, in nonlinear chromatography, in the model of non-classical shallow water flows.

2. δ -Shocks in granular hydrodynamics. Nowadays problems related with granular gases are very attractive for experimental, numerical, and theoretical investigation (see [1], [2] and the references therein). So far there is no consensus on the description of these type of media. In contrast to ordinary gases, granular gases are dilute assemblies of hard spheres which lose energy at collisions. In such gases a *local density can significantly increase while a local pressure can fall drastically*. A description of these phenomena is provided by the Navier-Stockes granular hydrodynamics which is derivable, under certain assumptions, from the basic theory. In [5], [6] (see also [2; p.60-75]), the following hydrodynamics system of granular gas

$$\rho_t + \nabla \cdot (\rho U) = 0,$$

$$(\rho U)_t + \nabla \cdot (\rho U \otimes U + I \rho T) = 0,$$

$$T_t + \nabla \cdot (UT) + (\gamma - 2)T \nabla \cdot U = -\Lambda \rho T^{3/2},$$

(1)

was studied, where I is the identity matrix, \otimes is the tensor product of vectors, ρ is gas density, U is velocity, T is temperature, $p = \rho T$ is pressure; γ is the adiabatic index (if n = 2 then $\gamma = 2$, and if n = 3, then $\gamma = 5/3$), Λ is a constant connected with the energy of collision processes. As was proved in [5], [7], solutions of system (1) generically lose the initial smoothness within a finite time. Moreover (see [5], [6]), system (1) can admit a solution which contains δ -function in the density ρ : $\rho(x,t) = 2m_*(t)\delta(x) + \rho_*(x,t)$, and $m_*(t)$, $\rho_*(x,t)$ are smooth.

Here we shall consider some problems connected with δ -shocks in system (1). To deal with δ -shocks, we will use the *weak asymptotics method* developed in [3], [4] (see also [8]).

Let $\Gamma = \{(x,t) : S(x,t) = 0\}$ be a hypersurface of codimension 1 in $\{(x,t) : x \in \mathbb{R}^n, t \in [0,\infty)\} \subset \mathbb{R}^{n+1}$, $S \in C^{\infty}(\mathbb{R}^n \times [0,\infty))$, with $\nabla S(x,t)|_{S=0} \neq 0$ for any fixed t. Let $\Gamma_t = \{x \in \mathbb{R}^n : S(x,t) = 0\}$ be a moving surface in \mathbb{R}^n . Denote by $\nu = \frac{\nabla S}{|\nabla S|}$ the unit space normal to the surface Γ_t pointing from $\Omega_t^- = \{x \in \mathbb{R}^n : S(x,t) < 0\}$ to $\Omega_t^+ = \{x \in \mathbb{R}^n : S(x,t) > 0\}$. The time component of the normal vector $-G = \frac{S_t}{|\nabla S|}$ is the velocity of the wave front Γ_t along the space normal ν . For system (1) we consider the δ -shock type initial data

$$\begin{pmatrix} U^{0}(x), \rho^{0}(x), T^{0}(x), x \in \mathbb{R}^{n}; U^{0}_{\delta}(x), x \in \Gamma_{0} \end{pmatrix},$$
where $\rho^{0}(x) = \hat{\rho}^{0}(x) + e^{0}(x)\delta(\Gamma_{0}),$

$$(2)$$

and $U^0 \in L^{\infty}(\mathbb{R}^n; \mathbb{R}^n)$, $\hat{\rho}^0, T^0 \in L^{\infty}(\mathbb{R}^n; \mathbb{R})$, $e^0 \in C(\Gamma_0)$, $\Gamma_0 = \{x : S^0(x) = 0\}$ is the initial position of the δ -shock wave front, $U^0_{\delta}(x)$ is the *initial velocity* of the δ -shock, $\delta(\Gamma_0) (\equiv \delta(S^0))$ is the Dirac delta function on Γ_0 .

3. Rankine–Hugoniot conditions. First, basing on [8] we introduce the integral identities, which give a *definition of* δ -shock wave type solution of the Cauchy problem (1), (2). This solution is a triple of distributions (U, ρ, T) and a hypersurface Γ , where $\rho(x, t)$ is represented as a sum

$$\rho(x,t) = \widehat{\rho}(x,t) + e(x,t)\delta(\Gamma),$$

 $U \in L^{\infty}(\mathbb{R}^n \times (0,\infty);\mathbb{R}^n), \ \widehat{\rho}, T \in L^{\infty}(\mathbb{R}^n \times (0,\infty);\mathbb{R}), \ e \in C(\Gamma), \ \text{and} \ \delta(\Gamma) \ (\equiv \delta(S))$ is the Dirac delta function concentrated on the surface Γ . Next, using the above integral identities and repeating the proof of [8; Theorem 9.1] almost word for word, we derive the corresponding Rankine–Hugoniot conditions.

4. Mass, momentum, and energy transport laws. Assume that a moving δ -shock wave front $\Gamma_t = \{x : S(x,t) = 0\}$ permanently separates \mathbb{R}_x^n into two parts $\Omega_t^{\pm} = \{x \in \mathbb{R}^n : \pm S(x,t) > 0\}$. Let (U, ρ, T) be compactly supported with respect to x. Denote by $M(t) = \int_{\Omega_t^- \cup \Omega_t^+} \rho(x,t) \, dx, \, m(t) = \int_{\Gamma_t} e(x,t) \, d\Gamma_t$, and $P(t) = \int_{\Omega_t^- \cup \Omega_t^+} \rho(x,t) U(x,t) \, dx, \, p(t) = \int_{\Gamma_t} e(x,t) U_{\delta}(x,t) \, d\Gamma_t$, masses and momenta of the region $\Omega_t^- \cup \Omega_t^+$ and the moving δ -shock wave front Γ_t , respectively, where e is a density of the wave front Γ_t , $U_{\delta} = \nu G = -\frac{S_t \nabla S}{|\nabla S|^2}$ is the δ -shock wave velocity. Let $W_{kin}(t) = \int_{\Omega_t^- \cup \Omega_t^+} \rho(x,t) |U(x,t)|^2 / 2 \, dx, \, w_{kin}(t) = \int_{\Gamma_t} e(x,t) |U_{\delta}(x,t)|^2 / 2 \, d\Gamma_t$, be the kinetic energies of the region $\Omega_t^- \cup \Omega_t^+$ and the moving wave front Γ_t , respectively.

Using technique of the papers [9], [8], we prove the theorem with gives the mass, momentum and energy balance relations between the area outside of the moving δ -shock wave front and this front, i.e., we derive connections between quantities M(t) and m(t), P(t) and p(t), $W_{kin}(t)$ and $w_{kin}(t)$.

5. Propagation of a δ -shock wave. Let S^0 be a given smooth function. Denote by $\Omega_0^{\pm} = \{x \in \mathbb{R}^n : \pm S^0(x) > 0\}$ the domains on the one side and on the other side of the hypersurface $\Gamma_0 = \{x \in \mathbb{R}^n : S^0(x) = 0\}$. In order to study the propagation of a singular front Γ_t starting from the initial position Γ_0 , we need to solve the Cauchy problem for system (1) with the following initial data

$$(U^{0}, \rho^{0}, T^{0}, U^{0}_{\delta}), \text{ where } U^{0} = U^{0+} + [U^{0}]H(-\Gamma_{0}), \rho^{0} = \rho^{0+} + [\rho^{0}]H(-\Gamma_{0}) + e^{0}(x)\delta(\Gamma_{0}), T^{0} = T^{0+} + [T^{0}]H(-\Gamma_{0}),$$

$$(3)$$

where $U^{0-}(x) = U^{0+}(x) + [U^0(x)]$, $\rho^{0-}(x) = \rho^{0+}(x) + [\rho^0(x)]$, $T^{0-}(x) = T^{0+}(x) + [T^0(x)]$; e^0 , $\rho^{0\pm}$, $T^{0\pm}$ are given functions, $U^{0\pm}$ are given vectors; $H(-\Gamma_0) \ (\equiv H(-S^0))$ is the Heaviside function. Since in the direction ν the characteristic equation of system (1) has repeated eigenvalues $\lambda = U \cdot \nu$, we assume that for the initial data (2) the geometric entropy condition holds: $U^{0+}(x) \cdot \nu^0|_{\Gamma_0} < U^0_{\delta}(x) \cdot \nu^0|_{\Gamma_0} < U^{0-}(x) \cdot \nu^0|_{\Gamma_0}$, where $\nu^0 = \frac{\nabla S^0(x)}{|\nabla S^0(x)|}$ is the unit normal of Γ_0 , U^0_{δ} is the *initial velocity* of the δ -shock.

Using the weak asymptotics method we describe the propagation of δ -shock wave, i.e., we construct a solution of the Cauchy problem (1), (3).

References

- [1] N. V. Brilliantov, T. Pöschel, *Kinetic theory of granular gases*, Oxford University Press, (2004).
- [2] G. Capriz, P. Giovine, P.M. Mariano (Eds.), Mathematical Models of Granular Matter, Lecture Notes in Mathematics, Vol. 1937, Springer, (2008).
- [3] V. G. Danilov and V. M. Shelkovich, Delta-shock wave type solution of hyperbolic systems of conservation laws, Quarterly of Applied Mathematics, 63, no. 3 (2005), pp. 401-427
- [4] V. G. Danilov and V. M. Shelkovich, Dynamics of propagation and interaction of delta-shock waves in conservation law systems, *Journal of Differential Equations*, **211** (2005), pp. 333-381.
- [5] I. Fouxon, B. Meerson, M. Assaf, and E. Livne, Formation of density singularities in ideal hydrodynamics of freely cooling inelastic gases: A family of exact solutions, *Phys. Fluids*, **19**, 093303 (2007), (17 pages).
- [6] I. Fouxon, B. Meerson, M. Assaf, and E. Livne, Formation of density singularities in hydrodynamics of inelastic gases, *Phys. Review*, E **75**, 050301(R) (2007), (4 pages).
- [7] O.S. Rozanova, Formation of singularities in solutions to ideal hydrodynamics of freely cooling inelastic gases, Preprint arXiv:1107.0365v1 [math.AP] 2 Jul 2011, to appear on Nonlinearity, 25 (2012),
- [8] V.M. Shelkovich, δ- and δ'-shock types of singular solutions to systems of conservation laws and the transport and concentration processes, Uspekhi Mat. Nauk, 63:3 (2008), 73-146. English transl. in Russian Math. Surveys, 63:3 (2008), pp. 473-546.
- [9] V.M. Shelkovich, Transport of mass, momentum and energy in zero- pressure gas dynamics, *Proceedings of Symposia in Applied Mathematics 2009*; Volume: 67. Hyperbolic Problems: Theory, Numerics and Applications Edited by: E. Tadmor, Jian-Guo Liu, and A.E. Tzavaras, AMS, 2009. pp. 929-938.

Critical thresholds on pressure-less Navier-Stokes equations with nonlocal viscocity

* * *

Changhui Tan University of Maryland ctan@cscamm.umd.edu

The global existence of strong solutions for Navier-Stokes equations is one of the most challenged problems in the study of partial differential equations. In this talk, we discuss about the pressure-less compressible Navier-Stokes equations with regularized nonlocal viscosity

$$\begin{cases} \rho_t + \nabla \cdot (\rho \mathbf{u}) = 0, \\ \mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u} = \int_{\mathbb{R}^n} \phi(\mathbf{x} - \mathbf{y}) (\mathbf{u}(\mathbf{y}, t) - \mathbf{u}(\mathbf{x}, t)) \rho(\mathbf{y}, t) d\mathbf{y}, \end{cases}$$

where the kernel ϕ is bounded and Lipschitz. The system is related to the hydrodynamic discription of the Cucker-Smale flocking model. Taking advantage of the non-locality of the viscosity, we establish critical thresholds for the initial profiles, which guarantee existence of global strong solutions for the system.

References

 S. Engelberg, H. Liu and E. Tadmor, Critical threshold in Euler-Poisson equations, Indiana Univ. Math. J., 50 (2001), pp. 109-157

- [2] S.-Y. Ha and E. Tadmor, From particle to kinetic and hydrodynamic descriptions of flocking, *Kinetic and Related Models*, 1(3) (2008), pp. 415-435.
- [3] H. Liu and E. Tadmor, Critical thresholds in convolution model for nonlinear conservation laws, SIAM J. Math. Anal. Vol.33, no. 4, pp. 930-945.
- [4] S. Motsch and E. Tadmor, A new model for self-organized dynamics and its flocking behavior, J. Stat. Phys, 144(5) (2011), pp. 923-947.

Joint work with: Eitan Tadmor (University of Maryland).

Decay property for symmetric hyperbolic systems with non-symmetric relaxation

Yoshihiro Ueda Kobe University, JAPAN ueda@maritime.kobe-u.ac.jp

In this talk, we consider the Cauchy problem for the first-order linear symmetric hyperbolic system of equations with relaxation:

$$A^{0}u_{t} + \sum_{j=1}^{n} A^{j}u_{x_{j}} + Lu = 0$$
⁽¹⁾

with $u|_{t=0} = u_0$. Here $u = u(t, x) \in \mathbb{R}^m$ over t > 0, $x \in \mathbb{R}^n$ is an unknown function, $u_0 = u_0(x) \in \mathbb{R}^m$ over $x \in \mathbb{R}^n$ is a given function, and A^j $(j = 0, 1, \dots, n)$ and L are $m \times m$ real constant matrices, where integers $m \ge 1$, $n \ge 1$ denote dimensions. Throughout this talk, it is assumed that all A^j $(j = 0, 1, \dots, n)$ are symmetric, A^0 is positive definite and L is nonnegative definite with a nontrivial kernel. Notice that L is not necessarily symmetric. For this general linear degenerately dissipative system it is interesting to study its decay structure under additional conditions on the coefficient matrices and further investigate the corresponding time-decay property of solutions to the Cauchy problem.

When the degenerate relaxation matrix L is symmetric, Umeda-Kawashima-Shizuta [5] proved the large-time asymptotic stability of solutions for a class of equations of hyperbolic-parabolic type with applications to both electro-magneto-fluid dynamics and magnetohydrodynamics. The key idea in [5] and the later generalized work [2] that first introduced the so-called Kawashima-Shizuta condition is to design the compensating matrix to capture the dissipation of systems over the degenerate kernel space of L. The typical feature of the time-decay property of solutions established in those work is that the high frequency part decays exponentially while the low frequency part decays polynomially with the rate of the heat kernel.

Unfortunately, when the degenerate relaxation matrix L is not symmetric, the theorems derived in [2,5] can not be applied any longer. In fact, this is the case for some concrete systems, for example, the Timoshenko system [1] and the Euler-Maxwell system [3,4], where the linearized relaxation matrix L indeed has a nonzero skew-symmetric part while it was still proved that solutions decay in time in some different way. Therefore, our purpose of this talk is to formulate some new structural conditions in order to extend the previous works to the general system (1) when L is not symmetric, which can include both the Timoshenko system and the Euler-Maxwell system.

References

 K. Ide and S. Kawashima, Decay property of regularity-loss type and nonlinear effects for dissipative Timoshenko system, *Math. Models Meth. Appl. Sci.*, no.18 (2008), pp.1001–1025.

- [2] Y. Shizuta and S. Kawashima, Systems of equations of hyperbolic-parabolic type with applications to the discrete Boltzmann equation, *Hokkaido Math. J.*, no.14 (1985), pp.249–275.
- [3] Y. Ueda and S. Kawashima, Decay property of regularity-loss type for the Euler-Maxwell system, preprint (2011), to appear on *Methods and Applications of Analysis*.
- [4] Y. Ueda, S. Wang and S. Kawashima, Dissipative structure of the regularity-loss type and time asymptotic decay of solutions for the Euler-Maxwell system, preprint (2011), to appear on SIAM Journal of Mathematical Analysis.
- [5] T. Umeda, S. Kawashima and Y. Shizuta, On the devay of solutions to the linearized equations of electromagneto-fluid dynamics, Japan J. Appl. Math., no.1 (1984), pp.435–457.

Joint work with: Shuichi Kawashima (Kyushu University), Renjun Duan (Chinese University of Hong Kong).

Hyperbolic differential-operator equations with the time differentiation in boundary conditions

Yakuv Yakubov Tel-Aviv University, Israel yakubov@post.tau.ac.il

We give an abstract interpretation in Hilbert spaces of such initial boundary value problems for hyperbolic equations that a part of boundary conditions may contain the differentiation on the time of the same (second) order as the equation. The well-posedness of these abstract problems in appropriate abstract functional spaces is proved. Moreover, we expand the unique solution to the series of eigenvectors of the corresponding spectral problem. Then, we show an application of the abstract results to second order partial (hyperbolic) differential equations. The latter, in fact, is a generalization of the classical Fourier method of separation of variables to the case when the boundary conditions may contain the (second order) differentiation on the time.

13.3 Thursday, 14.00–16.15, Via Bassi Rooms, first floor

A finite volume approximation of a 2 Layer system for growth of sandpile based on schemes for discontinuous flux for hyperbolic conservation laws

Aekta Aggarwal TIFR Centre for Applicable Mathematics, Bangalore, India aekta@math.tifrbng.res.in

We propose an explicit finite volume numerical scheme for a system of partial differential equations proposed in [3], a model for growing sandpiles under a vertical source on a flat bounded table, based on schemes for discontinuous flux for hyperbolic conservation laws. In such a system, an eikonal equation for the standing layer of the pile is coupled to an advection equation for the rolling layer. The model in one dimension is given by

$$v_t - (vu_x)_x = -(1 - |u_x|)v + f, \text{on}[0, 1] \times (0, T)$$
(1)

$$u_t = (1 - |u_x|)v, \text{in}[0, 1] \times (0, T)$$
(2)

with initial condition

$$u(x,0) = 0 = v(x,0) \forall x \in (0,1)$$

and boundary condition

$$u(0,t) = u(1,t) = 0$$

where, f is a given poisitive source.

The idea here is to include the source term f in the form of an integral with the flux term, i.e. $-(vu_x)_x$ and use the idea of well balanced schemes proposed by Mishra [2]. Since the equation (1) is a first order pde with discontinuous coefficient $u_x(x,t)$ in space variable, we approximate (1) using the idea of discontinuous flux for hyperbolic conservation laws proposed by Gowda and Adimurth [1]. Our schemes are monotone and can be extended to higher dimensions. We prove some basic estimates about the physical properties of the model. We compare our scheme and the results of the numerical experiments established in 1 and 2 dimension with the finite difference schemes proposed by Falcone and Vita[4]. Our schemes work for larger CFL.

References

- Adimurthi, J. Jaffre and G.D.Veerappa Gowda, Godunov type methods for conservation laws with flux function discontinuous in the space variable, SIAM J. of Numerical Analysis 42 (2004), pp. 179-208
- [2] Kenneth Hvistendahl Karlsen, Siddhartha Mishra and Nils Henrik Risebro, Well-balanced schemes for conservation laws with source terms based on a local discontinuous flux formulation, *Math. Comp.* 78 (2009), pp. 55-78.
- [3] K.P. Hadeler and C. Kuttler, Dynamical models for granular matter, Granular Matter 2 (1999), pp. 9-18.
- [4] M. Falcone and S. Finzi Vita, A finite-difference approximation of a two-layers system for growing sandpiles, SIAM 2006.

Joint work with: Second Author G D V Gowda (Professor, TIFR Centre For Applicable Mathematics)

Comparison of discontinuous Galerkin and finite difference for NWP

Slavko Brdar University of Freiburg, Freiburg i. Br., Germany slavko@mathematik.uni-freiburg.de

We compare operationally used numerical weather prediction (NWP) model COSMO of the German Weather Service and the university code DUNE for solving benchmark test cases that traditionally appear in the NWP community. The focus is on the efficiency and effectiveness, analysing advantages and pitfalls of the both codes with respect to the chosen test suite. The test suite includes the density current [5], the inertia gravity waves [4] and the linear hydrostatic mountain waves [1]. The governing equations are Euler equations in 2d, to which we add enough viscosity in order to ensure the grid convergent numerical solution. On the one side, the DUNE code uses high order conservative discontinuous Galerkin (DG) method for Euler equations without physical viscosity and the CDG2 method, recently introduced in [2], for viscous Euler equations. The time integration for DUNE is fully explicit Runge-Kutta scheme up to the third order. The COSMO code, on the other side, uses finite differences of second order for fast wave and of fifth order for slow waves. The time integration is the split-explicit scheme according to Klemp and Wilhelmson (1978). For the mountain wave test case we need to treat transparent boundary conditions. Both codes use sponge layer technique with similar damping functions. While the optimally expected convergence rate of the other two test cases is attained, the convergence rate for this case was shown to reduce to the first order.

- [1] Bonaventura, L., A semi-implicit semi-Lagrangian scheme using the height coordinate for a nonhydrostatic and fully elastic model of atmospheric flows. J. Comput. Phys., **158** (2000), 186-213
- [2] Brdar, S. and Dedner, A. and Klöfkorn, R., Compact and stable discontinuous Galerkin methods for convection-diffusion problems, SIAM J. Sci. Comp., 34 (2012), 263–282
- [3] Brdar, S. and Dedner, A. and Klöfkorn, R. and Kränkel, M. and Kröner, D., Simulation of geophysical problems with DUNE-FEM, in *Computational Science and High Performance Computing IV* (2011), pp. 93-106
- [4] Skamarock, W. C. and Klemp, J. B., Efficiency and accuracy of the Klemp-Wilhelmson time-splitting technique, Mon. Wea. Rev., 122 (1994), pp. 2623-2630
- [5] Straka, J. M. and Wilhelmson, R. B. and Wicker, L. J. and Anderson, J. R. and Droegemeier, K. K., Numerical solutions of a non-linear density current: A benchmark solution and comparison, *Int. J. Num. Meth. Fluids*, **17** (1993), pp. 1-22

Joint work with: Michael Baldauf (DWD, Offenbach am Main, Germany), Andreas Dedner (WMI, Warwick, UK), Robert Klöfkorn (IANS, Stuttgart, Germany)

Contact algorithms for cell-centered Lagrangian schemes

Guillaume Clair *CEA-DIF* gclair.recherche@gmail.com

We describe fundamental numerical features of multidimensional Riemann solvers for interface problems. In our sense, an interface can be viewed as a specific mathematical constraint. We illustrate this point of view in the framework of compressible fluid dynamics, specifically using advanced lagrangian cell-centered schemes based on a nodal velocity solver. We propose a new formulation of traditional nodal velocity solvers in order to solve constrained problems which are evidenced near an interface.

An example is the 2D impact of a lagrangian compressible fluid on a wall. At the time of impact, the normal component to the wall of the fluid velocity must cancel at the interface fluid-wall, constraining the fluid to slide on the interface. In this example, two different contact constraints (impact + sliding) apply on nodes belonging to the face that impinges on the wall.

In their actual formulation, traditional nodal velocity solvers are not capable to solve such problem. Most of them are based on the solving of a linear system to compute nodal velocities, which is inadequate to take into account constraints. The new multidimensional formulation of the nodal solver is based on a global constrained minimization procedure. Such procedure enables to incorporate many kind of constraints in the calculation of the nodal velocities, particularly impact and sliding. 1D and 2D numerical tests illustrate the potentialities of this new formulation.

- B. Després and C. Mazeran, Symmetrization of Lagrangian gas dynamics and Lagrangian solvers, Comptes Rendus Académie des Sciences (2003)
- [2] G. Carré, S. Del Pino, B. Després and E. Labourasse, A cell-centered Lagrangian hydrodynamics scheme on general unstructured meshes in arbitrary dimension, J. Comp. Phys., 228 (2009) pp 5160-5183

[2] P.H. Maire and B. Nkonga, Multi-scale Godunov-type method for cell-centered discrete Lagrangian hydrodynamics, J. Comp. Phys., 228 (2009), p. 799-821.

Joint work with: Bruno Després (UPMC Laboratoire Jacques-Louis Lions), Emmanuel Labourasse (CEA-DIF)

1D hemodynamic simulations thanks to numerical methods for Shallow Water system

- * * * -

Olivier, Delestre Laboratoire J.A. Dieudonné & Polytech Nice-Sophia, University of Nice delestre@unice.fr

We are interested in blood flow simulation with variable elasticity arteries thanks to a one dimensional conservative model (mass and momentum conservations):

$$\begin{cases} \partial_t A + \partial_x Q = 0\\ \partial_t Q + \partial_x \left(\frac{Q^2}{A} + \frac{1}{3\sqrt{\pi\rho}} k A^{3/2}\right) = \frac{A}{\sqrt{\pi\rho}} \left(\partial_x \mathcal{A}_0 - \frac{2}{3}\sqrt{A}\partial_x k\right) - C_f \frac{Q}{A}, \end{cases}$$

where A(x,t) is the cross-section area $(A = \pi R^2 \text{ with } R \text{ the radius of the arteria})$, Q(x,t) = A(x,t)u(x,t) the discharge, u(t,x) the mean flow velocity, ρ the blood density, k(x) the stiffness of the artery and $\mathcal{A}_0 = k\sqrt{A_0}$ where $A_0(x)$ is the cross section at rest.

We present here a well-balance finite volume scheme based on recent developments in shallow water equations context. We thus get a mass conservative scheme which also preserves the man at "eternal rest equilibrium" (*i.e.* Q = 0). This numerical method is validated on analytical tests.

References

- E. Audusse, F. Bouchut, M.-O. Bristeau, R. Klein and B. Perthame, A fast and stable well-balanced scheme with hydrostatic reconstruction for shallow water flows, SIAM J. Sci. Comput., 25(6) (2004), pp. 2050-2065
- F. Bouchut, Nonlinear stability of finite volume methods for hyperbolic conservation laws, and well-balanced schemes for sources, Birkhauser Basel, (2004)

Joint work with: Pierre-Yves, Lagrée (CNRS, Institut Jean le Rond d'Alembert, UPMC), Xiaofei, Wang (Institut Jean le Rond d'Alembert, UPMC).

______ * * * _____

Numerical Solution of the Two-Dimensional Advection Equation on Unstructured Grids with Logarithmic Reconstruction

Katharina Elsen Leibniz-Institute for Tropospheric Research, Permoserstr. 15, 04318 Leipzig, Germany katharina.elsen@gmx.net

265

There are numerous approaches for solving hyperbolic differential equations in context of finite volume methods. One popular approach is the limiter free Local-Double-Logarithmic-Reconstruction (LDLR) of Artebrant and Schroll. The aim of this work is to construct a two-dimensional reconstructing function based on the LDLR for solving the advection equation on unstructured grids. The new method should preserve the characteristics of the LDLR. That means in particular a reconstruction without use of limiters and with a small stencil of only the nearest neighbors of a particular cell. Also local extrema should be conserved while the local variation of the reconstruction within one cell should be under control.

We propose an ansatz which works on unstructured polygonal grids. To come up to this, an ansatz function with one logarithmic expression for each edge of the polygon is constructed. Required gradients at cell edge midpoints are determined by use of the Multi-Point-Flux-Approximation (MPFA) method. Further derivative information are obtained with help of special barycentric coordinates. All necessary integrals of the ansatz functions can be computed exactly. The new advection procedure is numerically evaluated with standard test cases from the literature on different unstructured quadrilateral grids.

Joint work with: Oswald Knoth (Leibniz-Institute for Tropospheric Research, Permoserstr. 15, 04318 Leipzig, Germany)

An entropy-satisfying fast and slow waves splitting method for the Baer & Nunziato two-phase flow model

Jean-Marc Hérard EDF R&D, Département MFEE, FRANCE. jean-marc.herard@edf.fr

In the present work, we consider a PDE model formulated in Eulerian coordinates where balance equations account for the evolution of mass, momentum and energy of each phase. For compressible one-dimensional flows, there are seven unknowns that describe the evolution of the two-phase flow: the velocities of each phase u_k (where $k \in \{1, 2\}$), the phasic densities ρ_k , the total energy of each phase E_k and finally the phase fractions α_k (knowing that $\alpha_1 + \alpha_2 = 1$). The model, which was first introduced by Baer & Nunziato in [1], reads

$$\begin{cases} \partial_t \alpha_1 + u_2 \partial_x \alpha_1 = 0, \\ \partial_t (\alpha_1 \rho_1) + \partial_x (\alpha_1 \rho_1 u_1) = 0, \\ \partial_t (\alpha_1 \rho_1 u_1) + \partial_x (\alpha_1 \rho_1 u_1^2 + \alpha_1 p_1) - p_1 \partial_x \alpha_1 = 0, \\ \partial_t (\alpha_1 \rho_1 E_1) + \partial_x (\alpha_1 \rho_1 E_1 u_1 + \alpha_1 p_1 u_1) - p_1 u_2 \partial_x \alpha_1 = 0, \\ \partial_t (\alpha_2 \rho_2) + \partial_x (\alpha_2 \rho_2 u_2) = 0, \\ \partial_t (\alpha_2 \rho_2 u_2) + \partial_x (\alpha_2 \rho_2 u_2^2 + \alpha_2 p_2) - p_1 \partial_x \alpha_2 = 0, \\ \partial_t (\alpha_2 \rho_2 E_2) + \partial_x (\alpha_2 \rho_2 E_2 u_2 + \alpha_2 p_2 u_2) - p_1 u_2 \partial_x \alpha_2 = 0. \end{cases}$$
(1)

The phasic total energies are given by $E_k = e_k(\rho_k, p_k) + \frac{u_k^2}{2}$, $k \in \{1, 2\}$, where $e_k(\rho_k, p_k)$ is the internal energy of phase k, assuming some equation of state.

In this work, we propose a fractional step method for computing approximate solutions of the Baer-Nunziato two-phase flow model. The scheme relies on an operator splitting method corresponding to a separate treatment of fast propagation phenomena due to the acoustic waves on the one hand, and slow propagation phenomena due to the fluid motion on the other. For each step of the splitting method, we provide a very simple and robust numerical treatment.

In addition to the preservation of positive values of the statistical phase fractions and densities, the scheme is proved to satisfy a numerical entropy inequality. We also provide some test-cases that assess the convergence of the method.

 M.R. Baer and J.W. Nunziato, A two-phase mixture theory for the deflagration -to-detonation transition (ddt) in reactive granular materials, *International Journal of Multiphase Flow*, **12** (1986), pp. 861-889

Joint work with: Frédéric Coquel (Ecole Polytechnique, CMAP), and Khaled Saleh (UMPC, LJLL and EDF R&D).

The numerical determination of Bryan's factor for a non-thin cylindrical shell

* * * -

Stephan V. Joubert Tshwane University of Technology joubertsv@tut.ac.za

When a vibrating structure is rotated, the vibrating pattern within the structure rotates at a rate proportional to the rate of rotation of the structure. This effect, observed in 1890 by G.H. Bryan [1], is utilised in the vibratory gyroscopes that navigate space shuttles, submarines and commercial jetliners. In a recent articles [2] and [3], expressions were derived for calculating Bryan's factor and vibration frequency in terms of eigenfunctions. These eigenfunctions were analytically derived using Helmholtz potential functions in [2]. In this paper we numerically determine these eigenfunctions for the first few circumferential numbers as well as numerical values for Bryan's factor and the eigenfrequency of vibration of a not necessarily thin cylindrical shell. The numerical routine used here is more robust than "thin shell" theory. Despite this robustness, the routine is easy enough for senior undergraduate students to understand and implement. Analytical solutions to the hyperbolic problems that arise from generalisations of the classical model of vibrating non-thin rods (such as the Midlin-Herrmann model (see [4])) are rare. This routine provides approximate solutions to a number of these models.

References

- [1] Bryan, GH, On the beats in the vibrations of a revolving cylinder or bell, *Proceedings of the Cambridge Philosophical Society*, 7 (1890), pp. 101-111.
- [2] Shatalov, MY, Joubert, SV, Coetzee, CE & Fedotov IA, Free vibration of rotating hollow spheres containing acoustic media, *Journal of Sound and vibration*, **322.** (2009), doi:10.1016/j.jsv.2008.11.020, pp. 1038 - 1047.
- [3] Joubert, SV, Shatalov, MY and Fay, TH, Rotating structures and Bryan's effect, American Journal of Physics, 77 (6) (2009), DOI: 10.1119/1.30888 77, pp. 520 - 525,
- [4] Fedotov, IA, Shatalov, MY & Tenkam, MH, Hyperbolic problems in the theory of longitudinal vibrations of non-thin rods, submitted to *HYP2012* (2012).

Joint work with: Temple H Fay (Tshwane University of Technology and The University of Southern Mississippi) & Michael Y. Shatalov (The South African CSIR and Tshwane University of Technology)

Relaxing the CFL Number of the Discontinuous Galerkin Method

* * * -

Lilia Krivodonova University of Waterloo, CANADA lgk@math.uwaterloo.ca Discontinuous Galerkin methods (DGM) have a Courant-Friedrichs-Lewy (CFL) number decreasing with the increase of the order of approximation p for convection dominated problems. This makes them computationally more expensive when compared with finite volume or finite difference methods. We propose a modification of the scheme that results in a family of high order methods which have a less restrictive CFL number. We show that in the standard DG method the dispersion and dissipation errors and the spectrum of the semi-discrete scheme are related to the [p/p+1] Pade approximation of exp(z) and exp(-z). This Pade approximant is responsible for both the superconvergent error in diffusion and dispersion $(O(h^{2p+2}) \text{ and } O(h^{2p+3}))$, respectively) and the small CFL number. We propose to modify the DGM so that the resulting rational approximation of the exponent corresponds to a spatial discretization operator with a smaller spectrum, i.e. a less restrictive CFL number. This is achieved by scaling the amount of the numerical flux contribution to the equations evolving solution coefficients in time. For the considered orders of approximation, the improvement in the CFL number ranges between two and five fold depending on how much modification is brought into the scheme. The interesting aspect of the new schemes is that the (p+1)st rate of convergence in the L^2 norm as well as the compact stencil of the traditional DGM are preserved. We show that for the same amount of work the new schemes are more efficient for smooth problems and considerably more accurate for problems with discontinuities.

References

- [1] L. Krivodonova and R. Qin. An Analysis of the Spectrum of the Discontinuous Galerkin Method, preprint at http://www.math.uwaterloo.ca/ lgk/research.html.
- [2] N. Chalmers, L. Krivodonova, and R. Qin. Relaxing the CFL Condition of the Discontinuous Galerkin Method, preprint at http://www.math.uwaterloo.ca/ lgk/research.html

Joint work with: Noel Chalmers (U. of Waterloo) and Ruibin Qin (U. Of Waterloo)

IMplicit-EXplicit (IMEX) schemes for 10-Moment Plasma Equations

Harish Kumar BACCHUS, INRIA Bordeaux, France harish.kumar@inria.fr

10-moment plasma equations are a generalized form of the ideal MHD equations in which the electrons and ions are considered separately and modeled using 10-moment flow equations. In addition to discretization of flux terms, a major difficulty in the design of efficient numerical algorithms for these equations is the presence of stiff source terms, particularly for realistic charge to mass ratios. In this work, we design implicit-explicit (IMEX) Runge-Kutta (RK) time stepping schemes for these equations. The numerical flux is treated explicitly with strong stability preserving (SSP)-RK methods and the stiff source term is treated implicitly using implicit Runge-Kutta methods. The special structure of the equations enable us to split the source terms carefully and ensure that only local (in each cell) equations need to be solved at each time step. Furthermore the resulting algebraic system of equations is solved exactly. Benchmark numerical experiments are presented to illustrate the efficiency of this approach.

* * * ------

Shock waves in quasi-thermal-incompressible materials

Andrea Mentrelli Department of Mathematics & Research Centre of Applied Mathematics (CIRAM), University of Bologna (Italy) andrea.mentrelli@unibo.it

Incompressibility is a useful idealization for materials characterized by an extreme resistance to volume changes. For pure mechanical problems, i.e. where no change in temperature is involved, an incompressible material is easily understood as a material whose density is constant; in this case the solutions of model equations for incompressible fluids are obtainable as the limit case of the corresponding models involving compressible fluids [1].

For thermomechanical problems, i.e. when the processes are not isothermal, the definition itself of incompressibility is not straightforward and several models have been proposed. The first model of incompressibility was characterized by the independence of all the constitutive equations on the pressure [2], which leads to the conclusion – in strike contrast with experimental evidence – that the density must be constant [3]. A second, less restrictive, model requires that the only constitutive function independent of the pressure is the specific volume [4]. Such a definition of incompressibility allows to avoid the problems raised by Müller's definition but it is still not satisfactory as in this model instabilities affect wave propagation: since the chemical potential is not concave, the sound velocity might become imaginary, therefore losing the hyperbolicity of the system of Euler equations.

In order to solve these inconveniences, a new model of incompressibility has recently been proposed by Ruggeri & Gouin [5]. According to this model, a material is called *quasi-thermal-incompressible* (QTI) if the specific volume, V, and the specific internal energy, ϵ , differ to order δ^2 from functions depending only on the temperature T:

$$V(p,T) = V_0 + \delta W(T) - \delta^2 U(p,T), \qquad \epsilon(p,T) = e(T) - \delta T W'(T) + \mathcal{O}(\delta^2),$$

where δ is a small dimensionless parameter, V_0 is a constant, W(T), U(p,T) and e(T) are constitutive functions chosen in agreement with thermodynamics restrictions. It is remarkable that QTI materials are compressible fluids that approximate incompressible fluids to order δ^2 in the sense of Müller's definition.

The purpose of the present work is to analyze wave propagation, in particular shock waves, is QTI materials. The limit case with $\delta \rightarrow 0$ (corresponding to incompressible materials) is going to be investigated as well.

References

- P. L. Lions, N. Masmoudi, Incompressible limit for a viscous compressible fluid, Journal de Mathématiques Pures et Appliquées 77, 585–627 (1998)
- [2] I. Müller, Thermodynamics, Pitman, London (1985)
- [3] H. Gouin, A. Muracchini, T. Ruggeri, On the Müller paradox for thermal-incompressible media, *Contin-uum Mech. Thermodyn.*, DOI: 10.1007/s00161-011-0201-1 (2012), in press
- [4] K. R. Rajagopal, M. Ruzika, A. R. Srinivasa, On the Oberbeck-Boussinesq approximations, Mathematical models and methods in applied sciences 6, 1157–1167 (1996)
- H. Gouin, T. Ruggeri, A consistent thermodynamical model of incompressible media as limit case of quasithermal-incompressible materials, *Internat. J. Non-Linear Mech.*, DOI:10.1016/j.ijnonlinmec.2011.11.005 (2012), in press

Joint work with: Tommaso Ruggeri (Department of Mathematics & Research Centre of Applied Mathematics (CIRAM), University of Bologna).

* * * -

Numerical scheme for a viscous Shallow Water system including new friction laws of second order

Ulrich Razafison

Université de Franche-Comté, Laboratoire de Mathématiques de Besançon, CNRS UMR 6623 ulrich.razafison@univ-fcomte.fr

Consider a 1D viscous Shallow Water model

$$\begin{cases} \frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0, \\ \frac{\partial (hu)}{\partial t} + \frac{\partial (hu^2)}{\partial x} + \frac{g}{2} \frac{\partial h^2}{\partial x} = S_f - gh \frac{\partial Z}{\partial x} + 4\mu \frac{\partial}{\partial x} \left(h \frac{\partial u}{\partial x}\right) \end{cases}$$
(1)

where h is the flow depth, u the flow velocity, Z topography variations, g the gravity acceleration, μ the viscosity of the fluid.

In (1), the novelty lives in the friction term S_f . A new model of second order friction term based on Darcy-Weisbach's or Manning's formula is proposed. It can be written into the form

$$S_f = -\frac{kh^{-\alpha}|u|u}{(1+\frac{k}{3\mu}|u|h^{1-\alpha})^2}.$$
(2)

If $\alpha = 0$ or $\alpha = \frac{1}{3}$, then a Darcy-Weisbach type formula or a Manning type formula is obtained respectively. The derivation of (1)–(2) originating from the free surface Navier-Stokes equations follows the same lines as in [1] and [3]. The key point is to prescribe at the bottom, stresses with a Darcy-Weisbach's or a Manning's formula.

In order to solve numerically system (1), a scheme based on finite volume method for hyperbolic system of conservation laws with source terms is suggested.

Following the same lines of [2], analytic solutions for (1)-(2) are proposed. These solutions provide a numerical validation of the scheme.

References

- [1] J.-F Gerbeau and B. Perthame, Derivation of viscous Saint-Venant system for laminar shallow water; numerical validation, *Discrete Contin. Dyn. Syst. Ser.*, **1(1)** (2001), pp. 89-102.
- [2] I. MacDonald, Analysis and computation of steady open channel flow, PhD thesis, University of Reading, (1996).
- [3] F. Marche, Derivation of a new two-dimensional viscous shallow water model with varying topography, bottom friction and capillary effects, *Eur. J. Mech. B Fluids*, **26(1)** (2007), pp. 49-63.

Joint work with: Olivier Delestre (Université de Nice-Sophia Antipolis, Laboratoire de Mathématiques J. A. Dieudonné), Carine Lucas (Université d'Orléans, Laboratoire MAPMO, CNRS UMR 2964)

* * * ·

Posters

Existence of positive solutions for a system of multi-point boundary value problems with p-Laplacian operator

Robabeh Sahandi Torogh

Department of Mathematics, Varamin-Pishva Branch, Islamic Azad University, Varamin, Iran sahandi_1352@yahoo.com

The paper is concerned with a multi-point boundary value problem system. we prove the existence of many positive solutions for

$$\begin{cases} (\phi_p(u'))' + q_1(t)f(t, u, v) = 0 & t \in (0, 1) \\ (\phi_p(v'))' + q_2(t)g(t, u, v) = 0 & \\ \end{bmatrix} \begin{cases} u(0) = \sum_{i=1}^n \alpha_i u(\xi_i) &, u(1) = \sum_{i=1}^n \alpha_i u(\eta_i) \\ v(0) = \sum_{i=1}^n \beta_i v(\xi_i) &, v(1) = \sum_{i=1}^n \beta_i v(\eta_i) \end{cases}$$

By using the fixed-point theorem of cone, we provide sufficient conditions under which the above system has positive solution.

- Bo Sun, WeiGao Ge, Existence and iteration of positive solutions for some p-Laplacian boundary value problems, *Nonlinear Analysis* 67 (2007), pp. 1820-1830.
- [2] Dingyong Bai, Yuantong Xu, Positive solutions and eigenvalue regions of two-delay singular systems with a twin parameter, Nonlinear Analysis, 66 (2007), pp. 2547-2564.
- [3] D. Ji, W. Ge, Multiple positive solutions for some p-Laplacian boundary value problems, Applied Mathematics and Computation 187 (2007) pp.1315-1325.
- [4] Hanying Feng, Weigao Ge, Multiplicity of symmetric positive solutions for multipoint boundary value problem with one-dimensional p-Laplacian, preprint (2007), to appear on Nonlinear Analysis, doi:10.1016/j.na.2007.08.075.
- [5] Huihui Pang, Hairong Lian, Weigao Ge, Multiple positive solutions for second-order four-point boundary value problem, preprint (2007), to appear on *Computers and Mathematics with Applications*, doi:10.1016/j.camwa.2007.03.014.
- [6] K. Deimling, Nonlinear Functional Analysis, Springer, New York, (1985).



Figure 1: Evolution of density distribution for shock interacting with helium cylinder, left: experimental results of Haas & Sturtevant [2], right: current numerical results, at $t = 32\mu s$, $52\mu s$, $62\mu s$ and $72\mu s$.

Index

Abdelrahman, Mahmoud, 90 Abenda, Simonetta, 206 Abreu, Eduardo, 233 Achdou, Yves, 198 Achleitner, Franz, 228 Adami, Riccardo, 135 Aggarwal, Aekta, 261 Amorim, Paulo, 145 Andreianov, Boris, 181 Aregba-Driollet, Denise, 174 Arora, Rajan, 158 Asakura, Fumioki, 235 Ascanelli, Alessia, 203 Audiard, Corentin, 167 Audusse, Emmanuel, 127 Auphan, Thomas, 214 Bae, Hantaek, 229 Bagnerini, Patrizia, 196 Ballew, Joshua, 148 Bauzet, Caroline, 229 Beauchard, Karine, 35 Benaissa, Abbes, 243 Benzoni-Gavage, Sylvie, 116 Berres, Stefan, 181 Berselli, Luigi, 118 Berthon, Christophe, 163 Bianca, Carlo, 243 Bokanowski, Olivier, 239 Borsche, Raul, 50 Boscarino, Sebastiano, 203 Boudin, Laurent, 77 Boulanger, Anne-Celine, 197 Boutin, Benjamin, 156 Brdar, Slavko, 262 Bressan, Alberto, 62 Briani, Maya, 123 Buerger, Raimund, 134 Cances, Clement, 53 Canic, Suncica, 105 Caravenna, Laura, 194 Carvalho, Filipe, 187 Castaneda, Pablo, 132 Castro Díaz, Manuel J., 52, 147 Cavalli, Fausto, 150 Chalons, Christophe, 130 Charles, Frederique, 140 Chemetov, Nikolai, 148 Chen, Guoxian, 121 Chen, I-Kun, 141 Chen, Xiuqing, 221 Chertock, Alina, 162

Chesnokov, Alexander, 252 Chiodaroli, Elisabetta, 117 Choi, Young-Pil, 58 Clair, Guillaume, 263 Coclite, Giuseppe Maria, 143 Coco, Armando, 170 Colombo, Rinaldo, 106 Corli, Andrea, 215 Corti, Paolo, 153 Crippa, Gianluca, 43

De Lellis, Camillo, 15 Debrabant, Kristian, 240 Delestre, Olivier, 264 Delis, Argiris, 191 Delle Monache, Maria Laura, 177 Després, Bruno, 146 Desveaux, Vivien, 103 Di Francesco, Marco, 152 di Ruvo, Lorenzo, 86 Dinu, Liviu, 172 Donatelli, Donatella, 69 Duan, Renjun, 221 Dudzinski, Michael, 201 Duran, Arnaud, 67

Elsen, Katharina, 264 Ernest, Jan, 157

Facchi, Giancarlo, 244 Fay, Temple, 245 Fedotov, Igor, 253 Feireisl, Eduard, 21 Ferretti, Roberto, 237 Festa, Adriano, 241 Fjordholm, Ulrik, 120 Foucher, Francoise, 189 Freistuhler, Heinrich, 224 Frid, Hermano, 155 Frolkovic, Peter, 199 Furtado, Frederico, 232

Gaggero, Mauro, 246 Garavello, Mauro, 217 Gazibo, Mohamed, 60 Gerard-Varet, David, 29 Gersbacher, Christoph, 115 Ghoshal, Shyam, 62 Giesselmann, Jan, 173 Girardin, Mathieu, 78 Gisclon, Marguerite, 137 Glimm, James, 57 Godlewski, Edwige, 136 Grunert, Katrin, 204 Guermond, Jean-Luc, 40 Guerra, Graziano, 216 Gusev, Nikolay, 49 Ha, Seung-Yeal, 30 Hadzic, Mahir, 193 Han, Ee, 253 Hantke, Maren, 187 Hashimoto, Itsuko, 171 Haspot, Boris, 139 Helzel, Christiane, 65 Herard, Jean-Marc, 265 Herty, Michael, 37 Hiltebrand, Andreas, 51 Hoewing, Johannes, 189 Hong, Zhenying, 218 Jaehn, Michael, 247 Jakobsen, Espen, 102 James, Francois, 157 Javeed, Shumaila, 186 Jiang, Song, 20 Jiu, Quansen, 99 Joubert, Stephan, 266 Junca. Stephane, 59 Junxia, Wei, 73 Kabil, Bugra, 222 Kaeppeli, Roger, 200 Kalise, Dante, 82 Kanso, Mohamed, 95 Karni, Smadar, 114 Kasimov, Aslan, 126 Kawski, Matthias, 143 Kemm, Friedemann, 74 Kenettinkara, Sudarshan Kumar, 132 Khe, Alexander, 192 Kissling, Frederike, 209 Klaiber, Andreas, 254 Klingenberg, Christian, 151 Kmit, Irina, 154 Knoth, Oswald, 80 Kogan, Irina, 145 Koksal, Mehmet, 247 Koley, Ujjwal, 48 Koottungal, Arun, 47 Koren, Barry, 208 Kotschote, Matthias, 50 Kraenkel, Mirko, 85 Krieger, Joachim, 44 Krivodonova, Lilia, 266 Kroeker, Ilja, 55 Kroener, Dietmar, 114 Kumar, Harish, 267 Kurganov, Alexander, 211

Lattanzio, Corrado, 34 Lee Keyfitz, Barbara, 138 Lemoine, Grady, 149 Li. Fucai. 71 Li, Mingjie, 255 Lie, Ivar, 54 Linares, Felipe, 154 Liu, Jian-Guo, 36 Lu, Yong, 176 Lukacova, Maria, 81 Luo, Jun, 66 Maerz, Thomas, 247 Mailybaev, Alexei, 192 Marchesin, Dan. 83 Marica, Aurora, 248 Masmoudi, Nader, 17 Mathis, Helene, 129 Matos, Vitor, 85 McMurry, Andrew, 153 Mentrelli, Andrea, 267 Mikula, Karol, 190 Mishra, Siddhartha, 16 Mitrovic, Darko, 89 Mueller, Thomas, 80 Najdi, Nadine, 249 Nakamura, Tohru, 255 Nakane, Kazuaki, 212 Neves, Wladimir, 206 Nguyen, Khai, 183 Noelle, Sebastian, 234 Noh, Se-Eun, 256 Nolte, Martin, 242 Nussenzveig Lopes, Helena, 69 Ohnawa, Masashi, 164 Okutmustur, Baver, 92 Ortleb, Sigrun, 201 Panov, Evgeniv, 88 Pelanti, Marica, 104 Penel, Yohan, 72 Peralta, Gilbert, 250 Perepelitsa, Mikhail, 29 Perrollaz, Vincent, 113 Pfaff, Sebastian, 94 Pizzocchero, Livio, 119 Plaza, Ramon, 193 Popov, Bojan, 210 Priuli, Fabio, 94 Puppo, Gabriella, 220 Raynaud, Xavier, 42 Razafison, Ulrich, 269 Reintjes, Moritz, 42

Lagoutiere, Frederic, 205

Renac, Florent, 109 Rispoli, Vittorio, 169 Roberts, Joseph, 165 Rodnianski, Igor, 19 Rodriguez-Bermudez, Panters, 56 Roe, Philip Lawrence, 79, 97 Rosini, Massimiliano, 179 Rouch, Olivier, 107 Rozanova, Olga, 236 Russo, Giovanni, 24 Rybicki, Martin, 188 Sahandi, Robabeh, 269 Sainte-Marie, Jacques, 207 Saleh, Khaled, 107 Savaré, Giuseppe, 110 Schuetz, Jochen, 75 Secchi, Paolo, 40 Seguin, Nicolas, 230 Semplice, Matteo, 122 Serre, Denis, 224 Sethian, James, 24 Sfakianakis, Nikolaos, 100 Shang, Peipei, 93 Shao, Zhi-Qiang, 110 Shatalov, Michael, 256 Shearer, Michael, 30 Shelkovich, Vladimir, 257 Shkoller, Steve, 39 Silva, Julio, 84 Sone, Yoshio, 20 Spinolo, Laura, 183 Spirito, Stefano, 119 Staffilani, Gigliola, 31 Storrøsten, Erlend Briseid, 168 Strani, Marta, 127 Sukys, Jonas, 123 Tadmor, Eitan, 111, 161 Taetz, Bertram, 159 Takata, Shigeru, 38 Tan, Changhui, 259 Terracina, Andrea, 61 Tesdall, Allen, 227 Texier, Benjamin, 225 Thein, Ferdinand, 250 Tkachev, Dmitry, 226 Tomar, Amit, 251 Tonon, Daniela, 195 Trakhinin, Yuri, 91 Trebeschi, Paola, 99 Trivisa, Konstantina, 70 Turpault, Rodolphe, 98 Twarogowska, Monika, 115 Ueda, Yoshihiro, 260

Ulusoy, Suleyman, 184 Vasseur, Alexis, 27 Waagan, Knut, 74 Waechtler, Johannes, 232 Wang, Jinhuan, 185 Wang, Yi, 140 Wang, Zhiqiang, 112 Weber, Franziska, 87 Wu, Kung-Chien, 142 Wu, Sijue, 19 Xu, Kun, 33 Yakubov, Yakov, 261 Yee, Helen, 68 Yu, Shih-Hsien, 35 Zanelli, Lorenzo, 177 Zanotti, Olindo, 89 Zeiler, Christoph, 125 Zuazua, Enrique, 21